Inventory Optimization for
Industrial Network Flexibility

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EWO Meeting
March, 2014
Motivation

Process networks describe the operation of chemical plants

- Raw material storage tanks hedge against supply variability
- Intermediate storage tanks hedge against production rates variability
- Finished product inventories hedge against demand variability

Inventories are necessary because of process uncertainty:

Holding inventory is expensive!

Need to trade-off between inventory and stock-out cost
Given:
- A process network with continuous processing units
- Uncertain supply, processing rates, and demand with known probabilistic description
- Candidate locations for storage units

Find optimal trade-off between inventory cost and service level by:
- Determine at which stages of the process to hold inventory
- Establish rules to deplete and replenish inventories
Operating policies in discrete-event simulation:

1. Satisfy demands \( D \) according to priorities using available supply \( S \) and production capacities \( R \)
   - Update \( D, S, R_1, R_2, \) and \( R_3 \)
2. Satisfy demands \( D \) according to priorities using inventories \( I \)
   - Update \( D, S, R_1, R_2, R_3, I_1, \) and \( I_2 \)
3. Replenish inventories according to priorities using left over supply \( S \) and production capacities \( R \)
   - Update \( S, R_1, R_2, R_3, I_1, \) and \( I_2 \)
4. Stop inventory replenishments at base-stock levels \( B \)
5. Repeat for next time-period

Inventory decisions connect the state of consecutive time-periods
Modeling decisions without foreseeing the future

**Policy:** rules to determine decisions as functions of the state of the system
- Decisions rules do not change as the information reveals
- Optimal policy $\rightarrow$ Optimal decisions $\rightarrow$ Optimal objective value

Anticipativity issues in inventory management

Inventories might be accumulated anticipating adverse realizations of random parameters:
- Use raw material inventories to anticipate *price increases* that have not been revealed
- Make-to-stock products anticipating unexpected *increases in demand* that will be realized
Multi-period Formulation (MILP)

**Minimize:** expected inventory cost + expected lost sales cost

**Subject to:**

- Mass balances

- Capacity constraints

- Operating policies (logic)

- Bounds: Base-stock levels \((B_i \geq 0)\)
  Flows \((F_j \geq 0)\)
Challenges:

- Inventory levels depend on the history of random parameters
- Arbitrary distributions for uncertain parameters

Solution strategy: Sample-path optimization

Discrete-time samples of random parameters in a time-interval (0,T):

- Available supply: $S_t$
- Maximum processing rates: $R_t$
- Demand rates: $D_t$

For long time-horizons, the solution is a good estimate of the optimal solution

Minimum cost
Illustrative Example

Process network with failures (no storage):

(Straub & Grossmann, 1990)

Data

Supply: \( S \sim N(12,1) \) ton/day

Demand: \( D \sim N(7,1) \) ton/day

Probability of operation:

\[
\begin{align*}
\pi_1 &= 0.95 \\
\pi_2 &= 0.95 \\
\pi_3 &= 0.92 \\
\pi_4 &= 0.87
\end{align*}
\]

Mass balance coefficients:

\[
\begin{align*}
\alpha_1 &= 0.92 \\
\alpha_2 &= 0.90 \\
\alpha_3 &= 0.85 \\
\alpha_4 &= 0.75
\end{align*}
\]

Processing capacity:

\[
\begin{align*}
R_1 &= 5 \\
R_2 &= 5 \\
R_3 &= 7 \\
R_4 &= 9
\end{align*}
\]

Probability of satisfying demand

Expected stochastic flexibility: 0.81
**Illustrative Example**

**Process network with failures**

**Inventory rule:**
only replenish inventories when demand is fully satisfied

Cost coefficients:

<table>
<thead>
<tr>
<th>Production costs:</th>
<th>$\mu_1 = 0$</th>
<th>$\mu_2 = 0$</th>
<th>$\mu_3 = 0$</th>
<th>$\mu_4 = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inventory holding cost:</td>
<td>$h_1 = 5$ (ton-day)</td>
<td>$h_2 = 10$ (ton-day)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Penalty for unmet demand:</td>
<td>$p = $25$ / ton</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Optimal results** on 25,000 time-periods

<table>
<thead>
<tr>
<th></th>
<th>Base-stock 1</th>
<th>Base-stock 2</th>
<th>Exp. holding cost</th>
<th>Exp. penalty cost</th>
<th>Exp. total cost</th>
<th>Exp. Stoch. flexibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>No storage</td>
<td>0</td>
<td>0</td>
<td>$0$ / day</td>
<td>$7.25$ / day</td>
<td>$7.25$ / day</td>
<td>0.81</td>
</tr>
<tr>
<td>Storage</td>
<td>7.04</td>
<td>7.60</td>
<td>$2.81$ / day</td>
<td>$1.25$ / day</td>
<td>$4.06$ / day</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Expected flexibility **increases** from 0.81 to 0.97
Conclusions

Novelty:

- Specialized approach for inventory management in process networks
- Arbitrary distributions for the uncertain parameters
- Discrete-event simulation principles in an optimization framework
- Operating policies are modeled with logic constraints

Impact for industrial applications:

- Historical data can be used directly for inventory optimization
- Cost reductions can be achieved by implementing optimal strategies for inventory management in process networks
- Process resilience can be improved with the addition of intermediate storage units