Extended Resource Task Network Formulation for Reactive Scheduling of a Mixed Batch/Continuous Process

Spring 2014 EWO Meeting

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Introduction
Project motivation

- Industrial application of optimization-based scheduling approaches

Process Flowsheet

BU1 \rightarrow T1 \rightarrow T2 \rightarrow CU1 \rightarrow T3 \rightarrow CU2

BU2 \rightarrow BU1

Product Storage

Int. prod. F (to make A, B, E)
Prod. A to E

- Problem characteristics
  - Mixed equipment types: batch and continuous
  - Multi-purpose plant: network structure and multiple products
  - Operating rules:
    - Equipment cleaning for product transitions
    - Replenishment task of CU2
  - Re-optimization of disrupted schedules: rescheduling capability
Introduction

Problem statement

- **Objective**
  - Meet product orders and maintain product inventory levels

- **Decision-to-make**
  - Batch units: number of batches and their timing (fixed size)
  - Tanks: inlet and outlet flow rates
  - Continuous units
    - CU\(_1\): processing rate and transition
    - CU\(_2\): processing rate, transition, and maintenance

- **Desired performance**
  - Generic modeling method
  - Fast solution
  - Ease of re-optimization
Modeling Framework

Resource Task Network (RTN) formulation

- A process can be represented as interactions between resources and tasks
  - **Resources**: Process units, materials, utilities, etc.
  - **Tasks**: Process operations, such as reaction, separation, and storage

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**Core (discrete-time) RTN equations**

- **Excess resource balances**:
  \[
  R_{r,t} = R_{r,t-1} + \sum_{i} \sum_{\theta=0}^{\tau_i} \mu_{i,r,\theta} N_{i,t-\theta} + \sum_{i} \sum_{\theta=0}^{\tau_i} \nu_{i,r,\theta} \xi_{i,t-\theta} + \Pi_{r,t}
  \]

- **Excess resource limits**:
  \[
  R_{r,t}^{min} \leq R_{r,t} \leq R_{r,t}^{max}
  \]
Modeling Framework

RTN and its extensions

- Advantages of discrete-time RTN
  - Concise and consistent model structure
  - Tight mixed integer linear program (MILP)
- RTN extensions*
  - Multi-extent resource balance

\[ R_{r,t} = R_{r,t-1} + \sum_i \sum_{\theta=0}^{\tau_i} \mu_{i,r,\theta} N_{i,t-\theta} + \sum_i \sum_m \sum_{\theta=0}^{\tau_i} \nu_{i,m,r,\theta} \xi_{i,m,t-\theta} + \Pi_{r,t} \]

Resource interaction routes of synchronized on/off:
→ Inlet/outlet flow rates of a buffer tank

- Resource limit balance:

\[ R_{r,t}^{max} = R_{r,t-1}^{max} + \sum_i \sum_{\theta=0}^{\tau_i} \alpha_{i,r,\theta} N_{i,t-\theta} + \sum_i \sum_m \sum_{\theta=0}^{\tau_i} \beta_{i,m,r,\theta} \xi_{i,t-\theta} \]

Time-varying resource limits
→ Emptying a tank with variable inventory level

Rescheduling is essential in practice
  ▶ Model Predictive Control (MPC) based approach for scheduling?
  ▶ Iterative optimization coupled with receding time horizon

Convert RTN model to state space form

Lifting: additional state variables to record decision history*

\[ N_{i,t,\theta} \triangleq N_{i,t} - \theta, \] Task \( i \) at time \( t \) has been running for \( \theta \) periods

State evolution: \[ \bar{N}_{i,t,\theta} = \bar{N}_{i,t-1,\theta-1}, \quad \bar{N}_{i,t,0} = N_{i,t} \]

Rewrite RTN equations, e.g., resource balance

\[ R_{r,t} = R_{r,t-1} + \sum_{i} \tau_i \sum_{\theta=0}^{\tau_i} \mu_{i,r,\theta} N_{i,t-\theta} + \sum_{i} \sum_{m} \sum_{\theta=0}^{\tau_i} \nu_{i,m,r,\theta} \xi_{i,m,t-\theta} + \Pi_{r,t} \]

Modeling Framework

State space RTN model

- Rescheduling is essential in practice
  - Model Predictive Control (MPC) based approach for scheduling?
  - Iterative optimization coupled with receding time horizon

- Convert RTN model to state space form

- Lifting: additional state variables to record decision history*

\[
\bar{N}_{i,t,\theta} \triangleq N_{i,t-\theta}, \quad \text{Task } i \text{ at time } t \text{ has been running for } \theta \text{ periods}
\]

State evolution:
\[
\bar{N}_{i,t,\theta} = \bar{N}_{i,t-1,\theta-1}, \quad \bar{N}_{i,t,0} = N_{i,t}
\]

- Rewrite RTN equations, e.g., resource balance

\[
R_{r,t} = R_{r,t-1} + \sum_{i} \sum_{\theta=0}^{\tau_i} \mu_{i,r,\theta} \bar{N}_{i,t} + \sum_{i} \sum_{m} \sum_{\theta=0}^{\tau_i} \nu_{i,m,r,\theta} \xi_{i,m,t} + \Pi_{r,t}
\]

Objective function evaluates process economic performance in

1. Order delivery
2. Product inventory maintenance
3. Operation smoothness (avoid excessive ons/offs for continuous units)

\[
\min \phi = \sum_o \sum_r \sum_{t \leq D_o} (-c_{o,r,t}^{order})(-\Pi_{r,t}) + \sum_r c_r^{slack} s_{r,H}^{\min} + \sum_r \sum_t c_{r,t}^{invt} E_{r,t}
\]

Model constraints

1. State space RTN: resource/limit balance, task history evolution
2. Auxiliary constraints: safety stock, order fulfillment

Solution approach

- Mixed-integer program: GAMS/Gurobi
- Rolling horizon scheme
Case Study
Nominal schedule design

- Three day schedule design with 1-hour grid

<table>
<thead>
<tr>
<th>Product demand</th>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prod. A</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>Prod. B</td>
<td></td>
<td>70</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Prod. C</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>Prod. D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prod. E</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
</tbody>
</table>

Model solution and statistics

<table>
<thead>
<tr>
<th>Var(binary) #</th>
<th>Cons. #</th>
<th>Opt. gap</th>
<th>CPU [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>134, 148(4, 380)</td>
<td>140, 382</td>
<td>2.6%</td>
<td>600</td>
</tr>
</tbody>
</table>

Optimal plant schedule of Day 1-3

Processing rates of continuous units
Case Study
Reactive schedule design

- Reschedule at $t = 24$ hr with three scenarios:

  1. Optimal schedule without process disruptions
  2. Optimal schedule with task delay in $BU_1$
  3. Optimal schedule with $T_3$ maintenance

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Conclusions

- **Novelty of work**
  - Generic framework of wide applicability
    - Model customization minimized for different application complications (storage policy, disturbance source, etc.)
    - Hybrid plant: Nott and Lee 1999, Castro et al. 2004
    - Reactive scheduling: Mendez and Cerda 2004, Li and Ierapetritou 2008
  - State space scheduling model of the extended RTN
    - Extended RTN model: Wassick and Ferrio 2011
    - State space STN model: Subramanian et al. 2012

- **Benefits and scope of industrial application**
  - Enable responsive schedule design for economic optimization
  - Satisfactory solution performance with thousands of discrete variables
    - Weekly schedule design of medium size processes
    - 7 units and 6 products in the case study example

Thank you
I am glad to take your questions