An Approximate Dynamic Programming Approach to Benchmark Practice-based Heuristics for Natural Gas Storage Valuation

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Joint work with Guoming Lai and Francois Margot
The Natural Gas (NG) Value Chain

Focus of this talk
NG “Warehouses”

A – Salt Caverns  B – Aquifers  C – Depleted Reservoirs
NG Storage Contracts

Storage operators lease their capacity (space and injection/withdrawal capacity) to shippers through storage contracts.

The valuation of these contracts is an important problem in practice.

The value of a storage contract is the value of the cash flows that the contract owner can realize during the contract lifetime (1-5 years).

Wild Goose Storage, Northern California (depleted Wild Goose natural gas field)

The following was retrieved on 4/8/2008:

SERVICES AND RATES: Our rates are 'market based' meaning they are fully negotiable, but our 'rack rates' (suggested retail prices) are currently as follows:

- Monthly Reservation Charges
  - Inventory ($/Dth) 0.03
  - Injection ($/Dth/day) 3.00
  - Withdrawal ($/Dth/day) 2.00
- Variable Charges ($/Dth) 0.04
- Fuel (approximately) 1%
NG Storage Facilities & Markets in North America

Locations of natural gas storage facilities

New York Mercantile Exchange (NYMEX) Trading Points

These are **forward** markets, about 40
There are also about 100 **spot** markets
NG Storage Contract Valuation

Shippers, in particular merchants such as **Sempra Commodities, BP Trading, Goldman Sachs**, etc., manage NG storage contracts as **real options** on NG prices.

They can be valued by applying risk neutral valuation ideas (Harrison and Kreps 1979, Harrison and Pliska 1981).

In principle, the **idea is simple**: Buy low, inject, store, withdraw, and sell high ...

... but there are practical **difficulties**

**Constraints** on minimum/maximum storage **space**

and injection/withdrawal **capacity**

Modeling the **evolution** of NG **prices**
NG Futures Contracts

NYMEX NG Futures Prices (Forward Curves) in November 2006

$/MMBtu

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**N-dimensional Black (1976) Model**

- This is a price model typically used in practice

- Each price in the forward curve is modeled as a driftless geometric Brownian motion driven by the standard Brownian motion increment $dZ_i(t)$ with constant maturity specific volatility $\sigma_i$

- The Brownian motions of maturities $i$ and $j$ are correlated with instantaneous correlation parameter $\rho_{ij}$

\[
\frac{dF(t, T_i)}{F(t, T_i)} = \sigma_i dZ_i(t), \ \forall i \in \mathcal{I} \\
dZ_i(t)dZ_j(t) = \rho_{ij} dt, \ \forall i, j \in \mathcal{I}, \ i \neq j
\]
Issues

Stochastic dynamic programming (DP) is the “natural” approach to the valuation of a NG storage contract
- Contract valuation = DP value function in the initial stage and state

Stochastic DP is intractable when a high dimensional price model is employed to describe the stochastic evolution of the NG price, such as the N-dimensional Black (1976) model used in practice

Thus, practitioners value natural gas storage heuristically
Research Questions

How good are these heuristics?
- This is not known because good upper bounds on the valuation of a NG storage contract are unknown

Can these heuristics be improved?

These are practically relevant questions
- This is an e-mail that I recently received from a VP of a major NG merchant company:

“Are you ever going to get a dynamic storage model built for the natural gas industry? We still need one!”
Practice-based Heuristics

- Deterministic DP model without and with reoptimization
  - Intrinsic value model, rolling intrinsic value model

- Linear program (LP) with spread options without and with reoptimization
  - Basket of spreads, rolling basket of spreads

Intrinsic Value Model: Deterministic DP

I: Intrinsic

\[ U^I_N(x_N; F_0) := 0, \quad \forall x \in \mathcal{X} \]
\[ U^I_i(x_i; F_0) := \max_{a \in \mathcal{A}(x_i)} r(a, F_{0,i}) + \delta U^I_{i+1}(x_{i-1} - a; F_0), \quad \forall i \in \mathcal{I}, \forall x \in \mathcal{X} \]

This model computes the intrinsic value of storage (the value of seasonality)

