Refinery Production Planning: Multiperiod MINLP with Nonlinear CDU Model

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Abstract

In a previous paper, Alattas, Grossmann & Palou-Rivera (2011) developed a single-period, nonlinear programing refinery planning model using the fractionation index (FI) for the crude distillation unit (CDU) equations. In this paper, the single period model is modified to a mixed-integer nonlinear programming (MINLP) model to determine the sequencing, changeovers and processing times of crude oils over multiple time periods. The MINLP equations include traveling salesman problem constraints to generate the crude oil sequences that maximize profit. Moreover, the disjunction for the fractionation index (FI) is formulated with mixed integer constraints as opposed to the Heaviside function formulation of the previous work. The resulting model is shown to be robust and relatively fast. When subcycles arise, they are eliminated by adding appropriate subtour elimination constraints. Examples with up to 5 crude oils and 6 weeks time horizon are presented to illustrate the application of the proposed model.

1. Introduction

Production Planning is an essential tool in today's petroleum refining industry. It aids in decision making and resource allocation to achieve business objectives through optimal production, distribution, sales and inventory management[1,2]. The refinery planning models differ in their levels of model complexity and sizes. They span the strategic level of single period, long term, crude-selection planning model to the operational multi-period, short term, crude-allocation-and-movement operation planning model[3,4,5]. The refining industry is one of the early adaptors of linear programming (LP) to address its planning and optimization needs[6]. The LP approach simplifies the inherent nonlinearity of the refinery processes to ensure simplicity, robustness and convenience of the models at the expense of true optimal and accurate solutions to the planning model. The need for more accurate nonlinear representations of the process units as well as integrated planning and scheduling refinery operation models are recognized as key future challenges in the field[5,7,8,9]. Multiperiod refinery planning models tend to utilize linear equations for the process units[10,11,12]. The nonlinear implementation for the process unit in multiperiod planning models rely mainly on empirical relations[13].

In our previous work[14], we presented the fractionation index (FI) model as a nonlinear model for the crude distillation unit (CDU), the front end of the refinery and an important process unit. The model benefited from the simple and crude-independent equations generated using the column-characteristic FI values and gave a more accurate and relatively fast model. The model was successfully integrated into a single period time horizon planning model for crude purchase decision. The natural next development for the robust nonlinear FI model is to integrate the scheduling element for an improved refinery-wide optimization. In this paper we extend the FI model to multiperiod implementation as a step toward a comprehensive planning and scheduling model. Our multiperiod model uses continuous time representation[15,16] addressing a planning and scheduling problem for continuous multiproduct plants. The approach used is to develop an accurate upper level planning incorporating changeovers, product inventories and periodic product demands, using the traveling salesman constraints to generate the processing sequence.

2. Problem Statement

Given is a configuration of the refinery whose operation is to be optimized over several time periods. The configuration, similar to the one shown in Figure 1[17], provides information on the major separation, conversion and blending units, along with the refinery feed streams, product streams and interconnections between the different units. Refineries differ in the number or types of process units they have depending on the quality of crude oils they process and the product slates. In this paper, the configuration of Figure 1 is used, where the crude is separated into different cuts in the crude distillation unit (CDU). Each cut is sent to a different conversion or treatment unit (reformer, catalytic cracking or hydrotreatment units) or to product blending.

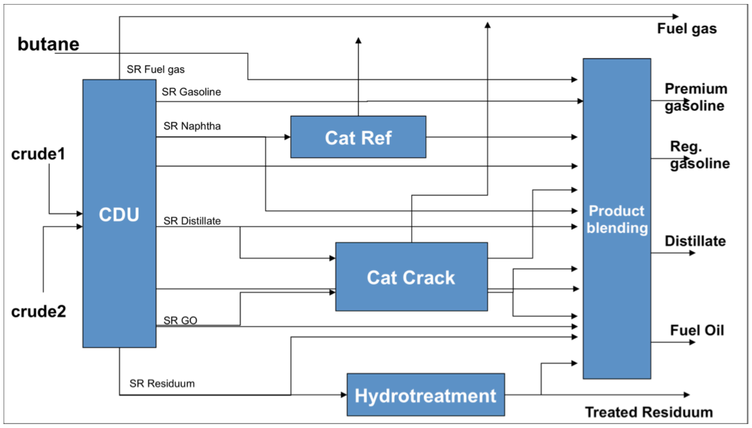

Figure 1 Complex Refinery Configuration

In addition to the configuration, the following information is assumed to be given

