Models for Designing Resilient Supply Chain Networks for Chemicals and Gases

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Outline

1. Introduction
2. The Model
3. Example
4. Improve p-robustness
5. Conclusion
Supply chain networks are usually designed as though they function in normal mode all the time.

But disruptions are a significant factor.

Result in significant cost increases:
  - Increased transportation costs
  - Penalty for unmet demands
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**Objective:** Develop model for designing supply chains that perform well when no disruptions occur but not too badly when disruptions occur.

Focus on packaged gases, but model is generic.
Disruptions

- Suppose disruptions can occur
- We must design supply chain before we know what facilities will be disrupted
- Possible disruptions are described by *scenarios*
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Model Objectives

- Can we improve performance under the disruption scenarios by choosing better facilities?
- "Nominal" cost will increase
- But scenario costs will decrease
Can we improve performance under the disruption scenarios by choosing better facilities?

”Nominal” cost will increase

But scenario costs will decrease

Two-stage problem:

- **Stage 1**: Choose facility locations
- (scenario occurs)
- **Stage 2**: Assign flows
This is a robust optimization problem

Prevent performance from being ”too bad” in any scenario

Lots of approaches for robust optimization in the literature

- Min-max cost
- Min-max regret
- CVaR
- etc.
This is a robust optimization problem

- Prevent performance from being "too bad" in any scenario
- Lots of approaches for robust optimization in the literature
  - Min-max cost
  - Min-max regret
  - CVaR
  - etc.

Our approach:
- Minimize cost in nominal scenario (no disruptions)
- Subject to bound on cost in any other scenario
- "p-robustness"
Decisions

- Need to decide which plants and warehouses to open
- Customer locations are fixed
- Minimize total cost which is the sum of
  - Fixed cost (to build/lease facilities)
  - Transportation cost
Notation: Nodes and Arcs

- General network \((V, A)\)
  - \(V = \) set of nodes
  - \(A = \) set of arcs
  - Not necessarily plants, warehouses, customers
- \(V_0 \subseteq V = \) set of non-demand nodes
Notation: Nodes and Arcs

- General network $(V, A)$
  - $V$ = set of nodes
  - $A$ = set of arcs
  - Not necessarily plants, warehouses, customers

- $V_0 \subseteq V$ = set of non-demand nodes

- $N$ = number of products

- $b_{jn} = \text{supply of product } n \text{ at node } j \in V$
  - $> 0$ for supply nodes
  - $< 0$ for demand nodes
  - $= 0$ for transshipment nodes

- $k_{jn} = \text{capacity for product } n \text{ at node } j \in V_0$
  - Depends on type of product and type of storage (bulk, drum, tote)
Notation: Costs and Disruptions

- \( f_j \) = fixed cost to open node \( j \in V_0 \)
- \( d_{ijn} \) = cost to transport one unit of product \( n \) on arc \( (i,j) \in A \)
Notation: Costs and Disruptions

- $f_j = \text{fixed cost to open node } j \in V_0$
- $d_{ijn} = \text{cost to transport one unit of product } n \text{ on arc } (i,j) \in A$
- $S = \text{set of scenarios}$
  - $s = 0$ is nominal scenario
- $a_{js} = 1$ if node $j \in V_0$ is disrupted in scenario $s$, $a_{j0} = 0 \forall j$
  - Applies to non-demand nodes only
- $p_s = \text{desired robustness level in scenario } s \in S \setminus \{0\}$
  - The standard problem of minimizing the nominal cost without considering scenario costs can be obtained by setting $p = \infty$
- $c_s^* = \text{the optimal objective value for scenario } s \in S \setminus \{0\}$
Notation: Bill of Materials (BOM)

Define

- $\alpha_{mn}$: amount of product $n$ that is used to make 1 unit of product $m$, $m, n \in \mathbb{N}$.

