

Facility Location Models: An Overview

1

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Outline

2

- **Introduction**
- **Taxonomy of location models**
- **IP formulations for some classical models**
- **Algorithms**
- **Extensions**
- **Facility location / network design software**

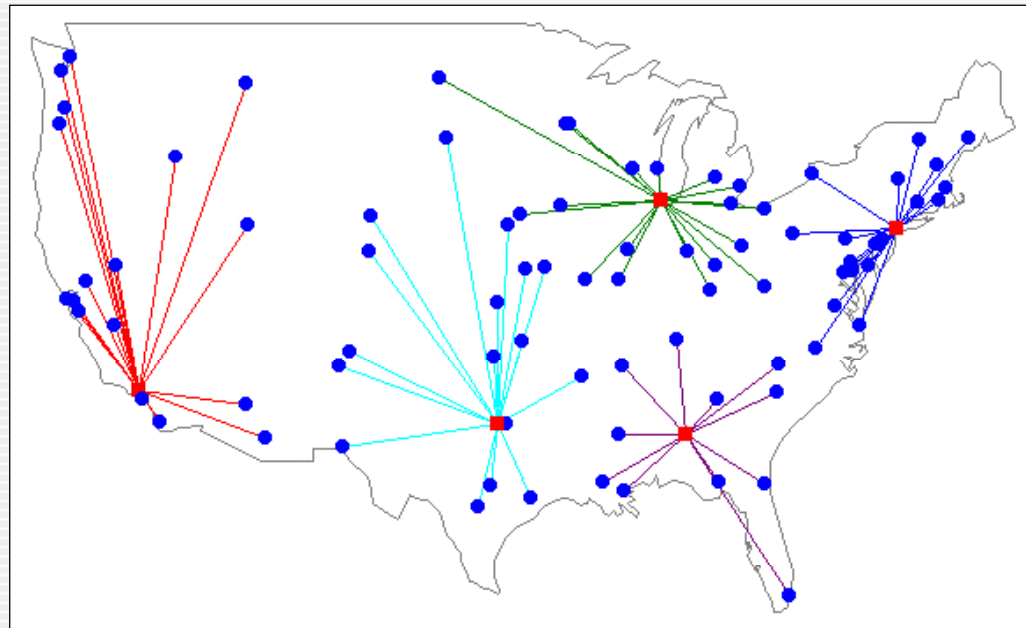
Introduction

3

Overview

4

- **Decide where to locate facilities**
 - (factories / warehouses / DCs / retail outlets / etc.)
- **To serve customers**
- **In order to achieve some balance between**
 - Cost
 - Service



Decisions

5

- **Usually 2 decisions to make:**
 - Where to locate?
 - Which customers are assigned/allocated to which facilities?
- **Sometimes referred to as “location–allocation models”**

Applications of Facility Location Models

6

- **Widely applied in public and private sectors:**
 - Emergency medical services (EMS) / fire stations
 - Airline hubs
 - Blood banks
 - Hazardous waste disposal sites
 - Fast-food restaurants
 - Public swimming pools
 - Schools
 - Vehicle inspection stations
 - Bus stops
 - etc.

Uses for Facility Location Models

7

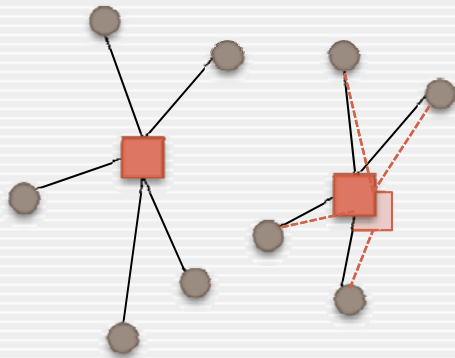
- **Also applied to “virtual facilities”:**
 - Wildlife reserves
 - Satellite orbits
 - Apparel sizes
 - Flexible manufacturing system tool selection
 - Location of bank accounts
 - Political party platforms
 - Product positioning
 - etc.
- **Sometimes arise as subproblems for other OR problems**
 - Vehicle routing

