Integrated Production Planning and Scheduling for Batch Operations

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Enterprise-wide Optimization Project
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AIM OF THE PROJECT

- Develop **Accurate Planning Models** for Batch Operated Plants using mixed integer optimization techniques by integrating planning and scheduling for the Dow Chemical Company.

INTEGRATION OF PLANNING AND SCHEDULING

ULTIMATE GOAL:
- Propose a novel decomposition algorithm to integrate planning and scheduling for Multisite Batch Reactors.
- Ensure optimality and consistency between the two levels.
Multi-Site Problem of the Dow Chemical Company

- Multiple sites
- Multiple markets
- Multiple products
- Price of each product
- Transportation costs to each customer
- Monthly demand forecasts for each product

- Batch reactors in parallel
- Raw material availabilities and costs
- Reactor batch times and batch sizes
- Sequence dependent change-over times
- Intermediate and final storage units

Maximize profit
- What products are shipped to each market from each site
- Total sales of each product in each site
- Monthly production quantities for each reactor
- Assignment of end products to storage tanks
**Focus:** Planning and Scheduling of a Single Site

**Production Site:**
- Raw material availability and Raw material costs
- Storage tanks with associated capacity
- Transportation costs to each customer
- Reactors:
  - Materials it can produce
  - batch sizes (lbs) for each material it can produce
  - operating costs ($/hr) for each material
  - *Sequence dependent clean out times* (hrs per transition for each material)
  - Time the reactor is available during a given month (hrs)

**Customers:**
- Monthly forecasted demands for desired products
- Price paid for each product

**Materials:**
- Raw materials, Intermediates, Finished products
- Unit ratios (lbs of needed material per lb of material produced)
PROBLEM STATEMENT

DETERMINE:

➤ PLANNING PERSPECTIVE:
  • Monthly production quantities
  • Amount of raw materials to be purchased
  • Monthly Inventory levels

➤ SCHEDULING PERSPECTIVE:
  • Assignments of products to available processing equipment
  • Detailed timing and sequence of production in each processing equipment
  • Daily Production and Inventory levels

OBJECTIVE:

To Maximize Profit.

Profit = Sales – Raw Material Costs - Operating Costs – Inventory Costs – Transition Cost – Transportation Costs
BASIC IDEAS OF THE MULTIPERIOD PLANNING MODEL

RELAXATION OF STATE-TASK NETWORK

*Detailed timing constraints are relaxed*

- **States**: Raw materials, intermediates, end products, (J)
- **Tasks**: Physical and Chemical transformations between adjacent states (i.e. reactions), (I)
- **Equipment**: Physical devices that will execute a given task (i.e. reactors), (L)

**KEY VARIABLES:**

\[
Y_{i,l,t} = \begin{cases} 
1, & \text{process } i \text{ is assigned to equipment } l \text{ at period } t \\ 
0, & \text{otherwise} 
\end{cases}
\]

\[W_{i,l,t}: \text{batch size of process } i, \text{ in unit } l \text{ at time period } t \text{ (lb)}\]

\[N_{i,l,t}: \text{number of batches of process } i \text{ in unit } l \text{ at time period } t\]

*Integrality requirements enforced on the number of batches*
AGGREGATE MILP PLANNING MODEL

MASS BALANCE FOR EACH CHEMICAL AT EACH TIME PERIOD:

\[ P_{j,t} + \sum_{i \in \text{SET}O} RHO_{i,j} \sum_{i \in N(1,L)} W_{i,l,t} = S_{j,t} + \sum_{i \in \text{SET}I} RHO_{i,j} \sum_{i \in N(1,L)} W_{i,l,t} + INV_{j,t} - INV_{j,t-1} \quad \forall j,t \]

- purchases
- production
- sales
- consumption
- change in inventory

\[ W_{i,l,t} \leq BOUND_{i,l} \cdot Y_{i,l,t} \quad \forall i,l,t \]

TIME BALANCE:

\[ \sum_{i \in N(1,L)} N_{i,l,t} \cdot TAU_{i} + \sum_{i} \text{TRA}_{i} \cdot Y_{i,l,t} - U_{l,t} \leq H_{l,t} \quad \forall l,t \]

- Number of batches
- Batch time
- Lower bounds for transitions
- Total available time

\[ N_{i,j,t} = \frac{W_{i,j,t}}{Q_{l}} \quad \forall i,l,t \]

- Integer number of Batches

OBJECTIVE FUNCTION:

Maximize:

\[ NPV = \sum_{t} \sum_{j} CP_{j,t} \cdot S_{j,t} - \sum_{t} \sum_{j} CR_{j,t} \cdot P_{j,t} - \sum_{i \in N(1,L)} \sum_{j \in M(1,J)} \sum_{l \in N(1,L)} \sum_{i} COP_{i,t} \cdot W_{i,l,t} - \sum_{i \in N(1,L)} \sum_{l \in N(1,L)} COPF_{i,t} \cdot Y_{i,l,t} \]