Reoptimizing this model at the beginning of each stage (month) with updated price information yields a state dependent policy

The value of this policy can be computed by Monte Carlo simulation
Spread Options

Time $T_0$ value of an $i$-$j$ option on the spread between prices $F_{ij}$ and $F_{ii}$

$$S_{0}^{i,j}(F_0) := \delta^i E \left\{ \left( \delta^{j-i} \alpha W \tilde{F}_{i,j} - \left( \alpha^I \tilde{F}_{i,i} + \delta^{j-i} c^W + c^I \right) \right)^+ \right| F_0 \right\}$$

The spread option is valued at time $T_0$

Buy 1 unit of NG at time $T_i$ if advantageous relative to selling it at time $T_j$

Sell any NG that was bought at time $T_i$

No exact closed form formulas available

We use a closed form approximation due to Kirk (1995)
## Portfolio of Spread Options

Think of a storage contract as a constrained portfolio of spread options

$q_{i,j}$: Notional amount of spread option $i$-$j$

<table>
<thead>
<tr>
<th>Injection Maturity</th>
<th>Withdrawal Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$q_{0,1}$</td>
</tr>
<tr>
<td></td>
<td>$q_{0,j}$</td>
</tr>
<tr>
<td></td>
<td>$q_{0,N-1}$</td>
</tr>
<tr>
<td>$i$</td>
<td>$q_{i,j}$</td>
</tr>
<tr>
<td></td>
<td>$q_{i,N-1}$</td>
</tr>
<tr>
<td>$N-2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$q_{N-2,N-1}$</td>
</tr>
</tbody>
</table>
### Spread Option-based LP

Use an LP to find the “best” constrained portfolio of spread options

\[
U^{LP}_0(\mathcal{F}_0) := \max_{q, x} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}, i < j} S^{i,j}_0(\mathcal{F}_0) q_{i,j} \quad \text{Portfolio value}
\]

**Inventory balance**
\[
\text{s.t. } x_{i+1} = x_i + \sum_{j \in \mathcal{I}, j > i} q_{i,j} - \sum_{j \in \mathcal{I}, j < i} q_{j,i}, \quad \forall i \in \mathcal{I}
\]

**Max inventory**
\[
x_i \leq \bar{x}, \quad \forall i \in \mathcal{I} \cup \{N\}
\]

**Max injection**
\[
\sum_{j \in \mathcal{I}, j > i} q_{i,j} \leq -C^I, \quad \forall i \in \mathcal{I} \setminus \{N - 1\}
\]

**Max withdrawal**
\[
\sum_{i \in \mathcal{I}, i < j} q_{i,j} \leq C^W, \quad \forall j \in \mathcal{I} \setminus \{0\}
\]

**Spread option notional**
\[
q_{i,j} \geq 0, \quad \forall i, j \in \mathcal{I}, \ i < j
\]

**Min inventory**
\[
x_i \geq 0, \quad \forall i \in \mathcal{I} \cup \{N\}
\]

(1) The optimal solution to this LP can be used to construct an admissible policy

(2) The value of this policy can be computed by Monte Carlo simulation

(3) The LP can be reoptimized at the beginning of each stage to obtain an improved policy
Our Approach

• Tractable approximate stochastic DP (ADP)
  • Information reduction
  • Value function approximation
  • Without and with reoptimization

• Upper bound
  • We use the value function of the ADP model without reoptimization in conjunction with the information relaxation and duality approach of Brown et al. (2008)
ADP Model

\[ U^\text{ADP}_N(x_N, s_N) := 0, \quad \forall x_N \in \mathcal{X} \]

\[ U^\text{ADP}_i(x_i, s_i) = \mathbb{E} \left[ u^\text{ADP}_i(x_i, s_i, \tilde{F}_{i+1}) \mid s_i, F_{0,i+1} \right], \quad \forall i \in \mathcal{I}, \quad (x_i, s_i) \in \mathcal{X} \times \mathbb{R}_+ \]

\[ u^\text{ADP}_i(x_i, s_i, F_{i+1}) := \max_{a \in \mathcal{A}(x_i)} r(a, s_i) + \delta \mathbb{E} \left[ U^\text{ADP}_{i+1}(x_i - a, \tilde{s}_{i+1}) \mid F_{i+1} \right] \]

This model yields a feasible policy with a basestock-type structure.