Taxonomy of Location Models

8

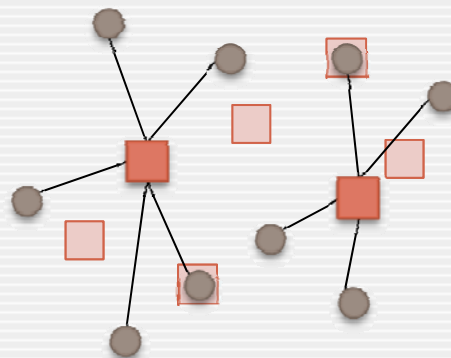
Topology

9



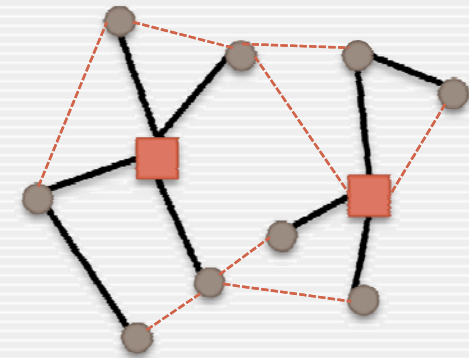
Continuous

- Locate anywhere on plane
- Continuous, non-linear optimization
- “Weber problem”



Discrete

- Locate at pre-defined points
- Integer programming



Network

- Locate anywhere on network
- Travel along arcs
- Integer programming
- **Hakimi property:** Optimal to locate at nodes
 - ✦ Holds for some (not all) problems

We'll consider **discrete problems**
(Network problems are a special case)

Distance Metric

10

- **Euclidean:** $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
- **Rectilinear / Manhattan:** $|x_1 - x_2| + |y_1 - y_2|$
 - (travel along streets)
- **Great circle:** accounts for Earth's curvature
- **Highway / network:** shortest path within network
 - (e.g., U.S. highway network)
- **Matrix:** distance between each pair given explicitly

- For sake of generality, we'll assume **matrix distances**
- Also, “distance” = “transportation cost”

Distance Objective

11

- **Total distance:** total distance between customers and their assigned facilities
 - (distance is usually demand-weighted)

$$\sum_{\text{customers}} \text{distance to assigned facility}$$

- **Maximum distance:** maximum distance between a customer and its assigned facility
 - (distance is usually unweighted)

$$\max_{\text{customers}} \{ \text{distance to assigned facility} \}$$

- **Coverage:** cust. is “covered” if distance \leq specified radius
- Can appear in objective function or constraints

Preventing Too Many Facilities

12

- **Fixed cost:** fixed (annual) cost to open/operate facility
 - Represents construction / leasing cost + overhead (lights, heat, security, etc.)
 - Independent of volume of demand served by facility
- **Restriction on # facilities:** require # of facilities $\leq P$ in constraints

Classical Models

13

- ***P*-median problem:** minimize demand-weighted distance
s.t. locate $\leq P$ facilities
- **Uncapacitated fixed-charge location problem (UFLP):**
minimize fixed cost + DWD
- ***P*-center problem:** minimize maximum distance
s.t. locate $\leq P$ facilities
- **Set covering location problem (SCLP):**
minimize # of facilities
s.t. cover all customers
- **Maximum covering location problem (MCLP):**
maximize covered demands
s.t. locate $\leq P$ facilities

Capacity

14

- **Most models also have a capacitated version**
 - Facilities have fixed throughput capacity
- **Capacity is usually an input**
- **But sometimes a decision variable**
 - Discrete choices (50,000 sq ft / 100,000 sq ft / 200,000 sq ft)
 - Continuous variable (cost is a function of capacity)

IP Formulations for Some Classical Models

15

Notation

16

- **Sets**

- $I = \{\text{customers}\}$
- $J = \{\text{potential facility sites}\}$

- **Parameters**

- $h_i =$ annual demand of customer $i \in I$
- $c_{ij} =$ cost to transport one unit from $j \in J$ to $i \in I$
- $f_j =$ fixed (annual) cost to open a facility at site $j \in J$

