Optimization under uncertainty

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Stochastic Programming (SP)
SP

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SP References


Two-stage SP

\[
\begin{align*}
\text{minimize}_{x,y} & \quad O_\lambda f(x, y, \lambda) \\
\text{subject to} & \quad h(x, y, \lambda) = 0 \\
& \quad g(x, y, \lambda) \leq 0
\end{align*}
\]
Decision framework

— Decisions $x$ are made (here & now)
— Stochastic vector $\lambda$ realizes in a scenario $\lambda_i$
— Given $x$, decisions $y_i(x,\lambda)$ are made for each realization of $\lambda$, $\lambda_i$ (wait & see)
Decision tree

1st stage decisions here & now

2nd stage decisions wait & see

Recourse
Scenario formulation

\[ \lambda = \begin{cases} 
\lambda_1 & \alpha_1 & \text{high} \\
\lambda_2 & \alpha_2 & \text{average} \\
\lambda_3 & \alpha_3 & \text{low} 
\end{cases} \]
Scenario formulation

minimize \( Z^s = \sum_{i=1}^{3} \alpha_i f(x_i, y_i, \lambda_i) \)

subject to

\( h(x_i, y_i, \lambda_i) = 0 \quad i = 1, 2, 3 \)
\( g(x_i, y_i, \lambda_i) \leq 0 \quad i = 1, 2, 3 \)
\( x_1 = x_2 = x_3 \)

Non-anticipativity

Here & Now

Expectation
Non-anticipativity constraints

\[ x_1 = x_2 = x_3 \]

Decisions cannot depend on the unknown future!
Node formulation

\[
\text{minimize } Z^S = \sum_{i=1}^{3} \alpha_i f(x, y_i, \lambda_i)
\]

subject to

\[
h(x, y_i, \lambda_i) = 0 \quad i = 1, 2, 3
\]
\[
g(x, y_i, \lambda_i) \leq 0 \quad i = 1, 2, 3
\]

Non-anticipativity constraints are implicit!
EVPI

Expected Value of the Perfect Information

Measure of the value of “perfect” information
EVPI

\[
\begin{align*}
\text{minimize} & \quad Z^P = \sum_{i=1}^{3} \alpha_i f(x_i, y_i, \lambda_i) \\
\text{subject to} & \quad f(x_i, y_i, \lambda_i) = 0 \quad i = 1, 2, 3 \\
& \quad g(x_i, y_i, \lambda_i) \leq 0 \quad i = 1, 2, 3
\end{align*}
\]

No non-anticipativity constraints:
we perfectly foresee the future
EVPI

$$EVPI = Z^S - Z^P$$

EVPI is non-negative
VSS
(only expectation)

Value of the Stochastic Solution

Measure of the relevance (gain) of using a stochastic approach
VSS

maximize_{x, y} \quad f(x, y, \lambda^{avg})

subject to \quad h(x, y, \lambda^{avg}) = 0
\quad g(x, y, \lambda^{avg}) \leq 0

Solution \quad x^D

Average!
minimize \( y_1, y_2, y_3 \) \[ Z^D = \sum_{i=1}^{3} \alpha_i f(x^D, y_i, \lambda_i) \]
subject to \[ h(x^D, y_i, \lambda_i) = 0 \quad i = 1, 2, 3 \]
\[ g(x^D, y_i, \lambda_i) \leq 0 \quad i = 1, 2, 3 \]

This problem decomposes by scenario
VSS

\[ Z^D = \sum_{i=1}^{3} \alpha_i f(x^D, y_i, \lambda_i) \]

We evaluate the “deterministic” solution in all scenarios.
\[ VSS = Z^D - Z^S \]

VSS is non-negative
Robust Optimization
Outline

• Why Robust Optimization (RO)?
• RO without recourse
• RO with recourse
• Scheduling energy and reserve
Why RO?

