Medium Term Planning & Scheduling under Uncertainty for BP Chemicals

Progress Report

Murat Kurt
Mehmet C. Demirci
Gorkem Saka
Andrew Schaefer

University of Pittsburgh

Norman F. Jerome
Anastasia Vaia

BP Chemicals
Outline

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   • PTA
   • By Products
   • Applications

2) Models
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   • Schematic Comparison
   • Extension to Stochastic Models

3) Results & Future Research
Products - PX

- Paraxylene (PX): Colorless, flammable liquid that has a sweet odor.
- Separated from a mixed xylene stream that results from the refining of petroleum.
- Areas of use:
  - Feedstock for the local manufacture of Purified Terephthalic Acid (PTA)
  - Sellable to the customers.

Mixed Xylenes

PX

fed into the PTA units / sold
Products - PTA

- Purified Terephthalic Acid (PTA): an aromatic acid.
- Primarily applied in the production of polyester
- The main raw material for PTA → PX.

- Production Chains:
By Products

- Benzene:
  - Used elsewhere by other BP companies.
  - Used for production of styrene.
  - Styrene can be converted into polystyrene
  - Polystyrene can be used as an insulating material in the construction industry

- Fuel Additives:
  - Used as an additive for petrol production
Applications

- Users:
  - BP's business units
  - Customers of BP’s business units

- Usage: Products as chemical intermediates are used in the manufacture of other downstream chemicals.

- Chemical intermediates
  - PX
  - PTA
  - Mixed xylenes
  - Benzene
  - Metaxylene
  - Toluene

- PX → PTA
  - Polyester fibers
  - Films
  - PET-bottles.

Products & Applications
Background

- Existing deterministic model for planning medium term operations
  - Monthly production
  - Inventory targets

- What proportion of demand should be satisfied from which inventory location?

- Types of businesses:
  - PX
  - PTA

- Deterministic model represents:
  - Global production assets & distribution system for these businesses
Background

- Deterministic model does not consider future uncertainties.

- Uncertainty lies behind the future forecasts, how can it be dealt with?
  - Use Stochastic Programming

- Contributions:
  
  Model extension to cover the probabilistic nature of future economies.
  - 2 stage Stochastic Program
  - Multistage Stochastic Program
Overview of New Models

- Three models have been analyzed
- In all of the models:
  - 5 scenarios 5 different economic views
  - Operating policy for the first month as a whole constitutes the first-stage decision variables
- Model 1
  - 2 stage Stochastic Linear Program
  - No integrality restrictions
- Model 2
  - Extension of Model 1 with piecewise linear variables
- Model 3
  - Full model (All integrality restrictions)
  - 2 stage Stochastic MIP
## Schematic Comparison of Models

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# An Extension to Stochastic Models

- **FIRST STAGE DECISIONS**
  - **Initial**: Decisions corresponding to the operating policy for the first time period
    - *Time Periods: 1*
  - **Generic**: Decisions corresponding to the operating policy for the first *i* time periods
    - *Time Periods: 1,...,*i*

- **SECOND STAGE DECISIONS**
  - Decisions corresponding to the operating policy for the remaining time periods
    - *Time Periods: 2,3,...*
  - Decisions corresponding to the operating policy for the remaining time periods
    - *Time Periods: i+1,...*
Results

- Models were solved with data that was close to the actual data used by BP

- Performance measures for the stochastic solution (Full 2-stage SMIP Model):
  - Expected value of perfect information (EVPI) : the maximum amount a decision maker would be ready to pay in return for complete information about the future
    - EVPI = $120,386
  - Value of stochastic solution (VSS): the possible gain from solving the stochastic model
    - VSS = $1,450,842
Near-term Research

- **Current Research**: Building all the models as 3 stage stochastic program
- Limitation of solving the extensive form on
  - Number of scenarios
  - Number of stages
- Implementation of the L-shaped method for solving multi-stage stochastic programs
- Extension of the new model to Multistage SMIP using Lagrangian relaxation of nonanticipativity constraints
  - Many such constraints, even with few scenarios/stages
  - Duality gap
  - Finding an approximate solution
Solving SP’s with non-anticipativity

- Decomposable by scenario
- Non-anticipativity constraints are linking constraints
- Lagrangian relaxation of linking constraints
- Large scenario trees → enormous # of possible nonanticipativity constraints

