Risk Management for Chemical Supply Chain Planning under Uncertainty

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Chemical Supply chain: an integrated network of business units for the supply, production, distribution and consumption of the products.
Motivation

• Chemical Supply Chain Planning
  - Costs **billions** of dollars annually
  - Always under various uncertainties and risks

• Objective: managing the risks in supply chain planning
Case Study

• Given
  • Minimum and initial inventory
  • Inventory holding cost and throughput cost
  • Transport times of all the transport links & modes
  • Uncertain customer demands and transport cost

• Determine
  • Transport amount, inventory and production levels

• Objective: Minimize Cost & Risks
Stochastic Programming

- **Scenario Planning**
  - A scenario is a *future possible outcome* of the uncertainty
  - Find a solution perform well for all the scenarios
- **Two-stage Decisions**
  - Here-and-now: Decisions ($x$) are taken **before** uncertainty $\omega$ resolute
  - Wait-and-see: Decisions ($y_\omega$) are taken **after** uncertainty $\omega$ resolute as “corrective action” - recourse
Stochastic Programming for Case Study

- **First stage decisions**
  - Here-and-now: decisions for the first month (production, inventory, shipping)

- **Second stage decisions**
  - Wait-and-see: decisions for the remaining 11 months

\[
\text{Minimize } E \left[ \text{cost} \right]
\]

Decision-making under Uncertainty
Objective Function

\[ E[Cost] = Cost_1 + \sum_s P_s \cdot Cost_{2s} \]

First stage cost

Cost_1 =

\[ \sum_k \sum_j \sum_t h_{k,j} I_{k,j,t} \]

Probability of each scenario

Inventory Costs

\[ \sum_k \sum_j \sum_t \sum_k' \sum_j' \sum_t \gamma_{k,k',j} F_{k,k',j,t} \]

Freight Costs

\[ \sum_k \sum_l \sum_j \sum_t \sum_l' \sum_j' \sum_t \gamma_{k,l,j} S_{k,l,j,t} \]

Throughput Costs

\[ \sum_k \sum_j \sum_t \sum_k' \sum_j' \sum_t \delta_{k,j} F_{k,k',j,t} \]

Demand Unsatisfied

\[ \sum_k \sum_l \sum_j \sum_t \sum_l' \sum_j' \sum_t \delta_{k,j} S_{k,l,j,t} \]

Second stage cost

Cost_{2s} =

\[ \sum_k \sum_j \sum_t h_{k,j} I_{k,j,t,s} \]

\[ + \sum_k \sum_k' \sum_j \sum_t \sum_j' \sum_t \gamma_{k,k',j,s} F_{k,k',j,t,s} \]

\[ + \sum_k \sum_l \sum_j \sum_t \sum_j' \sum_t \gamma_{k,l,j,s} S_{k,l,j,t,s} \]

\[ + \sum_k \sum_j \sum_t \sum_k' \sum_j' \sum_t \delta_{k,j,s} F_{k,k',j,t,s} \]

\[ + \sum_k \sum_l \sum_j \sum_t \sum_j' \sum_t \delta_{k,j,s} S_{k,l,j,t,s} \]

\[ + \sum_l \sum_j \sum_t \sum_j' \sum_t \eta_{l,j} S_{F_{l,j,t,s}} \]
Multiperiod Planning Model (Case Study)

- **Objective Function:**
  - Min: **Total Expected Cost**

- **Constraints:**
  - Mass balance for **plants**
  - Mass balance for **DCs**
  - Mass balance for **customers**
  - Minimum **inventory** level constraint
  - **Capacity** constraints for plants
Result of Two-stage SP Model

\[ E[\text{Cost}] = $182.32\text{MM} \]
Risk Management

• SP model: optimize expected cost (risk-neutral objective)
  ◆ Could not control variance, extreme values, etc.
  ◆ The following distributions have the same $E[\text{Cost}]=4.1$