- sales
- purchases
- variable operating costs
- fixed costs
**DETAILED SCHEDULING MODEL**

- **F1** → Reaction 1 → A → STORAGE
- **F2** → Reaction 2 → B → STORAGE
- **F3** → Reaction 3 → C → STORAGE

**DETAILS:**
- Several batch reactors operating in parallel
- Each reactor is connected to any final product storage tank.
- Raw materials and end products—end product of one process might be used as a raw material of another process.
- Once an end product is sent to the dedicated storage tanks, it can’t be fed back into the plant—requires modeling of intermediate storage

**Difficulties with STN/RTN Models or Sequential Models**
- Sequence dependent transition times are handled via the slot based representation.
- To handle intermediate storage, process that produces the intermediate is duplicated.
Problem Statement for the Single Site Scheduling

Given

- **Multiproducts** to be processed on a multipurpose batch plant with equipment in parallel.
- Time horizon subdivided into **weeks** at the end of which demands are specified.
- Transition times are **sequence dependent**.
- **Continuous** time representation is used.
- **Time slot** representation is used.

Decisions

- Number of batches of each product
- Amounts to be produced
- Product inventories
- Sequencing of products
- Assignments of batches to equipments

Objective

- Max Profit = Sales − Operating Costs − Inventory Costs − Transition Costs
Problem Formulation

Transition times across periods

Week t

Weekly slot assignment

Product i

Batch time

Change over

Slot l

Unit M

Equipment in parallel

Key variables for assignments:

$W_{i m t} = 1$: Product $i$ is produced in slot $l$ of unit $m$ of time period $t$

binary variable

$Y_{i m t} = 1$: slot $l$ of unit $m$ of time period $t$ is occupied.

continuous variable
Assumptions

- Batch times and batch sizes are fixed parameters.
- Each slot represents one batch, at most one batch is produced in each slot.

- Same product can be produced in more than one slot.

- Consecutive utilization of the slots is ensured by the model.

- Number of batches is a variable to be determined by the model, hence exact number of slots to be postulated is unknown.

- To avoid infeasible or suboptimal solutions, more than necessary number of slots are postulated at each time period. Hence some slots might be left unutilized.
Detailed Scheduling Model (MILP)

\[ Z = \sum_{i} \sum_{t} CP_{i,t} \cdot S_{i,t} - \sum_{i} \sum_{m} \sum_{l} COP_{i,m,l,t} \cdot XB_{i,m,l,t} - \sum_{i} \sum_{l} CINV_{i,l} \cdot INV_{i,t} - \sum_{i} \sum_{k} \sum_{m} \sum_{l} TRANS_{i,k} \cdot (Z_{i,k,m,l,t} + ZDELT_{i,k,m,l,t} + ZBELT_{i,k,m,l,t}) \]

\[ \sum_{i} W_{i,m,l,t} \leq 1 \quad \forall m,l,t \]

\[ \sum_{i} W_{i,m,l,t} \geq \sum_{i} W_{i,m,l+1,t} \quad \forall m,l,t \]

\[ PT_{i,m,l,t} = TA_{i,m} \cdot W_{i,m,l,t} \quad \forall i,m,l,t \]

\[ XB_{i,m,l,t} = R_{i,m} \cdot PT_{i,m,l,t} \quad \forall i,m,l,t \]

\[ Y_{m,l,t} = \sum_{i} W_{i,m,l,t} \quad \forall m,l,t \]

\[ Z_{i,k,m,l,t} \geq W_{i,m,l,t} + W_{k,m,l+1,t} - 1 \quad \forall i,k(k \neq l),m,l \neq N,t \]

\[ ZDELT_{i,k,m,l,t} \geq W_{i,m,l,t} + W_{k,m,l+1,t} - 1 \quad \forall i,k(k \neq l),m,l,t \neq HT \]

\[ ZBELT_{i,k,m,l,t} \geq W_{i,m,l,t} + W_{k,m,l+1,t} - 1 \quad \forall i,k(k \neq l),m,l = N,t < HT \]

\[ T_{e,m,t} = T_{s,m,t} + \sum_{i} PT_{i,m,l,t} + \sum_{i} \sum_{k} TAU_{i,k} \cdot Z_{i,k,m,l,t} + (\sum_{i} \sum_{l} \sum_{k} TAU_{i,k} \cdot ZDELT_{i,k,m,l,t}) \cdot (1 - Y_{m,l,t}) + (\sum_{i} \sum_{l} \sum_{k} TAU_{i,k} \cdot ZBELT_{i,k,m,l,t}) \]

