A Novel Global Optimization Approach to the Multiperiod Blending Problem

Scott Kolodziej and Ignacio Grossmann
Department of Chemical Engineering
Carnegie Mellon University
Pittsburgh, PA

Kevin Furman and Nick Sawaya
ExxonMobil
Houston, TX

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Given Information

- **Time Horizon (Periods)**
- **Network Topology**
- **Supply Tanks**
  - Amount entering
  - Concentrations
- **Demand Tanks**
  - Amount withdrawn
  - Concentration limits
- **Initial Conditions**
  - Inventories
  - Concentrations
- **Economic Costs**
  - Network flow costs
  - Raw material costs
  - Profit for meeting demand

Determine

- Flows between which tanks in which time periods
- Inventories and concentrations for all tanks in each time period
- **Minimize total cost** of blending operation
Assumptions

• Supply concentrations are constant
• No simultaneous input/output to blending tanks
  – Avoids dynamic concentration changes
• Perfect mixing

Time Periods

• Generally hours to days
• Time periods are coupled by inventories
  – Requires simultaneous optimization over all periods
  – Example: Storage for excess demand in a later time period
Mathematical Model

Objective: Minimize cost

• Mass Balances
  – Overall Flows
  – Individual Components (Blending)
• Flow/Inventory Bounds
• Operational Constraints
• Demand Specifications

Variables

• Flows, Concentrations, and Inventories (Continuous)
• Existence/Nonexistence of Streams (Binary)

Complicating nonconvex bilinearities $F \cdot C$ and $I \cdot C$ appear in the individual component mass balances

Requires global optimization techniques

Resulting model is an MINLP
### GloMIQO Results

<table>
<thead>
<tr>
<th></th>
<th>GloMIQO</th>
<th></th>
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</thead>
<tbody>
<tr>
<td><strong>Tanks</strong></td>
<td>6</td>
<td>8</td>
<td>8</td>
<td><strong>GloMIQO still cannot close the gap in less than 2 hours</strong></td>
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<tr>
<td><strong>Time Periods</strong></td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
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<tr>
<td><strong>Wall Time (s)</strong></td>
<td>1771</td>
<td>&gt;7200</td>
<td>&gt;7200</td>
<td>&gt;7200</td>
<td>&gt;7200</td>
<td>&gt;7200</td>
<td>0 (Fail)</td>
</tr>
<tr>
<td><strong>LB</strong></td>
<td>13.359</td>
<td>47.246</td>
<td>7.179</td>
<td>13.830</td>
<td>54.147</td>
<td>9.226</td>
<td>22.718</td>
</tr>
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GloMIQO closes the gap faster than BARON ... but need a new approach!
Radix-Based Discretization

Discretize concentration $C$ using a disjunction for levels of precision $p$ to $P$

Let $C = \sum_{k=p}^{P} \lambda_k$

$$\bigvee_{j=0}^{9} \left[ \lambda_k = 10^k \cdot j \right] \quad \forall k \in \{p, \ldots, P\}$$

$$\lambda_k = \sum_{j=0}^{9} \lambda_{j, k} \quad \forall k \in \{p, \ldots, P\}$$

$$\lambda_{j, k} = 10^k \cdot j \cdot z_{j, k} \quad \forall k \in \{p, \ldots, P\}, j \in \{0, \ldots, 9\}$$

$$\sum_{j=0}^{9} z_{j, k} = 1 \quad \forall k \in \{p, \ldots, P\}$$

$$z_{j, k} \in \{0, 1\} \quad \forall k \in \{p, \ldots, P\}, j \in \{0, \ldots, 9\}$$

Convex hull for this disjunction
The range of the variable is discretized over a region determined by $p$ and $P$. This region must encapsulate the full range of the variable being discretized.
Given that the bilinear product $u = F \cdot C$ can be replaced by this set of constraints using exact linearization:

\[
\begin{align*}
C &= \sum_{k=p}^{9} \sum_{j=0}^{9} 10^k \cdot j \cdot z_{j,k} \\
u &= \sum_{k=p}^{9} \sum_{j=0}^{9} 10^k \cdot j \cdot \hat{F}_{j,k} \\
\hat{F}_{j,k} &\leq F^U \cdot z_{j,k} \quad \forall \ k \in \{p, \ldots, P\}, j \in \{0, \ldots, 9\} \\
\sum_{j=0}^{9} \hat{F}_{j,k} &= F \quad \forall \ k \in \{p, \ldots, P\} \\
\sum_{j=0}^{9} z_{j,k} &= 1 \quad \forall \ k \in \{p, \ldots, P\} \\
z_{j,k} &\in \{0, 1\} \quad \forall \ k \in \{p, \ldots, P\}, j \in \{0, \ldots, 9\}
\end{align*}
\]

Discretization replaces bilinearity with mixed-integer linear constraints. The MINLP model can be reformulated as an MILP approximation.
Lower Bounding

• RBD yields an **upper bound**
• What about a **lower bound**?

![Diagram showing a variable ΔC between 0 and 10^p (the size of the “gaps”).]

• We introduce ΔC as a **slack variable**
  – ΔC ranges between 0 and 10^p (the size of the “gaps”)
  – **Relaxed RBD constraints** can be added to “fill the gap” and relax the original problem

*Kolodziej, Castro and Grossmann (2012)*
Algorithm 1

• Solve the MILP discretized problem for upper bound
• Solve the MILP relaxed problem for lower bound
• If gap < $\varepsilon$, stop. Otherwise, let $p = p - 1$ and repeat

Algorithm 2

• Solve the MILP relaxed problem for upper bound
• Fix the process binary variables
• Solve the resulting NLP with an NLP solver for a lower bound
Algorithm 2

- Computational Results

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Remarks

• Using a base 2 discretization yields the smallest problem sizes.

  Performance is slightly better in most cases.

• Discretizing over flows instead of concentrations slower performance.

  Trend might be reversed for larger number of properties