- Crude oils. The availability of crude oils are specified, along with their properties and compositions, typically in the form of the crude assays. Several processing parameters, property values and indices are generated.
- Refinery units. There is information on the capacities of these units and minimum feed requirements. The required coefficients for yield and property calculations are generated from the units and crude oil information,
- Product slates. The list of the products produced by the refinery and their possible constituting streams are given. The required product qualities are also specified, along with minimum or maximum demands. The demands are specified at the end of each time period.
- Product inventory. In this work, we consider only the final product storage. The cost of storing these products is specified along with any initial inventory at the start of period 1.
- Changeovers. Due to the variation in crude qualities, changeovers are sequence dependent. The changeovers are specified in terms of duration and cost.
- Planning horizon. The horizon and the period duration are also specified. We consider 4 to 6 weeks time horizon, with 1 week time periods.

Based on the above information, the objective is to determine the operation that maximizes profit (or equivalently minimizes cost) in terms of the following decisions:

- What crude oil to process and in which time period
- The quantities of these crude oils to process
- The sequence of processing them

- The rate of processing those crude oils and the processing duration
- The refinery products produced, their quantities and storage requirement

3. Mathematical Formulation

The mathematical formulation for the multiperiod refinery planning model builds on the previous work of Alattas, Grossmann & Palou-Rivera (2011)[14] and Erdirik-Dogan & Grossmann (2008)[16]. The formulation can be broken into three layers. The inner layer is the fractionation index model (FI) which is the nonlinear CDU model. The middle layer is the refinery planning model, which in this work is a linear model. The last layer is the multiperiod extension, which is an MINLP model.

In contrast to the single period model[14], the FI and planning model equations include additional indices for the processed crude *cr* at time period *t*.

3.1. FI Model

The FI model is based on the fractionation index introduced by Geddes[18]. It allows calculating the yield and cut point temperature of the CDU cuts using the column-characteristic FI values. The FI model represents the CDU as a series of fractionation unit, as shown in Figure 2.

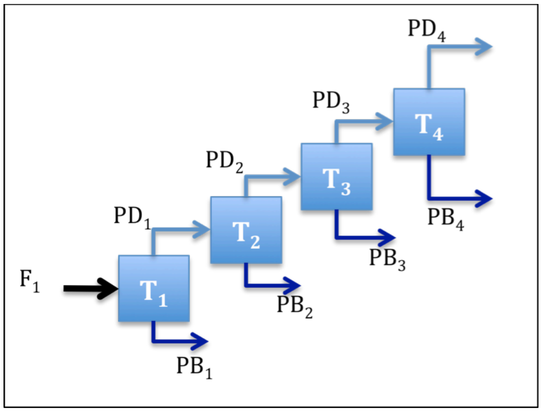

Figure 2 CDU Representation for the FI Model

The NLP model for the CDU starts with a mass balance around each unit *j* and component *i*. Every unit yields the top product $PD_{j,i,cr,t}$ feeding the next unit and the bottom product $PB_{j,i,cr,t}$, which is the product crude cut. There is also a summation equation for each type of stream over its set of constituent components *i*'s

$$F_{j,i,cr,t} = PD_{j,total,cr,t}\ x_{PD,j,i,cr,t} + PB_{j,total,cr,t}\ x_{PB,j,i,cr,t} \qquad \forall j,i,cr,t \qquad (1)$$

$$F_{j+1,i,cr,t} = PD_{j,total,cr,t}\ x_{PD,j,i,cr,t} \qquad \forall j,i,cr,t \qquad (2)$$

$$PB_{j,i,cr,t} = PB_{j,total,cr,t}\ x_{PB,j,i,cr,t} \qquad \forall j,i,cr,t \qquad (3)$$

$$PD_{j,i,cr,t} = PD_{j,total,cr,t}\ x_{PD,j,i,cr,t} \qquad \forall j,i,cr,t \tag{4}$$
$$F_{j,total,cr,t} = \sum_i F_{j,i,cr,t} \qquad \forall j,i,cr,t \tag{5}$$
$$PD_{j,total,cr,t} = \sum_i PD_{j,i,cr,t} \qquad \forall j,i,cr,t \tag{6}$$
$$PB_{j,total,cr,t} = \sum_i PB_{j,i,cr,t} \qquad \forall j,i,cr,t \tag{7}$$

