Planning and scheduling of spectrally selective tinted glasses production

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Aim of the project

- Development of an **MILP model** for the **integrated planning and scheduling** of the production line of spectrally selective glasses
  - **capture the essence** of the process that **is not considered** in the Master Production Schedule (MPS)
    - cullet management
    - changeovers driven by SKUs
  
- that may lead to **infeasible schedules**

- develop a production tool to perform systematic choices of production schedules based on performance criteria
Aim of the project

- Development of an **MILP model** for the **integrated planning and scheduling** of the production line of spectrally selective glasses
  - capture the essence of the process that is not considered in the Master Production Schedule (MPS)
    - cullet management
    - changeovers driven by SKUs
  that may lead to infeasible schedules
  - develop a production tool to perform systematic choices of production schedules based on performance criteria

- **Conduct economic studies**
  - Evaluate the **maximum profitability** of the production line
  - Analyze the possibility of producing **more products** in the same production line
  - Try to evaluate the **production cost** and **profit contribution** of each product
Process description

Continuous process

Raw materials → Refiner → Tin bath → Coater → Annealing → Ribbon → Products

Characteristics

- sequence dependent transitions
- long transition times (order of days)
- high cost transitions
- no transition time between substrate and substrate with coating
- coater cleaning
- process does not stop

Spectrally selective tinted glasses

<table>
<thead>
<tr>
<th>Substrate</th>
<th>Coated*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solexia</td>
<td></td>
</tr>
<tr>
<td>Caribia</td>
<td></td>
</tr>
<tr>
<td>Azuria</td>
<td></td>
</tr>
<tr>
<td>Solarbronze</td>
<td></td>
</tr>
<tr>
<td>Solargray</td>
<td></td>
</tr>
<tr>
<td>Graylite</td>
<td></td>
</tr>
</tbody>
</table>

* Reflective coated glass
Continuous process

The products are defined by stock keeping unit (SKU), among others characterized by:

- **Dimensions**
- **Thickness**
- **Orientation**
- **Quality**

**Color**
Stage I - Simplified model

Assume that the products are only defined by colors and coating, and

- production rates
- maximum and minimum inventory levels
- demand
- selling prices

are based on the amount (ton) of colors.

Define strategy for the integrated planning and scheduling, this may involve:

- decomposition approach
- forward rolling horizon algorithm
- Lagrangean decomposition
Project complexity stages

Stage I - Simplified model

Stage II - Detailed model

1 - Include cullet storage, recycling, and sales

Cullet: glass produced during transitions, broken, defects

2 - Define the products by SKU, and consider SKUs inventory and sales levels

At present, both are not considered by the MPS
Problem statement

Given

- deterministic product **demands** over time
- initial, minimum, and maximum **inventory** levels of products
- **production rates**
- sequence dependent **transitions**
- operating, inventory, and transition **costs**
- **selling prices**

Determine

- amounts to be produced
- production times
- sequence of production
- inventory levels
- sales

That **maximize profit**
Model

Assumptions

Process

- Neglect of transition times between substrate and substrate with coating
- Equal production times of substrate and substrate with coating in the same time period, to clean the coater
- Minimum run lengths for the total of substrate and substrate with coating
- Maximum inventory capacity

<table>
<thead>
<tr>
<th>Substrate</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Solexia</td>
<td>Green</td>
</tr>
<tr>
<td>Caribia</td>
<td>Cyan</td>
</tr>
<tr>
<td>Azuria</td>
<td>Cyan</td>
</tr>
<tr>
<td>Solarbronze</td>
<td>Brown</td>
</tr>
<tr>
<td>Solargray</td>
<td>Gray</td>
</tr>
<tr>
<td>Graylite</td>
<td>Black</td>
</tr>
</tbody>
</table>
Assumptions

Formulation

- slot based continuous time representation
- variable length slots
- one product per slot
- products may take more than one slot
- fixed number of slots per time period
- transition times across time periods (extension of Erdirik-Dogan and Grossmann (2007))
MILP Model

