Centralized versus Distributed Manufacturing: A Continuous Location-Allocation Problem

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Motivation: Rethinking of traditional manufacturing

Distributed Manufacturing – geographically dispersed network of facilities
- Exploit new technology and modularity
- Attend new requirements of the market
- Logistical aspects

Potential applications
- Biomass supply chain (ethanol production)
- Shale gas supply chain (gas processing plants)
- Electric power generation (distributed power)
Motivation: Rethinking of traditional manufacturing

Tradeoff: Capital cost vs. Transportation Cost
• Potential advantages of Distributed Manufacturing
• Economy of scale favors large-scale production

Need for a general framework that captures the tradeoff and design best network
• Evaluate cost of centralized versus distributed manufacturing
• Address higher level planning problems

Problem formulated as Capacitated Multi-facility Weber problem\(^1\)
• Determine location in continuous 2-D space for new facilities in relation to the location of existing facilities

Background: The Weber Problem

The Original Weber Problem (1909)\(^2\)
- 2 suppliers, 1 market, and 1 facility
- Fixed points not colinear
- Euclidean distances
- Find facility location in 2-D space

Capacitated Multi-facility Weber Problem
- Facilities to be installed have maximum capacity
- Cooper (1972) was the first to attempt this problem\(^3\)
  - Exact method: can only be applied for very small-problems
  - Heuristic method: Alternate the solution of the transportation and location problems until convergence. Do not guarantee optimality
- Sherali, Al-Lougani, Subramanian (2002) developed a Branch-and-Bound Algorithm\(^4\)
- Several heuristic methods\(^5\)

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\(^3\) L. Cooper, "The Transportation-Location Problem," 1972.
Problem statement

Given:

• A set of suppliers $i$, a set of consumer markets $j$, and their respective fixed location, availability and demand

• $M$ potential distributed and $N$ potential centralized set of $k$ single-product facilities, and their corresponding maximum capacity and conversion rate (unknown location)

• Investment, operating and transportation costs

Find:

• Number, type and 2-D location of facilities to design a manufacturing network that minimizes the cost

Continuous variables: $x_k, y_k, f_{ik}, f_{kj}, f_k, D_{ik}, D_{kj}$

Boolean variables: $Z_k, Z_{ik}, Z_{kj}$
General Disjunctive Programming (GDP) Formulation

\[
\text{Min Cost} = \sum_k Z_k \cdot f_k + \sum_i \sum_k Z_{ik} \cdot f_{ik} + \sum_k \sum_j Z_{kj} \cdot f_{kj}
\]

\[
\begin{align*}
Z_k &= ff_k + vf_k \cdot f_k \\
0 &\leq f_k \leq mc_k \\
0 &\leq x_k \leq x_k^u \\
0 &\leq y_k \leq y_k^u
\end{align*}
\]

\[
\begin{align*}
\neg Z_k &\quad \text{Cost}_k = 0 \\
f_k &= 0 \\
x_k &= 0 \\
y_k &= 0
\end{align*}
\]

, \forall k \in K

\[
\begin{align*}
\text{Choice of facility}
\end{align*}
\]

\[
\begin{align*}
\text{Choice of link supplier/facility}
\end{align*}
\]

\[
\begin{align*}
\text{Choice of link facility/market}
\end{align*}
\]

\[
\begin{align*}
\text{Distance supplier/facility}
\end{align*}
\]

\[
\begin{align*}
\text{Distance facility/market}
\end{align*}
\]

\[
\begin{align*}
\text{Logic constraints}
\end{align*}
\]

\[
\begin{align*}
\text{Availability of raw-material}
\end{align*}
\]

\[
\begin{align*}
\text{Mass balances}
\end{align*}
\]

\[
\begin{align*}
\text{Market demand}
\end{align*}
\]

\[
\begin{align*}
D_{ik} &\geq \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2} \\
0 &\leq f_{ik} \leq f_{ik}^u \\
R &\leq D_{ik} \leq D_{ik}^u
\end{align*}
\]