The component distribution at each stage *j* is based on the light key LK_j and heavy key HK_j components for each unit based on the initial and end boiling points relative to each cut. The components lighter than the light key are only obtained in the top product stream, while the ones heavier than the heavy key are only obtained in the bottom product stream.

$$PD_{j,i,cr,t} = F_{j,i,cr,t},\ PB_{j,i,cr,t} = 0 \qquad \forall j,cr,t,i < LK_j \tag{8}$$
$$PB_{j,i,cr,t} = F_{j,i,cr,t},\ PD_{j,i,cr,t} = 0 \qquad \forall j,cr,t,i > HK_j \tag{9}$$

The splits of the distributed components are calculated using the FI parameters. For each unit, there are two FI values, one for the rectifying section and another for the stripping section. Choice of the correct value is based on the calculated temperature and the particular component. The rectifying FI (FI_r) is used if the temperature is greater than the component boiling temperature; otherwise, the stripping FI (FI_s) is used. The following disjunction represents the FI choice,

$$\begin{bmatrix} \neg Y_{i,j,cr,t} \\ \gamma_{i,j,cr,t} = FIr_j \\ T_{b,i} \leq T_{j,cr,t} \end{bmatrix} \bigvee \begin{bmatrix} Y_{i,j,cr,t} \\ \gamma_{i,j,cr,t} = FIs_j \\ T_{b,i} \geq T_{j,cr,t} \end{bmatrix} \qquad \forall j,i,cr,t \tag{10}$$

where $Y_{i,j,cr,t}$ is the Boolean (binary) decision variable, $\gamma_{i,j,cr,t}$ is a placeholder for the FI value, $T_{b,i}$ is the component boiling point and $T_{j,cr,t}$ is the cut point temperature.

In our previous work[14], the Heaviside function was used for this disjunction, yielding the following FI equation for the distributed components,

$$PB_{j,i,cr,t} = \frac{F_{i,j,cr,t}}{\frac{PD_{total,j,cr,t}}{PB_{total,j,cr,tpr}}\alpha_{i,j}^{\beta(T_{j,cr,t})} + 1} \qquad \forall j,cr,t,LK_j \leq i \leq HK_j \tag{11}$$

or the simplified form substituting the equilibrium constant for the relative volatility[19] ,

$$PB_{j,i,cr,t} = \frac{F_{i,j,cr,t}}{\frac{PD_{total,j,cr,t}}{PB_{total,j,cr,tpr}}K_{i,j}^{\beta(T_{j,cr,t})} + 1} \qquad \forall j,cr,t,LK_j \leq i \leq HK_j \tag{12}$$

where

$$\beta(T_{j,cr,t}) = \frac{FIr_j}{1+e^{4(T_{b,i}-T_{j,cr,t})}} + \frac{FIs_j}{1+e^{-4(T_{b,i}-T_{j,cr,t})}} \qquad \forall j, cr, t, LK_j \le i \le HK_j \tag{13}$$

As opposed to our previous planning model above, the model presented in this paper corresponds to an MINLP. Therefore, we consider exact representation of the disjunction (10) with linear mixed integer constraints using convex hull as in (14) and big M formulation as in (15) and (16)[20],

$$\gamma_{i,j,cr,t} = FIr_j * (1 - Y_{i,j,cr,t}) + FIs_j * Y_{i,j,cr,t} \qquad \forall j, cr, t, LK_j \le i \le HK_j \tag{14}$$

$$T_{b,i} + M_L * Y_{i,j,cr,t} \le T_{j,cr,t} \qquad \forall j, cr, t, LK_j \le i \le HK_j \tag{15}$$

$$T_{j,cr,t} \le T_{b,i} + M_U * (1 - *Y_{i,j,cr,t}) \qquad \forall j, cr, t, LK_j \le i \le HK_j \tag{16}$$