Stochastic programming exhibits two drawbacks:

1. Uncertain parameters are difficult to characterize using distribution functions

2. The number of scenarios needed to describe the uncertain parameters is large, leading to optimization problems that may become intractable
References


RO without recourse

$$\min_{x} \max_{w} f(x, w)$$

s.t.

$$g_i(x, w) \leq 0, \forall i,$$

$$x \in \mathcal{X},$$

$$\forall w \in \mathcal{W}$$
RO without recourse

\[ \min_{\mathbf{x}, z} \ z \]

s.t. \[ \max_{\mathbf{w}} \ \{ f(\mathbf{x}, \mathbf{w}), \mathbf{w} \in \mathcal{W} \} \leq z , \]

\[ \max_{\mathbf{w}} \ \{ g_i(\mathbf{x}, \mathbf{w}), \mathbf{w} \in \mathcal{W} \} \leq 0, \ \forall i , \]

\[ \mathbf{x} \in \mathcal{X} \]
RO without recourse

Under certain conditions over the robust set:

<table>
<thead>
<tr>
<th>Deterministic problem</th>
<th>Robust counterpart</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>Larger LP</td>
</tr>
<tr>
<td>MILP</td>
<td>Larger MILP</td>
</tr>
<tr>
<td>NLP</td>
<td>Larger NLP</td>
</tr>
</tbody>
</table>
RO with recourse

• Make scheduling decisions (min)
• Uncertainty realizes (max)
• Make operation (recourse) decisions (min)
RO with recourse

\[
\begin{align*}
\min_{x} \quad & \max_{w} \quad \min_{y} \quad f(x, w, y) \\
\text{s.t.} \quad & h^R(x, w, y) = 0, \\
& g^R(x, w, y) \leq 0, \\
& y \in \mathcal{Y}, \\
\text{s.t.} \quad & w \in \mathcal{W}, \\
\text{s.t.} \quad & h^P(x) = 0, \\
& g^P(x) \leq 0, \\
& x \in \mathcal{X}
\end{align*}
\]
RO with recourse: Example

\[
\begin{align*}
\min_{x_1, x_2} & \quad 30x_1 + 150x_2 \\
\max_{w_1, w_2} & \quad w_1 y_1 - w_2 y_2 \\
\text{s.t.} & \quad y_1 \leq 100x_1, \\
& \quad y_2 \leq 100x_2, \\
& \quad y_1 + y_2 = 100, \\
& \quad y_1, y_2 \geq 0, \\
\text{s.t.} & \quad 2w_1 + w_2 \geq 16, \\
& \quad w_1 \geq 3, \\
& \quad w_2 \geq 5.
\end{align*}
\]
RO with recourse

• Make scheduling decisions “x” with a prognosis of the future
• The uncertainty “w” realizes
• Make operation (recourse) decisions “y”
Power system applications
ISO

• ISO market clearing: large-scale, stochastic?

Maximize Expected Social Welfare

subject to:

Market equilibrium

Producer constraints

Consumer constraints
Producer

• Offering by non-strategic producers: stochastic

Maximize Expected Profit

subject to:

Producer constraints
Stochastic producer

- Offering by non-dispatchable producers: stochastic

  Maximize Expected Profit

subject to:

  Producer constraints
Producer

- Futures market involvement (forward contracts and options)

Maximize Expected Profit

subject to:

Producer constraints

Contracting constraints
Producer

• Insurances

If selling through forward contracts and the production units fail... an insurance is advisable
Consumer

- Consumer energy procurement

Maximize Expected Cost

subject to:

Consumer constraints
Contracting constraints
Producer

• Capacity investment by non-dispatchable producers

![Diagram showing upper and lower level decision-making with market clearing for different load and wind conditions.](image-url)
TSO

- Transmission capacity investment
TSO

• Transmission capacity investment

Upper-Level

Trade Maximization

subject to

Lines built

Lower-Level

Social Welfare Maximization
(Market Clearing)
ISO

• Transmission maintenance

Upper-Level

Security Maximization

subject to

Lower-Level

Social Welfare Maximization (Market Clearing)

Lines in maintenance

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A. J. Conejo, The Ohio State University
Conclusions

(Electrical) Energy problems are important!
Conclusions

How many coal plants are currently being built in planet Earth?
Conclusions
If renewables are considered:

- Major uncertainty: stochastic production facilities

- No such thing in the past (just demand uncertainty)
- No such thing in models for industry (production facilities are generally deterministic)
Conclusions
If renewables are considered:

- Complex uncertainty: multiple dependencies

- Spatial correlations (i) among production facilities, (ii) among demands, and (iii) among demands and production facilities.

- Temporal correlations for demands and production facilities
Conclusions
If renewables are considered:

• **Multi-stage** modeling is a must: future investment cost in stochastic sources is highly uncertain: the technology is not mature
  – No two-stage stochastic models
  – No adaptive robust optimization
Thank you