Production Planning for Batch Operations

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Motivation and Introduction

✓ Goal: To develop production planning models
  ✓ Determine the available production capacity accurately
  ✓ Accounting for sequence-dependent changeovers

✓ Real world problem:
  ✓ Specialty Chemicals and Plastics business within Dow Chemical

✓ Business challenges
  ✓ Introduction of new products
  ✓ Cost pressures

✓ Flexibility increases the complexity in the planning process
✓ Accurate production capability offers significant competitive advantage
Problem Statement

Materials:
- Raw materials, Intermediates, Finished products
- Unit ratios (lbs of needed material per lb of material produced)

Production Site:
- Raw material availability and Raw material costs
- Storage tanks with associated capacity
- Reactors:
  - Materials it can produce
  - Batch sizes (lbs) for each material it can produce
  - Operating costs ($/hr) for each material
  - **Sequence dependent change-over times** (hrs per transition for each material pair)
  - Time the reactor is available during a given month (hrs)

Customers:
- Monthly forecasted demands for desired products
- Price paid for each product
Problem Statement

DETERMINE THE PRODUCTION PLAN:

- Production quantities
- Inventory levels
- Number of batches of each product
- Assignments of products to available processing equipment
- Sequence of production in each processing equipment

OBJECTIVE:

To Maximize Profit.
Profit = Sales – Costs
Costs = Operating Costs – Inventory Costs – Transition Costs
Proposed MILP Planning Models

**OPTION A. (Relaxed Planning Model-RP)**
Constraints that **underestimate the sequence-dependent** changeover times
=> **Weak** upper bounds

**OPTION B. (RP*)**
Simplification of Relaxed Planning Model
Tightening transition constraints are neglected

**OPTION C. (Detailed Planning Model-DP)**
**Sequencing constraints** for accounting for changeovers rigorously
=> **Tight** upper bounds

**OPTION D. (Relaxed Detailed Planning Model-DP*)**
Relaxation of DP
Number of batches are treated as continuous variables

**OPTION E. (Rolling Horizon Approach- RH)**
Forward rolling horizon algorithm
✓ Intermediate storage tank and the dedicated storage tank aggregated into a single tank
✓ Aggregate mass balances for the intermediates
RELAXED PLANNING MODEL (RP)

- Mass Balances on State Nodes
- Time Balance Constraints on Equipment
- Objective Function
Key Variables for the Model (RP)

$$YP_{i,m,t}$$ : the assignment of products to units at each time period

$$NB_{i,m,t}$$ : number of each batches of each product on each unit at each period

$$FP_{i,m,t}$$ : amount of material processed by each task

Products: A, B, C, D, E, F → Reactor 1 or Reactor 2 or Reactor 3

Reactor 1
- T1: A, A, A, B, D, B, E, B
- T2: F, B, F, B, B, C, D
- T3: F, B, F, E, C, D

Reactor 2
- T1: F, B
- T2: F, B, B, C, D
- T3: F, B, E, C, D

Reactor 3
- T1: F, B
- T2: F, E
- T3: C, D

$$YP_{A,\text{reactor 1, time } 1} = 1$$
$$NB_{A,\text{reactor 1, time } 1} = 3$$

$$YP_{B,\text{reactor 2, time } 2} = 1$$
$$NB_{B,\text{reactor 2, time } 2} = 2$$
Proposed Planning Model RP

Mass Balance and Assignment Constraints:

\[ \text{Bound}_{imt} = \frac{H_t}{BT_{im}} \cdot Q_{im} \]

- Maximum lbs of product i produced on unit m at time period t if product i is assigned throughout the time period
- Largest number that the task can be repeated

\[ \text{FP}_{imt} \leq \text{Bound}_{imt} \cdot \text{YP}_{imt} \]
- Bound on production levels of product i on unit m at time period t.

\[ \text{FP}_{imt} \geq Q_{im} \cdot \text{YP}_{imt} \]
- Sets production level to zero if product i is not assigned on unit m at time period t.

\[ \text{NB}_{i,m,t} = \frac{\text{FP}_{i,m,t}}{Q_{i,m}} \]
- Number of batches of product i in unit m at time t \((\text{INTEGER})\)

Mass balance on each state node:

\[ P_{jt} + \sum_{i \in PS_j} \rho_{ji} \sum_{m \in MT_i} \text{FP}_{imt} = S_{jt} + \sum_{i \in CS_j} \bar{\rho}_{ji} \sum_{m \in MT_i} \text{FP}_{imt} + \text{INV}_{jt} - \text{INV}_{jt-1} \]