Notice that at $Y_{i,j,cr,t}=0$ (false) that $\gamma_{i,j,cr,t}=FI_{r,j}$ and $Tb_i \le T_{j,cr,t}$ while at $Y_{i,j,cr,t}=1$ (true) that $\gamma_{i,j,cr,t}=FI_{s,j}$ and $Tb_i \ge T_{j,cr,t}$. Since the components are listed in the order of increasing boiling point, the following constraint is included

$$Y_{i,j,cr,t} \le Y_{i+1,j,cr,t} \qquad \forall j, cr, t, LK_j \le i \le HK_j \tag{17}$$

The FI equations (the original and the simplified forms) become,

$$PB_{j,i,cr,t} = \frac{F_{i,j,cr,t}}{\frac{PD_{total,j,cr,t}}{PB_{total,j,cr,t}} \alpha_{i,j,cr,t}^{\gamma_{i,j,cr,t}} + 1} \qquad \forall j, cr, t, LK_j \le i \le HK_j \tag{18}$$

and

$$PB_{j,i,cr,t} = \frac{F_{i,j,cr,t}}{\frac{PD_{total,j,cr,t}}{PB_{total,j,cr,t}} K_{i,j,cr,t}^{\gamma_{i,j,cr,t}} + 1} \qquad \forall j, cr, t, LK_j \le i \le HK_j \tag{19}$$

Equations (14)- (19) are an improvement from the original FI model. We examine the impact of this improvement later in the discussion section.

The equilibrium constant and vapor pressure are calculated as follows[14],

$$K_{j,i,cr,t} = \frac{Pv_{j,i,cr,t}(T_{j,cr,t})}{P} \qquad \forall j, i, cr, t \tag{20}$$

$$Pv_{j,i,cr,t} = Exp((PVA_i - \frac{PVB_i}{T_{j,cr,t} + PVC_i - 273.15}) * 2.303) \qquad \forall j, i \in HC, cr, t \tag{21}$$

$$\begin{aligned}
Pv_{j,i,cr,t} &= Pc_i * Exp([-5.96346 * (1 - Tr_{j,i,cr,t}) + 1.17639 * (1 - Tr_{j,i,cr,t})^{1.5} \\
&- 0.559607 * (1 - Tr_{j,i,cr,t})^3 - 1.319 * (1 - Tr_{j,i,cr,t})^6]/Tr_{j,i,cr,t} \\
&+ \omega_i * [-4.78522 * (1 - Tr_{j,i,cr,t}) + 0.413999 * (1 - Tr_{j,i,cr,t})^{1.5} \\
&- 8.91239 * (1 - Tr_{j,i,cr,t})^3 - 4.98662 * (1 - Tr_{j,i,cr,t})^6]/Tr_{j,i,cr,t}) \qquad \forall j, i \in PsC, cr, t
\end{aligned} \tag{22}$$

The separation temperature is the arithmetic average of the initial boiling point and end boiling points relevant to each cut,

$$T_{j,cr,t} = \frac{TE_{j,cr,t} + TI_{j,cr,t}}{2} \qquad \forall j, cr, t \tag{23}$$

Moreover, the temperature decreases along the CDU column from bottom to top, which is expressed as,

$$T_{j,cr,t} \geq T_{j+1,cr,t} \qquad \forall j, cr, t \tag{24}$$

3.2. The LP Planning Model

The next layer in the model formulation is the planning model. This includes the set of equations for calculating the product yields from each process unit, except the CDU, as well as stream properties calculations and product blending.

The yield of process unit *k* is calculated using a coefficient multiplied by the unit feed *l'*, to give the unit product stream *l*

$$ST_{l,k,cr,t} = a_{k,l,l',cr} * ST_{l',k,cr,t} \qquad \forall\, k, cr, t \tag{25}$$

The requirement for capacity of unit *k* is ensured using the following constraint,

$$\sum_{l} ST_{l,k,cr,t} \leq capacity_{k} \qquad \forall\, k, cr, t \tag{26}$$

The interconnections between the process units are managed using several splitters and mixers. The splitters use the following equation,

$$ST_{l',sep,cr,t} = \sum_{l} ST_{l,sep,cr,t} \qquad \forall\, sep, cr, t \tag{27}$$

while the mixers use the following equation,

$$\sum_{l} ST_{l,mix,cr,t} = ST_{l',mix,cr,t} \qquad \forall\, mix, cr, t \tag{28}$$