Transitions across time periods

\[ \tau_{i,k} T R T_{i,k,t} = \tau e_{i,k,t} + \tau s_{i,k,t} \quad \forall i, k, \forall t \in T \setminus \{tm\} \]

\[ T e_{l,m,t} + \sum_{i} \sum_{k} \tau e_{i,k,t} = H T_{t} \quad \forall l, t \in T \setminus \{tm\} \]

\[ T s_{l,t+1} = \sum_{i \in l} \sum_{i \in l} \tau s_{i,k,t-1} + H T_{t-1} \quad l = 1, \forall t \in T \setminus \{tm\} \]

\[ \tau e_{i,k,t} \leq \tau_{i,k} T R T_{i,k,t} \quad \forall i, k, \forall t \in T \setminus \{tm\} \]

\[ \tau s_{i,k,t} \leq \tau_{i,k} T R T_{i,k,t} \quad \forall i, k, \forall t \in T \setminus \{tm\} \]

Coating application, and coater cleaning

\[ Y O P_{i,t} - Y O P_{k,t} \geq 0 \quad \forall i, k \in F A M \]

\[ \sum_{l} \theta_{i,l,t} \geq \sum_{l} \theta_{k,l,t} - M (1 - Y O P_{k,t}) \quad \forall i, k \in F A M, \forall t \]

Minimum run lengths

\[ \sum_{l \in L} \sum_{i \in L} \sum_{k \in F A M} \theta_{k,l,t} \geq M R T_{i} \cdot Y O P_{i,t} \quad \forall i \in COLOR, \forall t \]

Maximum inventory capacity

\[ \sum_{i} I N V O_{i,t} \leq I N V_{\text{max}} \quad \forall t \]
**MILP Model**

Maximize profit:

\[ Z^P = \sum_{i} \sum_{t} M P_{i,t} S_{i,t} - \sum_{i} \sum_{t} C I N V_{i,t} A_{i,t} - \sum_{i} \sum_{t} C O P_{i,t} X_{i,t} - \sum_{i} \sum_{t} c_{i,k}^{\text{trans}} Z_{i,k,l,t} - \sum_{i} \sum_{t} c_{i}^{\text{trans}} Z_{i,k,l,t} \]

subject to:

\[ \sum_{l} W_{i,l,t} = 1 \quad \forall i, t \]

\[ 0 \leq \tilde{\theta}_{i,l,t} \leq H_t W_{i,l,t} \quad \forall i, l, t \]

\[ \theta_{i,t} = \sum_{l} \tilde{\theta}_{i,l,t} \quad \forall i, t \]

\[ X_{i,l,t} = r_i \theta_{i,l,t} \quad \forall i, l, t \]

\[ \tilde{X}_{i,l,t} = \sum_{l} X_{i,l,t} \quad \forall i, l, t \]

\[ \sum_{k} T R T_{i,k,t} = W_{i,l,t} \quad \forall i, l = \bar{l}, t \in T \setminus \bar{t} \]

\[ \sum_{l} T R T_{i,l,t} = W_{i,l,t+1} \quad \forall i, l = \bar{l}, t \in T \setminus \bar{t} \]

\[ \sum_{k} Z_{i,k,l,t} = W_{i,l,t} \quad \forall i, t, l \in L \setminus \bar{l} \]

\[ \sum_{k} Z_{i,k,l,t} = W_{k,l+1,t} \quad \forall i, t, l \in L \setminus \bar{l} \]

\[ I N V_{i,t} = I N V_{I_i} + \sum_{l} r_i \theta_{i,l,t} \quad \forall i, t = 1 \]

\[ I N V_{i,t} = I N V_{O_{i,t-1}} + \sum_{l} r_i \theta_{i,l,t} \quad \forall i, t \neq 1 \]

\[ I N V_{O_{i,t}} = I N V_{i,t} - S_{i,t} \quad \forall i, t \]

\[ A_{i,t} \geq I N V_{O_{i,t}} H_t + r_i \theta_{i,t} H_t \quad \forall i, t \]