• Use risk measures to control the possible outcome
  ◆ Variance (Mulvey et al., 1995)
  ◆ Upper partial mean (Ahmed and Sahinidis, 1998)
  ◆ Probabilistic financial risk (Barbaro et al., 2002)
  ◆ Downside risk (Eppen et al., 1988)
Risk Management using Variance

Objective: Managing the risks by reducing the variance (robust optimization)
Risk Management using Variance

• Goal Programming Formulation
  - New objective function: Minimize $E[\text{Cost}] + \rho \cdot V[\text{Cost}]$
  - Different $\rho$ can lead to different solution

\[ E[\text{Cost}] + \rho \cdot V[\text{Cost}] \]
\[ = \text{Cost}_1 + \sum_s p_s \cdot \text{Cost}_2 s + \rho \cdot \sum_s p_s [\left( \sum_{s'} p_{s'} \cdot \text{Cost}_2 s' \right) - \text{Cost}_2 s]^2 \]

- Expected Cost
- Variance of all the scenarios
- Weighted coefficient
Case Study – Robustness vs. Cost

![Graph showing the relationship between cost, variance, and robustness.](image-url)
Case Study – Variance Reduction

Risk Management

E(Cost)=$182.24 MM, \rho=0
E(Cost)=$183.14 MM, \rho=1.5E-4
Risk Management via Variability Index

- **First Order Variability index**
  - Convert NLP to LP by replacing two norm to one norm

\[
\text{Min: } E[\text{Cost}] + \rho \cdot \sum_s p_s (E[\text{Cost}] - Cost_s)^2
\]

\[
\text{Min: } E[\text{Cost}] + \rho \cdot \sum_s p_s |E[\text{Cost}] - Cost_s|
\]
• Linearize the absolute value term
  • Introducing a first order non-negative variability index $\Delta$

Min: $E[\text{Cost}] + \rho \cdot \sum_s p_s \cdot (E[\text{Cost}] - \text{Cost}_s + 2\Delta_s)$

s.t. $\Delta_s \geq \text{Cost}_s - E[\text{Cost}]$

$\Delta_s \geq 0$

Min: $E[\text{Cost}] + \rho \cdot \sum_s p_s |E[\text{Cost}] - \text{Cost}_s|$
Risk Management

Upper Partial Mean

- Reduce undesirable penalty
  - Using positive variability index $\Delta$ by goal programming

\[
\begin{align*}
\text{Min: } & E[\text{Cost}] + \rho \cdot V[\text{Cost}] \\
& \text{(Desirable + undesirable penalty)} \\
\text{Min: } & E[\text{Cost}] + \rho \cdot \sum_s p_s \cdot \Delta_s \\
\text{s.t. } & \Delta_s \geq Cost_s - E[\text{Cost}] \\
& \Delta_s \geq 0 \\
& \text{(Only undesirable penalty)}
\end{align*}
\]
Risk Management

Efficient Frontier – Robustness vs. Cost

- Cost ($MM)
- Variance ($MM^2)

Graph showing the trade-off between cost and variance for different values of ρ.
Results – Variability Index Reduction

Risk Management

E(Cost)=$182.24MM, ρ=0
E(Cost)=$185.87MM, ρ=2
**Objective:** modify the cost distribution in order to satisfy the preferences of the decision maker – manage the probabilistic financial risk

- OR...
- Increase these?
- Reduce these probability?
Financial Risk Management Model

- Probabilistic Financial Risk
  - Probability of exceeding certain target $\Omega$
    \[
    \text{Risk}(x, \Omega) = \Pr[\text{Cost}(x) > \Omega] = \sum_s p_s Z_s(x, \Omega)
    \]
  
  - Binary variables
    \[
    Z_s(x, \Omega) = \begin{cases} 
    1 & \text{if } \text{Cost}_s > \Omega \\
    0 & \text{otherwise}
    \end{cases}
    \]
  