Stage 1

Stage 2

Stage 3

Scenario 1

Scenario 2

Scenario 3

Scenario 4

Products & Applications

Models

Results & Future Research
Long-Term Research: Finding Bounds on Multistage SMIPs

- Birge (1982) provided a method for finding bounds on a 2-stage stochastic linear program

- Sandikci (2006) extended this to 2-stage SMIPs

- Gets much better bounds than SLP relaxation (which is how every stochastic branch-and-bound algorithm gets bounds)

- We believe it will extend to multistage stochastic SMIPs (such as the BP problem)
Finding Bounds on Multistage SMIPs

- Computing the expected value of the optimal objective values of (P_ξ) is called the wait-and-see (WS) solution, i.e.,

\[ WS = E_ξ [\min_x z(x, ξ)] \]

\[ = E_ξ [z(\bar{x}(ξ), ξ)]. \]

- The optimal objective value to the (RP) problem is called the here-and-now solution, i.e.,

\[ RP = \min_x E_ξ [z(x, ξ)]. \]

- Proposition 1 (Madansky, 1960): \( WS \leq RP. \)
Size-k Group Subproblem

Let $\xi$ be a discrete random vector with finite support $\Xi = \{\xi^0, \xi^1, \ldots, \xi^K\}$ and $P\{\xi = \xi^k\} = p_k$ for $k = 0, 1, \ldots, K$. Suppose scenario $\xi^0$ is the reference scenario. Given a set $\Gamma_k$ for any $k = 1, 2, \ldots, K$, then the size-$k$ group subproblem is defined as:

\[
GR(\Gamma_k) \quad \min \ z_{\{0, \Gamma_k\}} = c^T x + p_0 q^T \xi^0 y_{\xi^0} + (1 - p_0) \left( \sum_{i \in \Gamma_k} p_i \sum_{j \in \Gamma_k} p_j q^T \xi^i y_{\xi^i} \right)
\]

s.t. $Ax = b,$

\[
T_{\xi^0} x + W_{\xi^0} y_{\xi^0} = h_{\xi^0},
\]

\[
T_{\xi^i} x + W_{\xi^i} y_{\xi^i} = h_{\xi^i}, \quad i \in \Gamma_k,
\]

$x \in R_+^{n_1-k_1} \times Z_+^{k_1},$

$y_{\xi^0} \in R_+^{n_2-k_2} \times Z_+^{k_2},$

$y_{\xi^i} \in R_+^{n_2-k_2} \times Z_+^{k_2} \quad i \in \Gamma_k.$
Sum of Group Expected Values

**Proposition 2 (Birge, 1982):** Let $\xi$ be a discrete random variable with finite support whose cardinality is $K$. Then,

$$WS \leq SPEV \leq RP,$$

- Sum of group expected values with $k$ scenarios, $SGEV(k)$, is defined as:

$$SGEV(k) \equiv \frac{1}{C_k^K(1-p_0)} \left[ \sum_{\Gamma_k \in \mathcal{P}_k(S)} \sum_{i \in \Gamma_k} p_i z_{\{0, \Gamma_k\}}^* \right],$$

- where $z_{\{0, \Gamma_k\}}^*$ is the optimal objective value of the problem $GR(\Gamma_k)$. 
Bounds – Sandikci (2006)

Proposition 3: $WS \leq SPEV = SGEV(1)$

$\leq SGEV(2)$

$\vdots$

$\leq SGEV(K-1)$

$\leq SGEV(K) = RP.$

- **Computations:**
  - Three test problems from SIPLIB.
    - SIZES (Jorjani et al. 1995): 2 instance SIZES5 and SIZES10
    - DCAP (Ahmed and Garcia 2003): 9 instance dcap233_m, dcap243_m, dcap342_m, m = 200, 300, 500.
  - LP relaxation comparison: relaxing 2nd-stage, relaxing both stages.
How Well does this Work for 2 stages?

- We compared the strength of the relaxation for SLP relaxations and SGEV.
- For the SIZES and DCAP problems, the CPU times ranged from 0 seconds to 4-5 minutes.
- The SLP gaps ranged from 37% to 83%.
- The SGEV gaps ranged from 1.3% to 5.0%.
- If our experience in deterministic IP is any guide, this is a very significant improvement.
- We believe that in multistage SMIP the relative improvement will be larger.