\[ TE_{mm,t} \leq TS_{m,t} + BIGW \cdot (1 - YY_{m,t}) \quad \forall ll,l,mm,m \text{ (mm} \neq m) \]

\[ TS_{m,t} \leq TE_{mm,t} + BIGW \cdot (YY_{m,t}) \quad \forall ll,l,mm,m \text{ (mm} \neq m) \]

\[ \begin{bmatrix} YY_{ll,t} \text{ \_ \_ \_ \_ \_ \_} \\ A\text{\_A,mm,}t \end{bmatrix} = \begin{bmatrix} XB_{A',mm,}t \text{ \_ \_ \_ \_ \_ \_} \\ AA_{A',mm,}t \text{ \_ \_ \_ \_ \_ \_} \end{bmatrix} \]

\[ \begin{bmatrix} \neg YY_{ll,t} \text{ \_ \_ \_ \_ \_ \_} \\ AA_{A',mm,}t \end{bmatrix} = 0 \]

\[ \alpha \cdot XB_{B,m,l,t} \leq INV_{A',t-1} + \sum_{ll} XB_{A',mm,}t + \sum_{ll} AA_{A',mm,}t \]

\[ INV_{i,t-1} + \sum_{m} \sum_{l} XB_{i,m,l,t} = S_{i,t} + INV_{i,t} \quad \forall t,i \neq A' \]

\[ INV_{i,t} + \sum_{m} XB_{i,m,l,t} = \alpha \cdot \sum_{m} \sum_{l} XB_{B,m,l,t} + INV_{i,t} \quad \forall t,i = A' \]
Linearization of the Timing Constraint

\[ T_{e_{m,l,t}} = T_{s_{m,l,t}} + \sum PT_{i,m,l,t} + \sum TAU_{i,k} \cdot Z_{i,k,m,l,t} + (\sum TAU_{i,k} \cdot ZDEL_{i,k,m,l,t}) \cdot (1 - Y_{m,l+1,t}) + (\sum TAU_{i,k} \cdot ZBEL_{i,k,m,l,t}) \]

- Batch time
- Transitions within the time period
- Transitions across time periods
- Transitions across time periods if the last slot is being utilized

\[ Y_{it} = 1 \quad Y_{2t} = 1 \quad Y_{3t} = 1 \quad Y_{4t} = 0 \quad Y_{5t} = 0 \]

Using Convex Hull Representation

\[ \begin{bmatrix} Y_{m,l+1,t} \\ TX_{m,l,t} \end{bmatrix} \vee \begin{bmatrix} \neg Y_{m,l+1,t} \\ TX_{m,l,t} = 0 \end{bmatrix} \]

\[ TRT_{m,l,t} = TRT1_{m,l,t} + TRT2_{m,l,t} \]
\[ TX_{m,l,t} = TRT1_{m,l,t} \]
\[ TRT1_{m,l,t} \leq BIGM \cdot Y_{m,l+1,t} \]
\[ TRT2_{m,l,t} \leq BIGM \cdot (1 - Y_{m,l+1,t}) \]
• Production of the intermediate to be used as a raw material for the end products should be completed before the production of that particular end starts!

**Key variable:**

$Y_Y^{ll, l, l', m, m', t} = \begin{cases} 1, & \text{slot } ll \text{ of unit } m1 \text{ is completed before slot } l \text{ of unit } m2 \text{ starts} \\ 0, & \text{otherwise} \end{cases}$

**Determining the location of the slots relative to slot $l$ of unit $M2$ at time $T$:**

$TE_{mm, ll', l, t} \leq TS_{m, l, t} + BIGW \cdot (1 - Y_Y^{ll, l, l', m, m, t}) \quad \forall ll, l, mm, m, t (mm \neq m)$

$TS_{m, l, t} \leq TE_{mm, ll', l, t} + BIGW \cdot Y_Y^{ll, l, l', m, m, t} \quad \forall ll, l, mm, m, t (mm \neq m)$

**Defining the availabilities of the slots:**

$\begin{bmatrix} Y_Y^{ll, l, mm, m, t} \\ A_A^{A, mm, ll', l, t} = XB^{A, mm, ll', l, t} \end{bmatrix} \lor \begin{bmatrix} \neg Y_Y^{ll, l, mm, m, t} \\ A_A^{A, mm, ll', l, t} = 0 \end{bmatrix}$

**Using Convex Hull Representation:**

$XB^{A, mm, ll', l, t} = XB1^{A, mm, ll', l, t} + XB2^{A, mm, ll', l, t}$

$AA^{A, mm, ll', l, t} = XB1^{A, mm, ll', l, t}$

$XB1^{A, mm, ll', l, t} \leq BIGU \cdot Y_Y^{ll, l, l', mm, m, t}$

$XB2^{A, mm, ll', l, t} \leq BIGU \cdot (1 - Y_Y^{ll, l, l', mm, m, t})$
Example and Preliminary Results

- Determine **plan and schedule** for 3 product, 2 reactors plant for a planning horizon of 3 **weeks** so as to maximize **profit**.

