Progress Report – BP Case Study

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Problem Description

- Medium-term operations planning model that produces a plan for monthly production, inventory targets and decisions on which demands should be satisfied from which inventory location.
- Forecasts of the future economic environment are used.
- BP’s model contains representation of the global production assets and distribution system for their PX and PTA businesses.
Problem Description

• Our Aim: Extend this model so that it will explicitly account for the uncertainty in the forecasts.
Products - PX

- Paraxylene (PX) is a colourless, flammable liquid that has a sweet odour. It is separated from a mixed xylene stream that results from the refining of petroleum.

- Can be used as a feedstock for the local manufacture of Purified Terephthalic Acid (PTA) or can be sold to customers.
Products - PTA

• BP is the largest PTA producer in the world.

• PTA is an aromatic acid, primarily applied in the production of polyester. The main raw material for PTA is PX.
Models

• Two models have been analyzed
• In both of the models
  – 5 scenarios are tested which represent 5 different economic views
  – Integrality restrictions are relaxed
Model 1

- Initial approach
- Only the shut-down decisions are in the first stage and all other decisions are in the second stage. First stage decisions are;
  - Number of days of operation of each unit running each valid feed for each break-point in a month
  - Total number of days running for a unit in a period summed over all feeds and break-point rates
  - Number of days spent shutdown
Model 2

• A more detailed model, enlarged first-stage decision space

• Operating policy for the first month as a whole constitutes the first-stage decision variables
  – production plan, days and rates running by unit, ending inventory levels etc.

• Almost twice the number of decision variables before
## Schematic Comparison of Two Models

<table>
<thead>
<tr>
<th>First-stage decisions</th>
<th>Second-stage decisions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model 1</strong></td>
<td><strong>Model 2</strong></td>
</tr>
<tr>
<td>Shut-down policy for the entire horizon (for all time periods)</td>
<td>Decisions corresponding to the operating policy for the first time period</td>
</tr>
<tr>
<td>INTEGRALITY IN THE FIRST-STAGE</td>
<td>INTEGRALITY IN THE FIRST-STAGE</td>
</tr>
<tr>
<td><em>Time Periods: 1,2,3,…</em></td>
<td><em>Time Periods: 1</em></td>
</tr>
<tr>
<td></td>
<td>Decisions corresponding to the operating policy for the remaining time period</td>
</tr>
<tr>
<td></td>
<td>INTEGRALITY IN THE SECOND-STAGE</td>
</tr>
<tr>
<td></td>
<td><em>Time Periods: 2,3,…</em></td>
</tr>
<tr>
<td>All remaining decisions for the entire horizon</td>
<td><strong>INTEGRALITY IN THE SECOND-STAGE</strong></td>
</tr>
<tr>
<td></td>
<td><em>Time Periods: 2,3,…</em></td>
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# Solving the Extensive Forms

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Constraints</td>
<td>9486</td>
<td>11086</td>
</tr>
<tr>
<td># of Variables</td>
<td>16340</td>
<td>20444</td>
</tr>
<tr>
<td># of Nonzeros</td>
<td>54736</td>
<td>84038</td>
</tr>
<tr>
<td>Time</td>
<td>~20 sec.</td>
<td>~30 sec.</td>
</tr>
</tbody>
</table>
Future Work

• Implement the L-shaped method for solving two-stage stochastic programs
  – This is not necessarily straightforward in a modeling language like AIMMS (although they say it is)
  – However, solving the extensive form will limit the number of scenarios and stages that can be considered
Multistage SMIP

• One approach is to solve the multistage SMIP using Lagrangian relaxation of nonanticapivity constraints
  – There will be many such constraints, even with few scenarios/stages
  – There will (surely) be a duality gap
  – Finding an exact solution for such a large problem is well beyond the scope of the current state of the art
Solving SPs with Nonanticipativity

• Such a model is decomposable by scenario, where nonanticipativity constraints are linking constraints
• Lagrangian relaxation of linking constraints
• For reasonably large scenario trees, the number of possible nonanticipativity constraints is enormous
Nonanticapitivity

Scenarios 1 and 2 are *indistinguishable* in stage 2.

Stage 1

\[ x^1(\omega^1) = x^1(\omega^2) = x^1(\omega^3) = x^1(\omega^4) \]

Stage 2

\[ x^2(\omega^1) = x^2(\omega^2) \]

Stage 3

Scenario 1

\[ x^3(\omega^1) \]

Scenario 2

\[ x^3(\omega^2) \]

Scenario 3

\[ x^3(\omega^3) \]

Scenario 4

\[ x^3(\omega^4) \]

Separable by scenario with nonanticapitivity constraints as linking constraints
Nested Benders’ for MSLP

• For a multi-stage SLP, much more is known
• While the nested Benders’ procedure gives an optimal answer, many computational questions remain
• The downside is that all recourse decisions must be continuous
Solving Multistage SPs

Nested Decomposition

• Built on the two-stage L-shaped method
• Extended to the multistage case by Birge
• The idea is to place cuts on $\mathcal{G}^{t+1}(x^t)$ and to add other cuts to achieve an $x^t$ that has a feasible completion in all descendant scenarios
• Successive linear approximations of $\mathcal{G}^{t+1}$
• Due to the polyhedral structure of $\mathcal{G}^{t+1}$, the process converges finitely