, \forall i \in I, \ k \in K

\[
\begin{align*}
D_{kj} &\geq \sqrt{(x_j - x_k)^2 + (y_j - y_k)^2} \\
0 &\leq f_{kj} \leq f_{kj}^u \\
R &\leq D_{kj} \leq D_{kj}^u
\end{align*}
\]

, \forall k \in K, \ j \in J

\[
\begin{align*}
Z_k \Rightarrow \bigvee_i Z_{ik} \\
Z_k \Rightarrow \bigvee_j Z_{kj} \\
\sum_k f_{ik} \leq a_i \\
\sum_i f_{ik} \cdot cv_k = f_k \\
f_k = \sum_j f_{kj} \\
\sum_k f_{kj} = d_j \\
Z_k, Z_{ik}, Z_{kj} \in \{\text{True, False}\} \\
, \forall i = i \in K, \ k \in K, \ j \in J
\end{align*}
\]
Illustrative example: ethanol production

Problem:

- 2 switchgrass suppliers
  - Supplier 1: $2,000/ton
  - Supplier 2: $2,200/ton
- 120 tons/week of switchgrass available per supplier
- 2 switchgrass suppliers
- 2 markets
- Demand of 33,444 gal/week of ethanol per market
- 3 potential facilities
  - 2 distributed
  - 1 centralized

$mc_w = 41,792 \text{ gal/week}$
$mc_c = 83,610 \text{ gal/week}$
$cv_k = 90\%$
Illustrative example: ethanol production

Intuitive answer: 1 centralized facility

$516,100/week
Illustrative example: ethanol production

Optimal network: 2 distributed facilities

- $f_{21} = 102.22$ ton/week
- $F_{12} = 30,767$ ton/week
- $F_{21} = 33,444$ gal/week
- $503,900$/week

-suppliers $i$
-markets $j$
-facilities $k$
### Illustrative example

#### Computational results

**Global Optimization:**

<table>
<thead>
<tr>
<th>METHOD</th>
<th>Cost ($10^3$/week)</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BARON</td>
<td>503.9</td>
<td>703.2</td>
</tr>
<tr>
<td>SCIP</td>
<td>503.9</td>
<td>656.7</td>
</tr>
<tr>
<td>Multiparametric Disaggregation⁶</td>
<td>503.9</td>
<td>1,045.6</td>
</tr>
<tr>
<td>Bilevel decomposition</td>
<td>503.9</td>
<td>36.1</td>
</tr>
</tbody>
</table>

**Convex relaxation (lower bound):**

<table>
<thead>
<tr>
<th>METHOD</th>
<th>Cost ($10^3$/week)</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McCormick</td>
<td>482.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Piecewise McCormick⁷ (16 partitions)</td>
<td>503.8</td>
<td>2065.5</td>
</tr>
<tr>
<td>Logarithmic Piecewise McCormick⁸ (16 partitions)</td>
<td>503.8</td>
<td>35.9</td>
</tr>
</tbody>
</table>

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⁷ P. M. Castro, “Tightening piecewise McCormick relaxations for bilinear problems,” 2014
Bilevel decomposition: Background

Global Logic-Based Outer Approximation (GLBOA)\textsuperscript{9}

- Non-convex GDP
- **Master problem** (MP): linear relaxation of the nonconvex GDP
  - Lower Bound

- **Subproblem** (SP): lower dimensional nonconvex NLP in which the Boolean variables are fixed in the GDP
  - Upper bound

- Every time MP is resolved, an integer cut is added to exclude fixed discrete variables already used