- We consider the production process in our model
- Raw materials are shipped to the plants and made into different products, then the products are shipped to the customers
- Demand of raw materials is calculated from demand of products and BOM
Notation: Decision Variables

- $X_j = 1$ if node $j \in V_0$ is open, 0 otherwise
- $Y_{ijns} =$ flow of product $n$ on arc $(i,j)$ in scenario $s$
Minimize nominal-scenario (no-disruption) cost:

\[
\text{minimize} \quad \sum_{j \in V_0} f_j X_j + \sum_{(i,j) \in A} \sum_{n=1}^{N} d_{ijn} Y_{ijn0} \quad (1)
\]
Constraints

Subject to:

\[
\sum_{j \in V_0} f_j X_j + \sum_{(i,j) \in A} \sum_{n=1}^{N} d_{ijn} Y_{jins} \leq (1 + p_s) c_s^* \quad \forall s \tag{2}
\]

\[
\sum_{(j,i) \in A} Y_{jins} - \sum_{(i,j) \in A} \sum_{m \in N} \alpha_{mn} Y_{ijms} = b_{jn} \quad \forall j, s, n \tag{3}
\]

\[
\sum_{(j,i) \in A} Y_{jins} \leq (1 - a_{js}) k_{jn} X_j \quad \forall j, n, s \tag{4}
\]

\[
X_j \in \{0, 1\} \quad \forall j \tag{5}
\]

\[
Y_{ijns} \geq 0 \quad \forall i, j, n, s \tag{6}
\]
minimize cost in nominal scenario
subject to cost in scenario $s \leq (1 + p)c^*_s$ \quad \forall s
flow balance constraints \quad \forall j, n, s
flow can’t exceed capacity, or 0 if disrupted \quad \forall j, n, s
$X_j$ integer
$Y_{ijns}$ non-negative
minimize \( c_0(X, Y) \)
subject to \( c_s(X, Y) \leq (1 + p)c^*_s(X^*, Y^*) \quad \forall s \)
\[(X, Y) \in \Omega\]
Solution Methods

• For now, we solve using an off-the-shelf MIP solver Xpress-MP
• Run times generally < 1 hour for problems with
  • 40 suppliers
  • 40 plants/warehouses
  • 80 customers
  • 5 scenarios
  • 10 products
• For bigger problems, we’ll need customized algorithms
  • Future research
Numerical Example

- 10 Suppliers
- 10 plants
- 20 customers
- 7 products (5 raw materials)
- 5 scenarios
  - Nominal scenario
  - 4 disruption scenarios
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- 10 Suppliers
- 10 plants
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- Costs
  - $f_j = 10,000$ on average for plants
  - $d_{ijn} = 500$ on average
  - Penalty cost for unmet demand = 10,000
- Desired robustness level $p = 0.1$
Numerical Example

- Minimizing Nominal Cost w/o bounds on scenario cost (Method 1)
- Minimizing worst-case cost (Method 2)
- Minimizing Nominal Cost with bounds on scenario cost (Method 3)
Numerical Example

Solution Comparison

Cost (X $1000)

Method1
Method2
Method3

Scenario1  Scenario2  Scenario3  Scenario4  Scenario5
Relative Regret

Suppose that

- the optimal objective value for scenario $s$ is $c_s^*$
- the best scenario cost for current solution is $c_s$
- Define the relative regret $R_s$ for scenario $s$ as

$$R_s = \frac{c_s}{c_s^*} - 1$$  \(7\)

- $R_s$ shows how much worse the current solution is compared to the optimal solution in that scenario
Finding the Maximum Relative Regret

We can find the maximum relative regret among scenarios for a given solution \((X, Y)\):

![Relative Regret Chart](chart.png)
Update the Scenario Upper Bounds

Recall

\[
\begin{align*}
\text{minimize } & \quad c_0(X, Y) \\
\text{subject to } & \quad c_s(X, Y) \leq (1 + p)c_s^*(X, Y) \quad \forall s \\
& \quad (X, Y) \in \Omega
\end{align*}
\]
If we reduced $p$ to a little below $R_{max}$, the current solution is no longer feasible.
We can find the maximum relative regret $R_{\text{max}}$ among scenarios for a given solution $(X, Y)$:

- If we reduced $p$ to a little below $R_{\text{max}}$, the current solution is no longer feasible.
- One can obtain solutions with smaller regret but greater nominal costs.
- Can we improve the regret without greatly increasing the nominal costs?
Example

- 10 Suppliers, 10 Plants, 20 Customers
- 10 Scenarios, 10 Products
- $f_j = 10,000$
- $d_{ijn} = 250$

![Nominal Cost vs Regret](image)
Example

- We can improve robustness without increasing the cost too much
When $p$ gets smaller, the run time becomes longer.
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Supply chain network design model that accounts for disruptions

Minimize nominal cost, subject to bound on cost in each scenario

Substantial improvements in robustness can be attained without large increases in nominal cost

Next steps:
- Empirical study
- Design algorithm
Questions?
Thank you!