Piecewise Linear Approximation and Branch-and-Refine Algorithm for PX Tank Sizing under Uncertainty Problem

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Vehicle Routing – Tank Sizing Problem

- **Tank Sizing**
  - Tank installation, upgrade & downgrade
  - Several available discrete tank sizes
  - Safety stock optimization for uncertainty

- **Vehicle Routing**
  - Several available truck sizes
  - Routing and timing decisions

- **Integration**
  - Tradeoff: operating cost vs. capital cost
  - Capture the effects of customer synergies and truck availability
  - Integration requires to solve “extended” routing problem for long term (e.g. years)
  - Integrated MILP model is very large
Complexity – 4 customer case

- 256 possible route, 230,448 binary variables in the MILP model for integrating tank sizing and vehicle routing
Modeling Challenges

• How to effectively integrate tank sizing with vehicle routing?
   Continuous approximation (CA) approach
    ▪ tradeoff capital and operating cost in the strategic level
    ▪ reduce most integer variables with some nonlinearities

• How to optimize the safety stocks for demand uncertainty?
   Employ stochastic inventory model
   Integrate safety stock optimization with tank sizing

• How to model the uncertainty of adding/losing customers?
   Two-stage stochastic programming
   A network structure for each scenario in each year
Optimal Safety Stock Level

\[ D \sim N(\mu, \sigma^2) \iff \text{Safety Stock} = z_\alpha \sigma, \quad P(z \leq z_\alpha) = \alpha \text{ (Service Level)} \]

\[ \text{Lead time } = T \iff D \sim N(T \cdot \mu, T \cdot \sigma^2) \iff \text{Safety Stock} = z_\alpha \sigma \sqrt{T} \]
“Cyclic” Inventory-Routing

- **Key Assumption:** each customer is replenished in a “cyclic” way with interval $T$

- **Required tank size** $\geq$ max. inv. = Safety Stock + demand rate $\times T$

$$z_\alpha \sigma \sqrt{T}$$
Routing & Replenishment in CAM

- \( T = \frac{R}{\text{ave. speed}} \)
  - \( T \) - replenishment interval
  - \( R \) - minimum distance to visit all the customers in a cluster once
  - Average travelling speed is known

- If only one trip for each replenishment
  - \( R = \text{TSP distance of the cluster & plant} \)

- If allowing multiple trips for replenishment
  - \( R = ? \)
CAM for Capacitated Routing Problems*

- **Bounds** for minimum routing distance $R$

\[
R \approx 2\left\lceil \frac{n}{q} \right\rceil r + \left(1 - \frac{1}{q}\right) \cdot \text{TSP}
\]

- $n$ – number of customers in the cluster
- $q$ – capacity, max. number of customers that can be visited in one trip
- $r$ – average distance from customers to the plant
- TSP – traveling salesman distance to visit all customers once

- **Examples**

  - Cluster 1: $q=1$, TSP=0, $r = 67$
    \[
    2\left\lceil \frac{n}{q} \right\rceil r + \left(1 - \frac{1}{q}\right) \cdot \text{TSP} = 2r = 2 \times 67 = 134\text{km}
    \]
  - Cluster 2: $q=1$, same as Cluster 1, $R = 4,400\text{km}$
  - Cluster 2: $q=2$, TSP=50, $r = 1,100$
    \[
    2\left\lceil \frac{n}{q} \right\rceil r + \left(1 - \frac{1}{q}\right) \cdot \text{TSP} = 2r + \frac{\text{TSP}}{2} = 2,225\text{km}
    \]

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Decomposition for Scenario Planning

Tank sizing decisions for 1st stage (int. var.);
Routing decisions for 2nd stage (cont. var.)

Cont. Approximation Model
+ safety stock (all customers)
Minimize total expected cost
(obtain tank sizing decisions)

Fix tank sizes
Safety stock

Customer Clustering
Select the first clustering solution

Detailed Routing Model
(obtain vehicle routing decisions)

Next clustering soln.

Termination?

Pro: avoid integer variables in the 2nd stage recourse

solve deterministic routing problem for each scenario (network structure)

Each scenario has a network structure, problem size increases as scenarios increase
Branch-and-Refine Algorithm

- Global Optimization for MINLPs with *only* Univariate Concave Terms
  - Piece-wise linear approximation (MILP) provides *global* lower bounds
  - Feasible solutions provide *upper bounds* – solving a reduced MINLP
  - Increasing the number of pieces as iteration number increases
• **Example 1:** 3-year planning horizon
  - 4 customers, 6 tank sizes, 6 types of trucks
    - *N14* will not lose by the end of Year 3
    - *N15* may lose in Year 1 with 30% chance
    - *N18* may lose in Year 2 with 40% chance
    - *N21* may lose in Year 3 with 50% chance
  
• **Scenario Tree:**
3 Existing Customer and 1 New Customer

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  - 8 scenarios for two-stage stochastic programming
    - S1 – Y1: *N14, N15, N18, N21*; Y2: *N14, N15, N18, N21*; Y3: *N14, N15, N18, N21*.
    - S2 – Y1: *N14, N15, N18, N21*; Y2: *N14, N15, N18, N21*; Y3: *N14, N15, N18, N21*.
    - S3 – Y1: *N14, N15, N18, N21*; Y2: *N14, N15, N18, N21*; Y3: *N14, N15, N18, N21*.
    - S4 – Y1: *N14, N15, N18, N21*; Y2: *N14, N15, N18, N21*; Y3: *N14, N15, N18, N21*.
    - S6 – Y1: *N14, N15, N18, N21*; Y2: *N14, N15, N18, N21*; Y3: *N14, N15, N18, N21*.
    - S7 – Y1: *N14, N15, N18, N21*; Y2: *N14, N15, N18, N21*; Y3: *N14, N15, N18, N21*.
    - S8 – Y1: *N14, N15, N18, N21*; Y2: *N14, N15, N18, N21*; Y3: *N14, N15, N18, N21*. 
Pareto Curves and Scenario Costs

- **CPU Times**
  - Directly solved with DICOPT – infeasible
  - Directly solved with BARON – > 7 days
  - B&R algorithm (CPLEX 12 + DICOPT) - 285s for all the 17 instances
Tank Sizes and Safety Stocks

- **Model Statistics**
  - MINLP: 288 dis. var., 2,484 cont. var., 4,056 cons.
  - MILP model: 408 dis. var., 2,724 cont. var., 4,344 cons.; Reduced MINLP: 48 dis. var., 2,484 cont. var., 4,056 cons. (5 iter.)
Optimal Routing Decisions

- Scenario 1 at Year 1, 90% Service Level
  - Number of visits:
    - \(N14\): 17; \(N15\): 17; \(N18\): 37; \(N21\): 18.
  - Three routes are used:
    - \(P \rightarrow N18 \rightarrow N15 \rightarrow N14 \rightarrow N21 \rightarrow P\).
    - \(P \rightarrow N18 \rightarrow N21 \rightarrow P\).
    - \(P \rightarrow N18 \rightarrow P\).

![Inventory profile from detailed routing](image1)

![Inventory under continuous approximation](image2)