- Purchases
- Production
- Sales
- Consumption
- Change in inventory
Transitions and Time Balance Constraints for RP

- Lower bounds for changeovers
- Sequencing constraints are neglected
- Introduce a minimum transition time for each assigned product

Parameter:

\[ TR_i = \min \{ \tau_{i,i'} \} \]

<table>
<thead>
<tr>
<th></th>
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<th>B</th>
<th>C</th>
</tr>
</thead>
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<td>5</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \sum_i N_B_{i,m,t} \cdot BT_{i,m} + \sum_i TR_i \cdot YP_{i,m,t} \leq H_t \quad \forall m, t \]

\[ \sum_i N_B_{i,m,t} \cdot BT_{i,m} + \sum_i TR_i \cdot YP_{i,m,t} - U_{m,t} \leq H_t \quad \forall m, t \]

- Over estimation of changeovers
- Can not model transitions across adjacent periods

\[ \sum_i N_B_{i,m,t} \cdot BT_{i,m} + \sum_i TR_i \cdot YP_{i,m,t} - U_{m,t} \leq H_t \quad \forall m, t \]
Transitions and Time Balance Constraints for RP

where \( U_{m,t} \):
\[
U_{m,t} \geq TR_{i,m} \cdot YP_{i,m,t} \quad \forall i \in I, m, t
\]
\[
U_{m,t} \leq \text{Max}_{i \in I} \left\{ TR_{i,m} \right\} \quad \forall m, t
\]
\[
U_{m,t} \leq \sum_{i \in I} TR_{i} \cdot YP_{i,m,t} \quad \forall m, t (**)
\]

(*) Redundant, tightens the formulation, neglecting may result in overestimation of the available production time.

**Transition Costs:**

Transition Cost = \[ \sum_{t} \sum_{m} \left( \sum_{i} TRC_{i} \cdot YP_{i,m,t} \right) - UT_{m,t} \]

where:
\[
TRC_{i} = \text{Min}_{i \in I} \left\{ C_{\text{trans},i} \right\}
\]
\[
UT_{m,t} \geq TRC_{i,m} \cdot YP_{i,m,t} \quad \forall i \in I, m, t
\]
\[
UT_{m,t} \leq \text{Max}_{i \in I} \left\{ TRC_{i,m} \right\} \quad \forall m, t
\]
\[
UT_{m,t} \leq \sum_{i \in I} (TRC_{i,m} \cdot YP_{i,m,t}) \quad \forall m, t (**)
\]

(**) Redundant, tightens the formulation, neglecting may result in underestimation of the transition cost.

**Maximize PROFIT:**

\[
Z^p = \sum_{j} \sum_{t} cp_{jt} \cdot S_{jt} - \sum_{j} \sum_{t} c_{jt}^{\text{inv}} \cdot INV_{jt} - \sum_{i} \sum_{m} \sum_{t} c_{it}^{\text{oper}} \cdot FP_{imt} - \sum_{t} \sum_{m} (\sum_{i} TRC_{i} \cdot YP_{i,m,t}) + UT_{m,t}
\]

Sales \hspace{1cm} Inventory costs \hspace{1cm} Variable operating costs \hspace{1cm} Transition costs
PLANNING MODEL (RP*)

- RP* same as RP but constraints (*) and (**) are neglected.
- Provides a valid but a weaker upper bound compared to RP.
- It can lead to overestimation of the available production time and underestimation of transition costs.
Generic Form of the Detailed Planning Model (DP)- Option C

DETAILED PLANNING MODEL (DP)

✓ Mass Balances on State Nodes

✓ Sequencing Constraints
  ✓ Sequence dependent changeovers determined
  ✓ Detailed timings of operations neglected

✓ Time Balance Constraints on Equipment

✓ Objective Function
Sequencing Constraints for DP

Sequence dependent changeovers:

✓ Sequence dependent changeovers within each time period:

1. Generate a cyclic schedule where total transition time is minimized.

KEY VARIABLE:

\[ ZP_{ii^{'mt}} \]: becomes 1 if product i is after product i’ on unit m at time period t, zero otherwise

\[ ZP_{P1, P2, M, T} = 1 \]

\[ ZP_{P2, P3, M, T} = 1 \]

2. Break the cycle at the pair with the maximum transition time to obtain the sequence.

KEY VARIABLE:

\[ Z ZP_{ii^{'mt}} \]: becomes 1 if the link between products i and i’ is to be broken, zero otherwise

\[ Z ZP_{P4, P3, M, T} \]
Changeovers within each period for DP

According to the location of the link to be broken:

- P2, P3, P4, P5, P1 → ZZP_{P1, P2, M, T} = 1
- P3, P4, P5, P1, P2 → ZZP_{P2, P3, M, T} = 1
- P4, P5, P1, P2, P3 → ZZP_{P3, P4, M, T} = 1
- P5, P1, P2, P3, P4 → ZZP_{P4, P5, M, T} = 1
- P1, P2, P3, P4, P5 → ZZP_{P5, P1, M, T} = 1

The sequence with the minimum total transition time is the optimal sequence within time period t.

\[ YP_{int} = \sum_{i^i} ZP_{ii^i mt} \quad \forall i, m, t \]
\[ YP_{i mt} = \sum_{i'} \sum_{i^i} ZP_{ii' mt} \quad \forall i', m, t \]
\[ YP_{int} \land \left[ \bigwedge_{i, m} \neg YP_{i mt} \right] \iff ZP_{int} \quad \forall i, m, t \]
\[ YP_{int} \geq ZP_{i, i, m, t} \quad \forall i, m, t \]
\[ ZP_{i, i, m, t} \leq YP_{i, i, m, t} \quad \forall i, i', m, t \]
\[ ZP_{i, i, m, t} \geq YP_{i, i, m, t} - \sum_{i'} YP_{i, i', m, t} \quad \forall i, m, t \]
\[ \sum_{i} \sum_{i'} ZZP_{ii' mt} = 1 \quad \forall m, t \]
\[ ZZP_{ii' mt} \leq ZP_{ii' mt} \quad \forall i, i', m, t \]

Generate the cycle and break the cycle to find the optimum sequence where transition times are minimized.

Having determined the sequence, we can determine the total transition time within each week.
Changeovers within each period for DP

1) generate the cycle

2) break the cycle to obtain the sequence

\[ \tau_{P4, P5} \quad \tau_{P5, P1} \quad \tau_{P1, P2} \quad \tau_{P2, P3} \]

\[ \tau_{P3, P4} \]

\[ TRNP_{m,t} = \tau_{P4, P5} + \tau_{P5, P1} + \tau_{P1, P2} + \tau_{P2, P3} + \tau_{P3, P4} - \tau_{P3, P4} \]

Total transition time within period \( t \) on unit \( m \)

\[ TRNP_{m,t} = \sum_i \sum_{i'} \tau_{i, i'} \cdot ZP_{i, i', m, t} - \sum_i \sum_{i'} \tau_{i, i'} \cdot ZZP_{i, i', m, t} \quad \forall m, t \]
Sequence dependent changeovers across adjacent time periods:

Transitions across adjacent weeks:

$$\text{ZZZ}_{i,i',m,t} \geq XL_{i,m,t} + XF_{i',m,t+1} - 1 \quad \forall i, i', m, t$$
The time balance constraint:

\[ \sum_{i} NB_{i,m,t} \cdot BT_{i,m} + TRNP_{m,t} + \sum_{i} \sum_{i'} ZZZ_{i,i',m,t} \cdot \tau_{ii'} \leq H_{t} \quad \forall m, t \]

- Summation of batch times of assigned products to unit m at period t.
- Total transition time within time period t.
- Transition time between period t and period t+1.
- Total available time for unit m.

Objective Function:

Maximize PROFIT:

\[ Z^{p} = \sum_{j} \sum_{t} c_{pj,t} \cdot S_{jt} - \sum_{j} \sum_{t} c_{inv}^{j} \cdot INV_{jt} - \sum_{i} \sum_{m} \sum_{t} c_{i,m,t}^{\text{oper}} \cdot FP_{i,m,t} - \sum_{i} \sum_{i'} \sum_{m} \sum_{t} c_{i,i',m,t}^{\text{trans}} \cdot ZP_{i,i',m,t} + \sum_{i} \sum_{i'} \sum_{m} \sum_{t} c_{i,i',m,t}^{\text{trans}} \cdot ZZP_{i,i',m,t} - \sum_{i} \sum_{i'} \sum_{m} \sum_{t} c_{i,i',m,t}^{\text{trans}} \cdot ZZZ_{i,i',m,t} \]

- Sales
- Inventory costs
- Variable operating costs
- Transition costs
Generic Form of Model (DP*) - Option D

RELAXED DETAILED PLANNING MODEL (DP*)

✓ DP* same as DP but *number of batches* are treated as continuous variables.

✓ Provides a valid upper bound on the profit.

✓ Significantly reduces the computational expense.
The proposed planning models may still be too expensive to solve.
Sequence of sub-problems that are solved recursively.
Provides a lower bound on the profit.