  - Big-M constraints
    \[
    \begin{align*}
    \text{Cost}_s & \leq \Omega + M \cdot Z_s \\
    \text{Cost}_s & \geq \Omega - M \cdot (1 - Z_s)
    \end{align*}
    \]
Risk Management Model Formulation

Risk Objective → Min: \( \text{Risk}(x, \Omega) = \sum_s p_s Z_s \)

Economic Objective → Min: \( E[\text{Cost}] = \text{Cost1} + \sum_s P_s \cdot \text{Cost2}_s \)

s.t.

\[
\begin{align*}
\text{Cost1 + Cost2}_s & \leq \Omega + M \cdot Z_s \\
\text{Cost1 + Cost2}_s & \geq \Omega - M \cdot (1 - Z_s) \\
Ax &= b \\
W_s y_s &= h_s - T_s x \\
x &\geq 0, y_s \geq 0, z_s \in \{0, 1\}
\end{align*}
\]
Multi-objective Optimization Model

- An infinite set of alternative optimal solutions (Pareto curve)
- Pareto optimum: Impossible to improve both objective functions simultaneously

![Diagram of Pareto Curve](image)
Minimize: Cost + $\varepsilon \cdot \text{Risk}$

$\varepsilon$-constraint Method

$\varepsilon = 0.001$

Min Optimal Cost

Max Optimal Cost

Smallest Risk

Largest Risk
Pareto Curve: $E[\text{Cost}]$ vs. Risk

Note: Target at $188$ MM
Results for Probabilistic Risk Management

Risk = 0.08 (Min [Cost])
Risk = 0.02

Cost ($MM)
Downside Risk

- Definition: Positive Deviation
  - Binary variables are not required, pure LP (MILP -> LP)

\[
D\text{Risk}(x, \Omega) = \sum_s p_s \delta_s(x, \Omega)
\]

\[
\delta_s(x, \Omega) \geq \text{Cost}_s - \Omega, \quad \forall s
\]

\[
\delta_s(x, \Omega) \geq 0, \forall s
\]
**Risk Management**

**Downside Risk Model Formulation**

Risk Objective → Min: \( D\text{Risk}(x, \Omega) = \sum_s p_s \delta_s \)

**Economic Objective** → Min: \( E[\text{Cost}] = \text{Cost}_1 + \sum_s P_s \cdot \text{Cost}_2_s \)

s.t.

\[
\begin{align*}
\delta_s(x, \Omega) & \geq \text{Cost}_1 + \text{Cost}_2_s - \Omega \\
\delta_s(x, \Omega) & \geq 0 \\
Ax & = b \\
W_sy_s & = h_s - T_sx \\
x & \geq 0, y_s \geq 0, \ z_s \in \{0, 1\}
\end{align*}
\]
Risk Management

Results for Downside Risk Management

![Bar chart showing cost and probability distribution for Downside Risk Management. The chart includes two sets of bars: DRisk=36.92 (Min E[Cost]) and DRsik=5.38. The x-axis represents cost in millions of dollars (MM), and the y-axis represents probability. The chart highlights the probabilities for costs ranging from $170MM to $194MM.]
Simulation Framework

- **Stochastic Planner**: Solve Stochastic model and execute decisions for period $t$.
- **Deterministic Planner**: Solve Deterministic model and execute decisions for period $t$.
- **Period $t-1$**: Randomly generate demand and freight rate.
- **Period $t$**: Update information on the uncertain parameters (mean and variance) for period $t$.
- **Period $t+1$**: Update information on the uncertain parameters (only mean value) for period $t+1$.
Simulation Flowchart