Product blending is a mixer unit, but is distinguished by the final product streams *p*,

$$\sum_{l} ST_{l,k,cr,t} = ST_{p,cr,t} \qquad \forall\, p, cr, t \tag{29}$$

The product properties are checked against the product specification $PR_{r,p}$ using the following constraints

$$ST_{p,cr,t} * PR_{r,p,t} \leq \sum_{l',k} Pr_{r,l',k,cr,t} * ST_{l',k,cr,t} \qquad \forall p, cr, t \tag{30}$$

$$ST_{p,cr,t} * PR_{r,p,t} \geq \sum_{l',k} Pr_{r,l',k,cr,t} * ST_{l',k,cr,t} \qquad \forall p, cr, t \tag{31}$$

Typically the planning model will include checking the feedstock availability, meeting the product demands and the objective function. However, these constraints and equations are moved to the next layer of the model as part of the extension from single period to multi period time horizon.

3.3. MultiPeriod extension

The multi period layer is primarily based on the approach proposed by Eridirk-Dogan & Grossmann[16]. The idea is to determine the length of the processing times for each crude oil, and to use traveling salesman constraints to determine the sequence for processing crude in each time period and identifying the link to break and connect the sequence to the adjacent time periods. The objective function consists of maximizing the profit. When there are no subcycles in any time period, the changeovers are properly accounted for in each time period and any possible crossovers. Otherwise, subtour elimination constraints are added until all subcycles are eliminated. Also, since the duration of the changeovers is relatively modest, we assume for simplicity that changeovers cannot cross periods. The extension to handle this case can be readily handled (see Lima et al[21] and Kopanos et al[22]).The proposed MINLP model is described below.

3.3.1. Assignment

The binary variable $YP_{cr,t}$ is for deciding whether crude *cr* is processed at time period *t*. It is used in the following constraint to set the crude processing time $\theta_{cr,t}$ to zero if the crude oil is not selected. The length of the processing time is limited by the length of the time period given by *Ht,*

$$\theta_{cr,t} \leq Ht * YP_{cr,t} \qquad \forall cr, t \tag{32}$$

The maximum crude oil availability $AUFCr_{cr,t}$ or minimum requirements $ALFCr_{cr,t}$ are checked using the crude oil processing rate $FCr_{cr,t}$ multiplied by the processing time $\theta_{cr,t}$,

$$FCr_{cr,t} * \theta cr, t \geq ALFCr_{cr,t} \qquad \forall cr, t \tag{33}$$

$$FCr_{cr,t} * \theta cr, t \leq AUFCr_{cr,t} \qquad \forall cr, t \tag{34}$$

The crude processing rate is linked to the CDU model as follows,

$$FCr_{cr,t} = F_{1,total,cr,t} \qquad \forall cr, t \tag{35}$$

$$FCr_{cr,t} * zt_{i,cr,t} = F_{1,i,cr,t} \qquad \forall i, cr, t \tag{36}$$

The quantities of product p, $XP_{p,cr,t}$, are calculated by multiplying the production rate $ST_{p,cr,t}$ by the processing time $\theta_{cr,t}$,

$$XP_{p,cr,t} = ST_{p,cr,t} * \theta_{cr,t} \qquad \forall p, cr, t \tag{37}$$

$FCr_{cr,t}$ and $ST_{p,cr,t}$ are the two variables linking this outer MINLP layer, namely equations (33)-(37), to the remaining model equations from the planning and FI layers, namely equations (1)-(9) and (14)-(31).

3.3.2. Inventory

The inventory of product p, $Inv_{p,t}$, is accounted for at the end of each time period t, using the initial inventory at the start of the period $Invi_{p,t}$ and the product production $XP_{p,cr,t}$.

$$Inv_{p,t} = Invi_{p,t} + \sum_{cr} XP_{p,cr,t} \qquad \forall p, t \tag{38}$$

The initial inventory $Invi_{p,t}$ is the inventory of the previous period after any sales ($Sl_{p,t-1}$) at the end of that period,

$$Invi_{p,t} = Inv_{p,t-1} - Sl_{p,t-1} \qquad \forall p, t \tag{39}$$

For the purpose of the planning model, and since we are accounting for demand at the end of each period, the change of inventory is aggregated by multiplying the time period duration Ht times the inventory Inv. This represents the area under the curve, which is a conservative estimate for the purpose of the inventory cost.