- **3 Products, A,B,C**
- To produce 1 lb of “B”, 0.2lb of “A” is required.
- **2 Reactors, R1,R2**
- End time of each week is defined as due dates
- Demands are upper bounds

### Problem Data:

<table>
<thead>
<tr>
<th>Demand values (lb)</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>80,000.00</td>
<td>160,000.00</td>
<td>160,000.00</td>
</tr>
<tr>
<td>B</td>
<td>96,000.00</td>
<td>192,000.00</td>
<td>480,000.00</td>
</tr>
<tr>
<td>C</td>
<td>120,000.00</td>
<td>240,000.00</td>
<td>360,000.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Batch times (hrs)</th>
<th>Batch sizes (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>16.00</td>
</tr>
<tr>
<td>B</td>
<td>10.00</td>
</tr>
<tr>
<td>C</td>
<td>25.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transition times (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>
## Results

<table>
<thead>
<tr>
<th></th>
<th># of discr. Vrbs</th>
<th># of cont. vrbs</th>
<th># of eqns</th>
<th>Time (CPUs)</th>
<th>Solution ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planning Model</td>
<td>36</td>
<td>90</td>
<td>102</td>
<td>0</td>
<td>1,070,830.96</td>
</tr>
<tr>
<td>Scheduling Model</td>
<td>1,440</td>
<td>4,534</td>
<td>6,002</td>
<td>2,464</td>
<td>1,008,119.46</td>
</tr>
</tbody>
</table>

% 5.86 difference in the predicted profit!

<table>
<thead>
<tr>
<th>($)</th>
<th>Planning</th>
<th>Scheduling</th>
<th>Difference</th>
<th>%Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>1,070,830.96</td>
<td>1,008,119.47</td>
<td>62,711.49</td>
<td>5.86</td>
</tr>
<tr>
<td>Sales</td>
<td>1,788,320.00</td>
<td>1,693,280.00</td>
<td>95,040.00</td>
<td>5.31</td>
</tr>
<tr>
<td>InvCost</td>
<td>134.04</td>
<td>100.53</td>
<td>33.51</td>
<td>25.00</td>
</tr>
<tr>
<td>TransCost</td>
<td>1,035.00</td>
<td>1,380.00</td>
<td>-345.00</td>
<td>-33.33</td>
</tr>
<tr>
<td>OperCost</td>
<td>716,320.00</td>
<td>683,680.00</td>
<td>32,640.00</td>
<td>4.56</td>
</tr>
</tbody>
</table>
### Difference in prediction of Planning and Scheduling Models

#### Difference in the amounts produced in each time period:

<table>
<thead>
<tr>
<th></th>
<th>Planning- Production Values (lb)</th>
<th>Scheduling- Production Values (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T1</strong></td>
<td><strong>T2</strong></td>
<td><strong>T3</strong></td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>160,000.00</td>
<td>160,000.00</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>96,000.00</td>
<td>192,000.00</td>
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<tr>
<td><strong>C</strong></td>
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<td>240,000.00</td>
</tr>
</tbody>
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#### Difference in the sales values in each time period:

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<td><strong>C</strong></td>
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</tr>
</tbody>
</table>

#### Difference in the total amounts produced:

<table>
<thead>
<tr>
<th></th>
<th>Planning</th>
<th>Scheduling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total A Production (lb)</td>
<td>560,000</td>
<td>560,000</td>
</tr>
<tr>
<td>Total B Production (lb)</td>
<td>768,000</td>
<td>672,000</td>
</tr>
<tr>
<td>Total C Production (lb)</td>
<td>720,000</td>
<td>720,000</td>
</tr>
</tbody>
</table>

#### Difference in the total sales values:

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<tbody>
<tr>
<td>Total A Sales (lb)</td>
<td>400,000</td>
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<td>Total B Sales (lb)</td>
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<td>Total C Sales (lb)</td>
<td>720,000</td>
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</tr>
</tbody>
</table>
Results

Gantt Chart for the scheduling model for the demands as upper bounds case:

Week 1

Week 2

Week 3

Gantt Chart for the scheduling model for the demands as lower bounds case:

Week 1

Week 2

Week 3
Future Work

- Further investigate improvements for scheduling model.
- Develop bi-level decomposition algorithm for the single site problem to integrate planning and scheduling.
- Apply the proposed bi-level decomposition scheme to the entire system of sites and customers.
- Further decompose the problem via temporal Lagrangean decomposition so as to handle multiple sites.