1. **Calculate the real cost, store data, Set iter = iter+1, t =1**

2. **Solve the S/D model and implement the decision for current time period**

3. **Randomly generate demand and freight rate information**

4. **Update information**

5. **Move to next time period t = t+1**

6. **t=12 ?**
   - No: Go back to 3.
   - Yes: Go to Next iteration

7. **Reach iteration limit?**
   - Yes: STOP
   - No: Go back to 1.
Case Study

Simulation

Average 5.70% cost saving

Cost ($MM) vs. Iterations

- Stochastic Soln
- Deterministic Soln

Average 5.70% cost saving
### Problem Sizes

#### Toy Problem

<table>
<thead>
<tr>
<th></th>
<th>Deterministic Model</th>
<th>Two-stage Stochastic Programming Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10 scenarios</td>
</tr>
<tr>
<td># of Constraints</td>
<td>1,369</td>
<td>13,080</td>
</tr>
<tr>
<td># of Variables</td>
<td>3,937</td>
<td>37,248</td>
</tr>
<tr>
<td># of Non-zeros</td>
<td>8,910</td>
<td>85,451</td>
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</tbody>
</table>

#### Full Problem

<table>
<thead>
<tr>
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<th>Two-stage Stochastic Programming Model</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10 scenarios</td>
</tr>
<tr>
<td># of Constraints</td>
<td>6,373</td>
<td>61,284</td>
</tr>
<tr>
<td># of Variables</td>
<td>19,225</td>
<td>182,496</td>
</tr>
<tr>
<td># of Non-zeros</td>
<td>41,899</td>
<td>402,267</td>
</tr>
</tbody>
</table>

*Note: Problems with red statistical data are not able to be solved by DWS*
Algorithm: Multi-cut L-shaped Method

Two-stage SP Model

\[
\text{Min} \quad c^T x + p_1 q_1^T y_1 + p_2 q_2^T y_2 + \cdots + p_s q_s^T y_s \\
\text{s.t.} \quad A x + T_1 x + W_1 y_1 + T_2 x + W_2 y_2 + \cdots + T_s x + W_s y_s = b \\
\quad x \geq 0, \quad y_1 \geq 0, \quad y_2 \geq 0, \cdots \quad y_s \geq 0
\]

Master problem

Scenario sub-problems
**Algorithm: Multi-cut L-shaped Method**

**Standard L-shaped Method**

- **Solve master problem to get a lower bound (LB)**
  - $\min c^T x + \theta$
  - s.t. $Ax = b$
  - $\theta \geq e_l x + d_l$
  - $x \geq 0$

- **Solve the subproblem to get an upper bound (UB)**
  - $\min q_s^T y$
  - s.t. $Wy = h_s - T_s x$
  - $y \geq 0$

- **Add cut**
  - $e_l = \sum_s p_s \pi_s^T T_s$
  - $d_l = \sum_s p_s \pi_s^T h_s$

- **UB - LB < Tol ?**

- **Yes**
  - **STOP**

- **No**
Expected Recourse Function

- The expected recourse function $Q(x)$ is convex and piecewise linear
- Each optimality cut supports $Q(x)$ from below
Algorithm: Multi-cut L-shaped Method

Multi-cut L-shaped Method

Solve master problem to get a lower bound (LB)

Add cut

Add cut

Solve the subproblem to get an upper bound (UB)

UB – LB < Tol ?

STOP

\[ \min \quad c^Tx + \sum_s p_s \theta_s \]
\[ \text{subject to} \quad Ax = b \]
\[ \theta_s \geq e_{l,s} x + d_{l,s} \]
\[ x \geq 0 \]

\[ \min \quad q_s^T y \]
\[ \text{subject to} \quad Wy = h_s - T_s x \]
\[ y \geq 0 \]
Example

Algorithm: Multi-cut L-shaped Method

Cost ($MM)

Iterations

- Standard L-Shaped Upper_bound
- Standard L-Shaped Lower_bound
- Multi-cut L-Shaped Upper_bound
- Multi-cut L-Shaped Lower_bound
Conclusion

- **Current Work**
  - Develop a **two-stage stochastic programming** model for global supply chain planning under uncertainty. *Simulation* studies show that **5.70% cost saving** can be achieved in average.
  - Present four **risk management model**. Develop an **efficient solution algorithm** to solve the large scale stochastic programming problem.

- **Future Work**
  - **Capacity planning** under demand uncertainty.
Questions?