- **Decision variables**

- $x_j = 1$ if facility $j \in J$ is opened, 0 otherwise
- $y_{ij} = 1$ if facility $j \in J$ serves customer $i \in I$, 0 otherwise

P-Median Formulation

17

$$\min \sum_{i \in I} \sum_{j \in J} h_i c_{ij} y_{ij}$$

$$\text{s.t. } \sum_{j \in J} y_{ij} = 1 \quad \forall i$$

$$y_{ij} \leq x_j \quad \forall i, j$$

$$\sum_{j \in J} x_j = P$$

$$x_j \in \{0,1\} \quad \forall j$$

$$y_{ij} \in \{0,1\} \quad \forall i, j$$

Min demand-weighted distance
(transportation cost)

Satisfy all demands

Don't assign cust to closed facility

Locate P facilities

Integrality

UFLP Formulation

18

$$\min \sum_{j \in J} f_j x_j + \sum_{i \in I} \sum_{j \in J} h_i c_{ij} y_{ij}$$

Min fixed + transportation cost

$$\text{s.t. } \sum_{j \in J} y_{ij} = 1 \quad \forall i$$

Satisfy all demands

$$y_{ij} \leq x_j \quad \forall i, j$$

Don't assign cust to closed facility

$$x_j \in \{0,1\} \quad \forall j$$

Integrality

$$y_{ij} \in \{0,1\} \quad \forall i, j$$

Maximal Covering Formulation

19

$$\max \sum_{i \in I} h_i z_i$$

$$\text{s.t. } \sum_{j \in J} x_j = P$$

$$z_i \leq \sum_{j \in V_i} x_j \quad \forall i$$

$$x_j \in \{0,1\} \quad \forall j$$

$$z_i \in \{0,1\} \quad \forall i$$

Maximize covered demand

Locate P facilities

Definition of coverage

Integrality

where V_i = set of facilities that can cover customer i
 $z_i = 1$ if customer i is covered, 0 otherwise

Algorithms

20

Algorithms

21

- **Most facility location problems are NP-hard**
- **But many classical problems are “easy” computationally**
 - LP relaxations are often extremely tight
 - Sometimes integer solutions “for free”
- **Virtually every type of algorithm for discrete optimization has been applied to facility location**

Algorithms

22

- **Heuristics:**
 - Greedy add/drop
 - Swap
 - Neighborhood search
 - Metaheuristics
 - ✦ Genetic algorithms, tabu search, variable neighborhood search, simulated annealing, ant algorithms, bee algorithms, ...
- **Exact Algorithms:**
 - Branch and bound
 - Cutting planes
 - Benders decomposition
 - Column generation / Dantzig-Wolfe decomposition
 - Lagrangian relaxation

Lagrangian Relaxation for P -Median

23

$$\min \sum_{i \in I} \sum_{j \in J} h_i c_{ij} y_{ij}$$

$$\text{s.t. } \sum_{j \in J} y_{ij} = 1 \quad \forall i$$

$$y_{ij} \leq x_j \quad \forall i, j$$

$$\sum_{j \in J} x_j = P$$

$$x_j \in \{0,1\} \quad \forall j$$

$$y_{ij} \in \{0,1\} \quad \forall i, j$$

 RELAX

Lagrangian Subproblem

24

$$\min \sum_{i \in I} \sum_{j \in J} h_i c_{ij} y_{ij} + \sum_{i \in I} \lambda_i \left(1 - \sum_{j \in J} y_{ij} \right)$$

$$= \sum_{i \in I} \sum_{j \in J} (h_i c_{ij} - \lambda_i) y_{ij} + \sum_{i \in I} \lambda_i$$

$$\text{s.t. } y_{ij} \leq x_j \quad \forall i, j$$

$$\sum_{j \in J} x_j = P$$

$$x_j \in \{0,1\} \quad \forall j$$

$$y_{ij} \in \{0,1\} \quad \forall i, j$$

Facility- j Subproblem

25

- Subproblem is separable by j
- Suppose we open j ; need to solve

$$\begin{aligned} \min \quad & \sum_{i \in I} (h_i c_{ij} - \lambda_i) y_{ij} \\ \text{s.t.} \quad & y_{ij} \in \{0,1\} \end{aligned}$$

