Design of Supply Chains und the Risk of Facility Disruptions

EWO Meeting
March 13th, 2013
Garcia-Herreros, Wassick & Grossmann
Multiple commodities

Given:
• Reliable plant
• Candidate locations for DCs with risk of disruption
• Set of customer with deterministic demands for multiple commodities
• Set of scenarios and their associated probabilities

Minimize cost by:
• Selecting DCs among candidate locations
• Determining storage capacity for each commodity in selected DCs
• Allocating demands in every scenario
**Distribution Strategy**

Disruptions give rise to scenarios

DCs serve different customers in different scenarios

Rerouting produces a stochastic demand on DCs

![Diagram showing inventory levels and disruption scenarios](image-url)
Two-Stage Stochastic Programming

(Birge & Louveaux, 2011)

First stage decisions:
- Installation: $x_j \in \{0,1\}$
- Capacity: $c_{j,k} \in R^+$

Second stage decisions:
- Demand allocation: $y_{s,j,i,k} \in R^+$
- Unsatisfied demand: $w_{s,i,k} \in R^+$

Objective:
- Cost of first stage decisions
- Expected cost of second stage decisions
Basic Formulation

Minimize: Investment + Transportation to DCs + Transportation to customers + penalties

\[
\min \sum_{j \in J} \left[ f x_j + \sum_{k \in K} \nu c_{j,k} \right] + \chi \sum_{s \in S} \pi_s \sum_{k \in K} \left( \sum_{j \in J} \sum_{i \in I} a_{j,k} y_{s,j,i,k} + \sum_{i \in I} b_{j,i} D_{i,k} y_{s,j,i,k} \right) + p \sum_{i \in I} D_{i,k} w_{s,i,k}
\]

Subject to:

Demand satisfaction (\(\forall s \in S, \forall i \in I, \forall k \in K\)):

\[
\sum_{j \in DC} y_{s,j,i,k} + w_{s,i,k} = 1
\]

The inventory can be kept only at the installed DCs (\(\forall j \in DC, \forall k \in K\)):

\[
c_{j,k} - C_{\max}^x x_j \leq 0
\]

The demand satisfaction is restricted by the available inventory in every scenario (\(\forall s \in S, \forall j \in DC, \forall k \in K\)):

\[
\sum_{i \in I} D_{i,k} y_{s,j,i,k} - S_{t,s,j} c_{j,k} \leq 0
\]

Bounds:

\[
c_{j,k}, y_{s,j,i,k}, w_{s,i,k} \geq 0 ; \ x_j \in \{0,1\}
\]
Minimize: Investment + Transportation to DCs + Transportation to customers + penalties

\[
\begin{align*}
\min & \quad \sum_{j \in J} \left[ f x_j + \sum_{k \in K} \nu c_{j,k} \right] + \chi \sum_{s \in S} \pi_s \left( \sum_{j \in J} \left[ \sum_{l \in L} a_{j,l} y_{s,j,l,k} + \sum_{i \in I} b_{j,i} D_{i,k} y_{s,j,i,k} \right] + p \sum_{i \in I} D_{i,k} w_{s,i,k} \right)
\end{align*}
\]

Subject to:

Demand satisfaction (\( \forall s \in S, \forall i \in I, \forall k \in K \)):

\[
\sum_{j \in DC} y_{s,j,i,k} + w_{s,i,k} = 1
\]

The inventory can be kept only at the installed DCs (\( \forall j \in DC, \forall k \in K \)):

\[
c_{j,k} - C_{\max} x_j \leq 0
\]

The demand satisfaction is restricted by the available inventory in every scenario (\( \forall s \in S, \forall j \in DC, \forall k \in K \)):

\[
\sum_{i \in I} D_{i,k} y_{s,j,i,k} - S_{t,s,j} c_{s,k} \leq 0
\]

Tightening constraints (\( \forall s \in S, \forall j \in DC, \forall i \in I, \forall k \in K \)):

\[
y_{s,j,i,k} - x_j \leq 0
\]

Bounds:

\[
c_{j,k}, y_{s,j,i,k}, w_{s,i,k} \geq 0; \quad x_j \in \{0, 1\}
\]
Solution Strategy

Benders Decomposition

1. Initiate
2. Solve Benders master problem
   - Fix first-stage variables
   - Solve Benders subproblems
   - Determine design $(x_j, c_{j,k})$
3. Generate Benders cut based on subproblems multipliers
4. Determine lower bound (LB)
5. Determine distribution policy $(y_{S,j,k}, w_{S,k})$
6. Determine upper bound (UB)
7. Check $UB - LB < tol$
8. Terminate
Benders Decomposition