\textsuperscript{9} F. Trespalacios and I. E. Grossmann,”Cutting planes for improved global logic-based outer-approximation for the synthesis of process networks .”, 2015.
**Bilevel decomposition algorithm**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve MP (MILP)</td>
<td>Master problem: linear GDP relaxation provides a lower bound, and the selection of facilities to fix</td>
</tr>
<tr>
<td>Fix $Z_k = True$</td>
<td>• Distance constraints, which are convex, are linearized for a given discretization of space</td>
</tr>
<tr>
<td>Solve SP (nonconvex MINLP)</td>
<td>Subproblem: For the fixed alternative of facilities, the MINLP is solved with global solver to obtain an upper bound</td>
</tr>
<tr>
<td>Provides UB</td>
<td>• Potential links, which involve discrete variables, are still to be determined.</td>
</tr>
<tr>
<td>$UB - LB \leq \epsilon$</td>
<td>Integer cuts are added to the (MP)</td>
</tr>
<tr>
<td>Yes</td>
<td>Stop</td>
</tr>
</tbody>
</table>
Large-scale problems: Example 1

Problem

- 10 suppliers
- 120 \(\text{tons/week}\) of raw material available per supplier

- 10 markets
- Demand of 100 \(\text{tons/week}\) per market

- 12 potential facilities \((cv_k = 90\%)\)
  - 10 distributed \((mc_m = 100\text{tons/week})\)
  - 2 centralized \((mc_n = 1000\text{tons/week})\)
Large-scale problems: Example 1

Optimal network found: 10 distributed facilities

$28,991,000 /week

supplier i
market j
facilities k
Global Optimization:

<table>
<thead>
<tr>
<th>METHOD</th>
<th>Cost ($10^3/week)</th>
<th>Optimality gap (%)</th>
<th>CPU time (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BARON</td>
<td>29,054</td>
<td>21%</td>
<td>12*</td>
</tr>
<tr>
<td>SCIP</td>
<td>29,892</td>
<td>92%</td>
<td>12*</td>
</tr>
<tr>
<td>Bilevel Decomposition</td>
<td>28,991</td>
<td>9%**</td>
<td>4*</td>
</tr>
</tbody>
</table>

* Exceeded maximum CPU time
** Estimated gap

For the Bilevel Decomposition Algorithm, the master problem (MILP) was solved using CPLEX and the subproblem (nonconvex MINLP) was solved using BARON
Large-scale problems: Example 2

Problem

- 10 suppliers
- Different availability of raw material for each supplier

- 10 markets
- Demand of 100 tons/week per market

- 12 potential facilities ($cv_k = 90\%$)
  - 10 distributed ($mc_m = 100\, \text{tons/week}$)
  - 2 centralized ($mc_n = 1000\, \text{tons/week}$)
Large-scale problems: Example 2

Optimal network found: 2 centralized + 1 distributed facilities

$24,984,000 /week
### Large-scale problems: Example 2

#### Computational results

**Global Optimization:**

<table>
<thead>
<tr>
<th>METHOD</th>
<th>Cost ($10^3/week)</th>
<th>Optimality gap (%)</th>
<th>CPU time (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BARON</td>
<td>24,990</td>
<td>0.6%</td>
<td>12*</td>
</tr>
<tr>
<td>SCIP</td>
<td>25,181</td>
<td>7.6%</td>
<td>12*</td>
</tr>
<tr>
<td>Bilevel Decomposition</td>
<td><strong>24,984</strong></td>
<td><strong>0.2%</strong></td>
<td><strong>4</strong></td>
</tr>
</tbody>
</table>

* Exceeded maximum CPU time
** Estimated gap

For the Bilevel Decomposition Algorithm, the master problem (MILP) was solved using CPLEX and the subproblem (nonconvex MINLP) was solved using BARON.
Conclusions

Nonconvex GDP reformulated as an MINLP

Commercial global solvers can solve small problems fairly easy

Computationally expensive to solve large-scale problems
  - Bilevel decomposition algorithm
    - Although at this point it cannot rigorously solve the large-scale problems to optimality, provides superior results
    - Potential to be improved
Future work

Develop new cuts to tighten the relaxation
  • Improve the performance of both the solvers and the algorithm

Rethink master problem formulation
  • So as it can be solved to optimality faster

Apply formulation to different problem structures
  • Investigate how the network configuration is affected by changes in the parameters
  • Explore which conditions favor distributed and/or centralized manufacturing networks.

Apply the model to biomass and electric power systems supply chain