- Easy—solve by inspection:

- Would set $y_{ij} = 1$ iff $h_i c_{ij} - \lambda_i < 0$
- “Benefit” of opening j is

$$\beta_j = \sum_{i \in I} \min\{0, h_i c_{ij} - \lambda_i\}$$

- Open P facilities with smallest β_j
- This gives lower bound
- Obtain upper bound from heuristic
- Update λ and repeat

Extensions

26

Other Flavors

27

- **Obnoxious facilities**
 - Obnoxious location: Maximize distance from facilities to customers
 - Dispersion: Maximize distance among facilities
- **Competitive location**
 - Multiple players try to capture demand by locating facilities
- **Multi-objective models**
 - Account for multiple stakeholders' objectives
- **Hub location**
 - Flows from facilities to customers but *also* among facilities
 - Quadratic objective
- **Dynamic location**
 - Facilities are located over time, or move over time

Uncertainty

28

- **Types of randomness:**
 - Demand-side: Randomness in demands, costs, etc.
 - Supply-side: Randomness in supply (e.g., disruptions)
- **Approaches to uncertainty:**
 - Stochastic programming: min expected cost
 - Robust optimization: minimax cost, minimax regret, CVaR, etc.
- **Modeling approaches:**
 - Scenario formulations
 - Interval uncertainty

Integrated Models

29

- Incorporate tactical / operational costs into strategic (facility location) decisions
 - Inventory
 - Routing
- Integrated location-inventory model
 - Daskin, Coullard, and Shen (*Ann OR*, 2002)
 - Objective function includes two concave terms:
 - ✦ Inventory economies of scale (EOQ)
 - ✦ Risk pooling (safety stock)
 - Constraints are same as UFLP
 - Solve via Lagrangian relaxation
 - ✦ Subproblem solved in $O(|I| \log |I|)$ time for each j
 - ✦ (UFLP: $O(|I|)$ time for each j)

Network Design

30

- **Multi-echelon facility location models**
 - Make open/close decisions for multiple tiers
 - Geoffrion and Graves (*MS*, 1974)
- **Generalization: network design problems**
 - Usually locate arcs in the network
 - ✦ But locating nodes is equivalent
- **Rich literature on network design**
 - e.g., Magnanti and Wong (*TS*, 1984)

Facility Location / Network Design Software

31

Software Packages

32

- **LogicNet / LogicNet Plus**
 - Originally LogicTools
 - Now ILOG Supply Chain Applications, part of IBM Consulting
- **SAILS**
 - INSIGHT
- **SAP, Oracle modules**

Capabilities

33

- **Key decisions:**
 - Facility locations, capacities, capabilities, volumes
 - Distribution lanes (yes/no, volumes)
 - Make vs. buy
- **Key features:**
 - Data import
 - Output report export
 - GUI, GIS
 - Optimization solver
 - Rating engine
 - What-if scenarios

Recent Developments

34

- **New features:**
 - “Green”
 - Risk management
 - Taxes
 - Seasonality
 - Cost vs. service level tradeoffs
- **Things most current software can’t do (very well):**
 - Non-linearities (e.g., quantity discounts)
 - Open/close decisions on arcs for larger models
 - Close integration with inventory modeling
 - Uncertainty
 - ✦ Optimization under multiple scenarios
 - ✦ Risk-averse objectives

Further Reading

35

- **Textbooks on facility location**
 - Daskin (1995)
 - Drezner (1995)
 - Drezner and Hamacher (2001)
- **Review articles / book chapters**
 - Discrete location: Current, Daskin, and Schilling (D&H book, 2001)
 - Continuous location: Drezner, et al. (D&H book, 2001)
 - Stochastic location: Snyder (*IIE Trans*, 2006)
 - Location with disruptions: Snyder, et al. (INFORMS Tutorial, 2006)
 - Location-inventory models: Shen (*JIMO*, 2007)

Questions?

36

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