The “true” value of this policy can be evaluated by Monte Carlo simulation.

It yields a lower bound on the value of storage.
ADP-based Upper Bound

ADP based penalties (based on Brown et al. 2008)

\[ p_i^{ADP}(x_i, a; F_{i+1}, s_{i+1}) := U_{i+1}^{ADP}(x_i - a, s_{i+1}) \]

\[ -\mathbb{E} \left[ U_{i+1}^{ADP}(x_i - a, \tilde{s}_{i+1}) \mid F_{i+1} \right], \]

\[ \forall i \in \mathcal{I}, (x_i, a) \in \mathcal{X} \times \mathcal{A}(x_i) \]

Forward curves sample path

\[ \mathcal{G} := (F_0, F_1, \ldots, F_{N-1}) \]

DUB: Dual Upper Bound

\[ U_N^{DUB}(x_N; \mathcal{G}) := 0, \forall x_N \in \mathcal{X} \]

\[ U_i^{DUB}(x_i; \mathcal{G}) = \max_{a \in \mathcal{A}(x_i)} r(a, s_i) - p_i^{ADP}(x_i, a; F_{i+1}, s_{i+1}) + \delta U_{i+1}^{DUB}(x_i - a; \mathcal{G}), \]

\[ \forall i \in \mathcal{I}, x_i \in \mathcal{X} \]

DUB

\[ V_0^{DUB}(\mathcal{F}_0) := \mathbb{E} \left[ U_0^{DUB}(x_0; \tilde{\mathcal{G}}) \right] \]

DUB can be computed by Monte Carlo simulation
Summary of Bounds

• Practice-based lower bounds
  – Intrinsic: I
  – Reoptimized I: RI
  – LP
  – Reoptimized LP: RLP
  – **NOTE**: In the following charts you will also see LPN and RLPN; these are modified LP/RLP models

• Our lower bounds
  – ADP
  – Reoptimized ADP: RADP

• Our dual upper bound: DUB
Instances: Forward Curves and Volatilities

2006 NYMEX Data – Maximum number of maturities = 24 (months)
Instances: Operational Parameters

- Minimum inventory: 0
- Scaled maximum inventory: 1

- Injection and withdrawal capacity parameters (fraction of maximum inventory)

<table>
<thead>
<tr>
<th>Case</th>
<th>Injection Capacity (units/month)</th>
<th>Withdrawal Capacity (units/month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
<td>0.30</td>
</tr>
<tr>
<td>2</td>
<td>0.30</td>
<td>0.60</td>
</tr>
<tr>
<td>3</td>
<td>0.45</td>
<td>0.90</td>
</tr>
</tbody>
</table>

- Number of stages (months): 12 or 24

- We generate 24 instances by varying the relevant parameters
Valuation Results: 24 Stage Instances w/o Reoptimization
Valuation Results: 24 Stage Instances w/ Reoptimization

(a) Spring

(b) Summer

(c) Fall

(d) Winter

% of DUB

Capacity Pair

Radp, RI, RLP, RLPN
## Computational Requirements: 24 Stage Instances

<table>
<thead>
<tr>
<th></th>
<th>CPU Seconds</th>
<th>Overall average</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Before Reoptimization</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ADP</td>
<td>140.01</td>
<td>0.54</td>
</tr>
<tr>
<td>I</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>LP</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td><strong>With Reoptimization</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RADP</td>
<td>595.73</td>
<td>23.90</td>
</tr>
<tr>
<td>RI</td>
<td>23.90</td>
<td>278.74</td>
</tr>
<tr>
<td>RLP</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(1) RI achieves the best balance between valuation quality and speed

(2) Our LB with reoptimization is slightly better than RI but slower
Conclusions

• Contributions
  – Developed new ADP lower bound and very tight upper bound for NG storage valuation using a price model employed in practice
  – Benchmarked practice-based heuristics
  – Shown that models used in practice perform very well provided that they are reoptimized

• Current developments
  – Does one really need N factors, i.e., the N-dimensional Black (1976) model? Are more parsimonious futures price models sufficient?
  – Generalization to more complex real options (assets), e.g., refineries and chemical complexes
Ex.: Real Option Management of a Refinery

At a basic level, an oil refinery processes crude oil into products, say, diesel, gasoline, and jet fuel, and maintains both input and output inventory.