Assignment and production

Time relations

Transitions

Demand

Transitions across time periods

Inventory

Coating application

Minimum run lengths

Maximum inventory capacity
Full model results

- 13 products
- time horizon of 6 and 10 months
- 6 slots per time period

<table>
<thead>
<tr>
<th>Variables</th>
<th>Equations</th>
<th>Continuous</th>
<th>Binary</th>
<th>CPU (s)</th>
<th>Profit (m.u.)</th>
<th>Gap (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 months</td>
<td>6,419</td>
<td>10,571</td>
<td>624</td>
<td>9,000</td>
<td>1.131</td>
<td>11.8</td>
</tr>
<tr>
<td>10 months</td>
<td>11,019</td>
<td>17,635</td>
<td>1,040</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Intel Xeon CPU 1.86GHz, 8Gb, GAMS/Cplex 10.2

Remarks:
- large scale model, even though we are still using the simplified model
- 6 months - Cplex stopped on the maximum time set, with a gap of 11.8%
- for 10 months no solution was obtained
- problem becomes intractable for long time horizons
Bi-level decomposition proposed by Erdirik-Dogan and Grossmann I.E. (2007)

**Upper level:** Decide which products are produced in each time period

**Lower level:** Solve the full model for a subset of products in each time period

```
Upper level
Assign products to time periods
Detailed schedule is neglected
Upper bound on profit

Add integer cuts

Assign a subset of products for each period

Lower level
Detailed schedule
Lower bound on profit

UB - LB \leq \varepsilon

N

Y

Stop
```
Case studies

Decomposition strategy

Case 1 - 6 months time horizon
- 13 products
- 6 and 13 slots per time period

Case 2 - 10 months time horizon
- 13 products
- 6 slots per time period
### Computational results

#### Algorithm performance

**Case 1 - 6 months**

<table>
<thead>
<tr>
<th>Iteration</th>
<th>6 slots</th>
<th>13 slots</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UB</td>
<td>CPU (s)</td>
</tr>
<tr>
<td>1</td>
<td>1.196</td>
<td>125.2</td>
</tr>
<tr>
<td>2</td>
<td>1.196</td>
<td>101.6</td>
</tr>
</tbody>
</table>

Intel Xeon CPU 1.86GHz, 8Gb, GAMS/Cplex 10.2

**Case 2 - 10 months**

<table>
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<th>Iteration</th>
<th>6 slots</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UB</td>
</tr>
<tr>
<td>1</td>
<td>1.601</td>
</tr>
</tbody>
</table>

* integrality gap 10.5%

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#### Remarks:

- A number of slots equal to the number of products is not needed because of the large transition times.
- The same profit was obtained for 6 and 13 slots for Case 1.
# Computational results

Comparison between full model solution and decomposition approach

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<tr>
<th>Variables</th>
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<th>CPU (s)</th>
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<tr>
<td><strong>Full model results</strong></td>
<td></td>
<td></td>
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<td>-</td>
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<tr>
<td><strong>Decomposition</strong></td>
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</tr>
<tr>
<td><strong>6 slots</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>6 months</td>
<td>6,498</td>
<td>10,615</td>
<td>580</td>
<td>126.7</td>
<td>1.111</td>
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<tr>
<td>10 months</td>
<td>11,149</td>
<td>17,696</td>
<td>979</td>
<td>9,000</td>
<td>1.363</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Intel Xeon CPU 1.86GHz, 8Gb, GAMS/Cplex 10.2

**Remarks:** 1.77% gap on the profit between full model and decomposition strategy.
Gantt charts

6 months, 6 slots
Profit = 1.111 m.u.

10 months, 6 slots
Profit = 1.363 m.u.

Remarks:
- We are considering sales \( \geq \) demand
- Long transition times
- Transition times on the beginning of time periods
Summary and future work

- **New model** for the planning and scheduling of spectrally selective tinted glasses production
- **Complex model** that requires special decomposition strategies
- Solutions for 6 months time horizon can be *easily obtained*
- 10 months time horizon **challenges current decomposition strategy**

Future work

- Reduce computational burden of the simplified model
- **Improve or develop** a new solution strategy, that can cope with the current model complexity
- Extend the model for **cullet management and SKU**