Multiperiod Blend Scheduling Problem

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**Motivation**

- The scheduling problem of blending operations arises frequently in the petrochemical industry.
- Large cost savings can be achieved if the correct blending decisions are taken.
- Although simple to represent, these models are highly nonconvex, leading to the need of global optimization techniques to find the optimal solution.
- The development of efficient solution methods that take care of the general case applied to large scale systems remains as a challenge…

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**Goal:** Develop tools and strategies aiming at improving the efficiency of the solution methods for the global optimization of the multiperiod blend scheduling problem.
General Problem Topology

The general case of a blending problem can be represented schematically as follows:

![Diagram of a blending problem topology]

### Remarks:
- Examples of the supply nodes are the tanks loaded by ships or the feedstocks downstream the CDU.
- Examples of the delivery nodes are the tanks feeding the CDU or the tanks delivering to final customers.

### Important Assumptions:
- The quality of each stream/inventory is constant for a given period.
- A tank can receive or deliver in a given period of time but not both.
- Supply tanks keep a constant quality.
- Delivery tanks keep the quality within a given range.
- Streams entering delivery tanks should satisfy a quality condition.
The **MINLP formulation** of the problem is easy to achieve, however, a more efficient representation can be obtained by using a GDP framework.

**MINLP Formulation**  
(Property Balance)

\[
S_{qi}^{b}I_{j} = S_{qi}^{a}I_{j-1} + \sum_{(j')\in E} S_{j'i}^{b}F_{j'}^{b} + \sum_{(j')\in E} S_{j'i}^{a}F_{j'}^{a} \quad \forall q \in Q, j \in J^{b}, t \in T
\]

\[
\sum_{(j')\in E} S_{j'i}^{b}F_{j'}^{b} - \sum_{(j')\in E} S_{j'i}^{a}F_{j'}^{a} \quad \forall j \in J^{a}, (j', j) \in E, t \in T
\]

\[
x_{j'i}^{b} + x_{j'i}^{a} \leq 1 \quad \forall j \in J^{a}, (j', j) \in E, t \in T
\]

\[
F_{j'i}^{b}x_{j'i}^{b} \leq F_{j'i}^{a}x_{j'i}^{a} \quad \forall (j', j) \in E, t \in T
\]

**Remarks:**
- GDP formulation has **reduced the number of bilinear terms**
- GDP formulation can be efficiently solved by **Logic Based OA methods**

**GDP Formulation**
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Outline of the Logic Based OA

Master Problem (Linear GDP)

NLP Subproblem

LGDP Master: This problem is obtained by relaxing the bilinear terms using the McCormick envelopes. It is solved as MIP using the Convex Hull reformulation.

NLP Subproblem: The set of boolean variables are fixed based on the result of the master problem. These are used to generate “on the fly” much smaller NLP formulations by getting rid of the disjuncts that are not active.

Iteration Step: Once a solution is obtained from the NLP subproblem a set of linear cuts are generated and passed to the master.

Remark:
This will not guarantee a global solution! (even if the NLP subproblem is solved with a global solver).
Solution Methods
Lagrangian decomposition
(Review)

Constraints are linked together by the inventory and composition variables

Duplicating inventory and composition variables and dualizing the correspondent equalities lead to a temporal decomposable structure
**Solution Methods**

**Lagrangian decomposition (Algorithm)**

**Outline of the Lagrangian decomposition method**

- Initialize $u_0$, BestLB, BestUB, $k$
- Solve LR1, Solve LR2, Solve LRN
- $LB = \sum LBR_i$
- If BestLB < LB, BestLB = LB
- Obtain UB (Solve local MINLP)
- Update Multipliers $u^{t+1} = u^t - \alpha^t \cdot \text{Error}$
- $\alpha^t = a / (b + k)$
- $k = k + 1$

$u_0$ represents the dual multipliers; 
**BestLB**, the best lower bound; 
**BestUB**, the best upper bound and $k$, the iter counter

Each subproblem (LRi) from the decomposition is solved

A lower bound for the original nonconvex problem can be obtained by adding up the solution of each (LRi) (i.e. $LB_{LRi}$)

Any local optimization algorithm can be used to find an UB. (e.g. The logic based outer approximation applied on the GDP formulation)
Case Study

The implementation of the Lagrangian Decomposition and Logic Based OA method has been tested in the following simple case.

**Network Description:**
- Two Supply, Intermediate and Delivery nodes
- Two properties transported
- Three time periods
Numerical Results

Logic Based AO

Representation of the nonzero flow streams in the different periods for the optimal solution

Remarks

- It stops after three iterations in a local solution ($Z = 12.52$)

- The solution of the NLP subproblem which leads to the optimal solution only contains six bilinear terms as opposed to thirty six bilinear terms (the number of bilinear terms in the NLP subproblem using a traditional MINLP formulation)
Numerical Results

Lagrangian Decomposition

Representation of the nonzero flow streams in the different periods for the global optimal solution

Remarks

- Forced to stop after 20 iterations (no improvement observed).
- Finds the global solution (Z = 14.22)
- The existence of the duality gap is due to the nonconvex nature of the problem
- By experience, Lagrangian decomposition applied to larger instances has shown smaller duality gaps

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Remarks

• Proposed a **GDP formulation** that aims at **reducing** the number of **nonconvex terms**
• Proposed a **Logic Based Outer Approximation** method that takes advantage of the logic structure of the problem
• Proposed a **Lagrangian decomposition** method to solve the problem to **global optimality**

Future Work

• Analyse the performance on **large scale problems**
• Combine LBOA with Lagrangian Decomposition. (i.e. Use **LBOA for upper bounding**)
• Analyse **best parameters** for LD approach