Benders Master problem

Adding one cut per commodity and scenario in every iteration

Minimize: Investment + Estimate of second-stage cost

$$\min \sum_{j \in J} \left[ f x_j + \sum_{k \in K} v c_{j,k} \right] + \sum_{s \in S} \sum_{k \in K} \theta_{s,k}$$

Subject to:

Benders cuts ($\forall \text{iter}, \forall k \in K, \forall s \in S$):

$$\theta_{s,k} \geq \sum_{i \in I} \lambda_{s,i,k}^{\text{iter}} - \sum_{j \in J} \mu_{s,j,k}^{\text{iter}} s_{t,s,j} c_{j,k} - \sum_{j \in J} \gamma_{s,j,i,k}^{\text{iter}} s_{t,s,j} x_j$$

The inventory can be kept only at the installed DCs ($\forall j \in DC$):

$$c_{j,k} - c_{\text{max}} x_j \leq 0$$

Bounds:

$$c_{j,k} \geq 0 ; x_j \in \{0,1\}$$
Benders Decomposition

Benders subproblem for scenario $s$

Fixing first-stage variables $x_j$ and $c_{j,k}$

Minimize: Transportation to DCs + Transportation to customers + penalties

$$
\min \chi \sum_{s \in S} \pi_s \left\{ \sum_{k \in K} \left( \sum_{j \in J} a_{j,k} y_{s,j,i,k} + \sum_{i \in I} b_{j,i,k} D_{i,k} y_{s,j,i,k} \right) + p \sum_{i \in I} D_{i,k} w_{s,i,k} \right\}
$$

Subject to:

Demand satisfaction ($\forall i \in I, \forall k \in K$):

$$
\sum_{j \in DC} y_{s,j,i,k} + w_{s,i,k} = 1
$$

The demand satisfaction is restricted by the available inventory in every scenario ($\forall j \in DC, \forall k \in K$):

$$
\sum_{i \in I} D_{i,k} y_{s,j,i,k} - St_{s,j} c_{j,k} \leq 0
$$

Tightening constraints ($\forall j \in DC, \forall i \in I, \forall k \in K$):

$$
y_{s,j,i,k} - x_j \leq 0
$$

Bounds:

$$
y_{s,j,i,k}, w_{s,i,k} \geq 0
$$
Supply chain network optimization:

- 1 Supplier
- 30 distribution centers with individual disruption probabilities between 0.5% and 1.5%
- 110 customers
- 61 commodities

<table>
<thead>
<tr>
<th>Number of simultaneous disruptions</th>
<th>Number of scenarios</th>
<th>Probability of all scenarios included</th>
<th>Number of constraints</th>
<th>Number of variables (binary)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>59.03%</td>
<td>3,820</td>
<td>10,113 (29)</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
<td>90.47%</td>
<td>63,154</td>
<td>251,103 (29)</td>
</tr>
<tr>
<td>2</td>
<td>436</td>
<td>98.50%</td>
<td>893,830</td>
<td>3,624,936 (29)</td>
</tr>
<tr>
<td>3</td>
<td>4090</td>
<td>99.83%</td>
<td>8,369,914</td>
<td>33,989,703 (29)</td>
</tr>
</tbody>
</table>
## Results

**Instance: 436 scenarios**

### Benders decomposition convergence

![Benders decomposition convergence graph](chart.png)

<table>
<thead>
<tr>
<th>Number of simultaneous disruptions</th>
<th>Number of selected DCs</th>
<th>Objective function (MM$)</th>
<th>Benders optimality gap</th>
<th>Upper bound in full problem (MM$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>117.347</td>
<td>0%</td>
<td>633.593</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>125.100</td>
<td>0%</td>
<td>244.445</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>127.485</td>
<td>0.82%</td>
<td>146.230</td>
</tr>
</tbody>
</table>
Conclusions

• Supply chain design for industrial purposes becomes a very large problem when the risk of disruptions is considered.

• The full problem is intractable. However, scenarios with several disruptions occurring simultaneously account for very small probabilities.

• Problems that consider only the most likely scenarios can give good designs with guaranteed bounds. However, they can still be very large.

• Decomposition strategies are necessary to solve the problem that yields a good supply chain design.

• Tightening constraints allow solving the Benders master problem with reasonable effort.