The inputs and outputs are traded both on physical and financial (futures) markets.

At the planning level, the refinery can be managed as a complex real option on price spreads (product prices – crude oil price) to maximize its market value during a given time horizon.
- Inventory to buy and sell, inventory to process.
- One can also account for customer demands.

This problem can be formulated as a large scale finite-horizon DP.

The optimal value function in stage $j$, $V_j(x,y_1,y_2,y_3,F_{ij},G_{1,ij},G_{2,ij},G_{3,ij})$, depends on
- $x$: crude oil inventory
- $y_i$: product i inventory, $i = 1, 2, 3$ for diesel, gasoline, and jet fuel
- $F_{ij}$: crude oil forward curve in stage $j$
- $G_{ij}$: product i forward curve in stage $j$
Math Programming Based ADP

Write the DP as an LP: This is standard

Postulate a value function approximation using basis functions

Reformulate the LP with the basis functions as decision variables, typically exploiting duality and decomposition (column generation and cutting planes)
- Adelman (2006)

This provides a suboptimal policy for the original problem, as well as a bound on its suboptimality

I am developing this approach to manage complex energy and commodity real options with my colleagues Francois Margot (OR) and Duane Seppi (Finance)

We welcome input from practitioners on
- This problem and other relevant business problems
- Data

We also welcome student interest
Supplemental Technical Slides
Model-based NG Storage Contract Valuation

Risk neutral valuation of a NG storage contract
- Dynamic optimization of inventory trading decisions with capacity constraints in the face of uncertain NG futures price dynamics
- Financial replication based on trading NG futures

Stochastic dynamic programming (DP) is the “natural” approach to this valuation problem

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>N – 1</th>
</tr>
</thead>
</table>

Stages

Periodic review, finite horizon model:
- **State**: the merchant’s inventory level and the forward curve
- **Decisions**: buy and inject, do nothing, or withdraw and sell
- **Constraints**: space and capacity limits
Some Notation

The forward curve has $N$ maturities 0, 1, ..., $N - 1$ indexed by set $I$

$F(t,T_i)$: time $t$ futures price for delivery at maturity $i$ (time $T_i$)

Shorthand notation $F_{i,j}$: time $T_i$ futures price for maturity at time $T_j$

$F_{i,i} = s_i$: spot price at time $T_i$

Set $I$ will be used to define the stages of different DPs
Per Stage Reward

$s$: spot price, $a$: action, $r(a,s)$: per stage reward function

\[
r(a, s) := \begin{cases} 
    (\alpha^I s + c^I) a & \text{if } a \in \mathbb{R}_- \\
    0 & \text{if } a = 0, \forall s \in \mathbb{R}_+ \\
    (\alpha^W s - c^W) a & \text{if } a \in \mathbb{R}_+ 
\end{cases}
\]

It never makes sense to buy and sell at the same time.
Space and Capacity Constraints

- Max Inventory
- Withdrawal Capacity
- Injection Capacity
- Max Inventory

Injection Capacity Constraint
Withdrawal Capacity Constraint
Space Constraint
Action
Inventory

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Intractable DP

Forward curve \((N - i)\)-dimensional vector (it includes the spot price \(s_i\)):

\[
\mathbf{F}_i := (F_{i,j}, j \in \mathcal{I}, j \geq i)
\]

Stochastic DP:

\[
V_N(x_N, \mathbf{F}_N) := 0, \forall x_N \in \mathcal{X}
\]

\[
V_i(x_i, \mathbf{F}_i) = \max_{a \in \mathcal{A}(x_i)} r(a, s_i) + \delta \mathbb{E}[V_{i+1}(x_i - a, \tilde{\mathbf{F}}_{i+1})|\mathbf{F}_i],
\]

\[
\forall i \in \mathcal{I}, (x_i, \mathbf{F}_i) \in \mathcal{X} \times \mathbb{R}_+^{N-i}
\]

This model is intractable since \(N\) is at least 12 in practice
This structure significantly facilitates the computation of the ADP optimal policy.