The detailed planning period (DP) moves as the model is solved in time.
Future planning periods include only underestimations for transition times (RP*).
In each iteration we fix the binary variables for assignment and sequencing variables.
Relaxed Planning Model (RP) is adequate if:

- Demand rates are low
- Transition times show low variance

*RP could lead to significant overestimation of the available production capacity if the above conditions are not true.*

Detailed Planning Model (DP) very powerful:

- Since the sequencing constraints are explicitly accounted for, it yields very tight upper bounds.
- In the absence of subcycles and for single stage production, it produces the identical solution as a detailed scheduling model would.
- Among all the instances we have solved so far, only one exhibited a subcycle.

*Trade-off between the extent of scheduling decisions incorporated and the size and the computational effort of the resulting problem.*
Examples
EXAMPLE 1 - 8 Products, 3 Reactors

✓ Determine the plan for 8 products, 3 reactors plant so as to maximize profit.

•8 Products, A,B,C,D,E,F,G,H
•All produced in a single stage.
•3 Reactors, R1,R2,R3
•End time of the week is defined as due dates
•Demands are lower bounds
For a planning horizon of 6 weeks:

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of Binary Variables</th>
<th>Number of Continuous Variables</th>
<th>Number of Equations</th>
<th>Time (CPU s)</th>
<th>Solution ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detailed Planning (DP)</td>
<td>864</td>
<td>1,327</td>
<td>1,483</td>
<td>1,667</td>
<td>11,819</td>
</tr>
<tr>
<td>Detailed Planning (DP*)</td>
<td>792</td>
<td>1,327</td>
<td>1,483</td>
<td>96</td>
<td>12,211</td>
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<tr>
<td>Relaxed Planning (RP)</td>
<td>144</td>
<td>349</td>
<td>553</td>
<td>1,75</td>
<td>13,460</td>
</tr>
<tr>
<td>Rolling Horizon (RH)</td>
<td>624</td>
<td>1,291</td>
<td>1,483</td>
<td>322</td>
<td>11,377</td>
</tr>
</tbody>
</table>

Schedule and Number of Batches obtained by DP

**Exact schedule!**

Schedule and Number of Batches obtained by DP*
EXAMPLE 1 - 8 Products, 3 Reactors

Comparison of Models for Planning Horizons of 6 to 24 Weeks for 5% Optimality Tolerance
EXAMPLE 2 - 15 Products, 6 Reactors, 48 Weeks

- Determine the **plan** for 15 products, 6 reactors plant so as to maximize **profit**.

- B, G and N are produced in 2 stages.
- 6 Reactors, R1, R2, R3, R4, R5, R6
- End time of the week is defined as due dates
- Demands are lower bounds
EXAMPLE 2 - 15 Products, 6 Reactors, 48 Weeks

For a planning horizon of 48 weeks for 6% optimality tolerance:

<table>
<thead>
<tr>
<th>method</th>
<th>number of binary variables</th>
<th>number of continuous variables</th>
<th>number of equations</th>
<th>time (CPU s)</th>
<th>solution ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>relaxed planning (RP)</td>
<td>2,592</td>
<td>5,905</td>
<td>9,361</td>
<td>362</td>
<td>224,731,683</td>
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<tr>
<td>rolling horizon (RH)</td>
<td>10,092</td>
<td>25,798</td>
<td>28,171</td>
<td>11,656</td>
<td>184,765,965</td>
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<tr>
<td>rolling horizon (RH**)</td>
<td>1,950</td>
<td>25,798</td>
<td>28,171</td>
<td>4,554</td>
<td>182,169,267</td>
</tr>
</tbody>
</table>

Variation of results from 6 to 48 weeks for 6% optimality tolerance:

![Graph showing variation of results from 6 to 48 weeks for 6% optimality tolerance](image1)

![Graph showing variation of results from 6 to 48 weeks for 6% optimality tolerance](image2)
Concluding Remarks and Summary

✔ Relaxed Planning Model (RP):
  ✔ Underestimates sequence-dependent changeover times and costs.
  ✔ Overestimates sales and profit.

✔ Detailed Planning Model (DP):
  ✔ Explicitly accounts for scheduling via sequencing variables and constraints.
  ✔ Very accurate production plans

✔ DP yields more realistic plans compared to RP but at the expense of increasing size and computational effort.

✔ For large problems and long time horizons without giving up the solution quality:
  ✔ Rolling Horizon Algorithm (RH): yields a lower bound on profit
  ✔ Relaxed Detailed Planning Model (DP*): yields an upper bound on profit