$$ArInv_{p,t} = Ht * (Invi_{p,t} + \sum_{cr} XP_{p,cr,t}) \qquad \forall p, t \tag{40}$$

3.3.3. Demand

The demand for product p is met using the sales variable $Sl_{p,t}$ at the end of each time period t. The following constraints are used to meet any minimum or maximum demand requirements, $DemandL_{p,t}$ and $DemandU_{p,t}$, respectively.

$$Sl_{p,t} \geq DemandL_{p,t} \qquad \forall p, t \tag{41}$$

$$Sl_{p,t} \leq DemandU_{p,t} \qquad \forall p, t \tag{42}$$

3.3.4. Sequence

The sequence of processing the available crude oils is modeled next. The new binary variable $ZP_{cr,ccr,t}$ is defined as 1 when crude cr is followed by crude ccr in time period t. For that to be true, both crude oils cr and ccr should be assigned to that time period and their $YP_{cr,t}$ variables are 1.

$$YP_{cr,t} = \sum_{ccr} ZP_{cr,ccr,t} \qquad \forall\, cr, t \tag{43}$$

$$YP_{ccr,t} = \sum_{cr} ZP_{cr,ccr,t} \qquad \forall\, ccr, t \tag{44}$$

These constraints represent the assignment constraints of the travelling salesman problem.

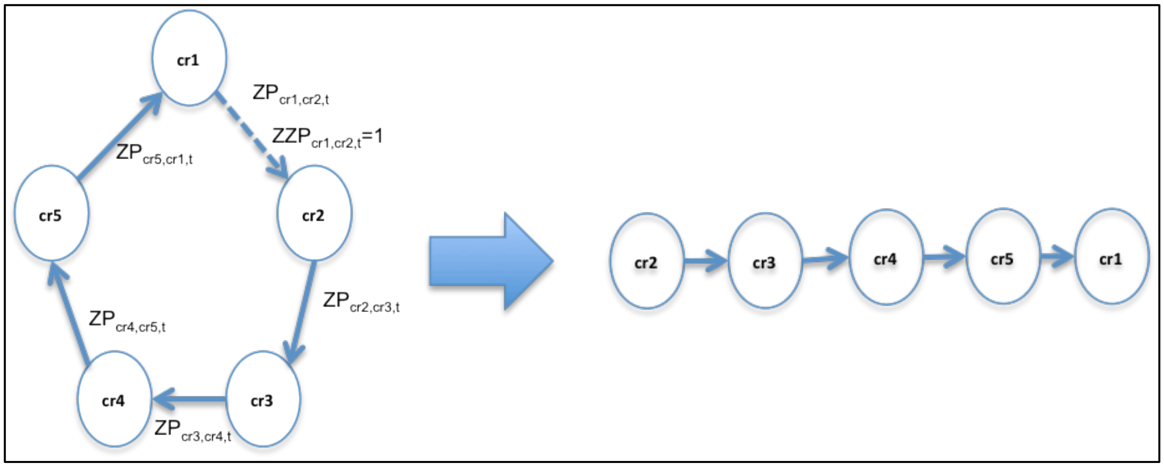

Figure 3 Crude oil processing cycle order and resulting optimal processing sequence

If there are no subcycles, this will create a cyclic sequence in each time period. To account for a single crude oil requiring a whole time period, the following constraints are used,

$$YP_{cr,t} \geq ZP_{cr,ccr,t} \qquad \forall\, cr, ccr, t \tag{45}$$

$$YP_{ccr,t} + ZP_{cr,cr,t} \leq 1 \qquad \forall\, cr \neq ccr, t \tag{46}$$

$$ZP_{cr,cr,t} \geq YP_{cr,t} - \sum_{ccr \neq cr} YP_{ccr,t} \qquad \forall\, cr, t \tag{47}$$

The link to be broken in each closed sequence is defined by the new binary variable $ZZP_{cr,ccr,t}$. Each time period will have one link broken using the following constraint,

$$\sum_{cr} \sum_{ccr} ZZP_{cr,ccr,t} = 1 \qquad \forall\, t \tag{48}$$

as long as the link exists, that is

$$ZZP_{cr,ccr,t} \leq ZP_{cr,ccr,t} \qquad \forall\, cr, ccr, t \tag{49}$$

To establish the sequence, the first and last crude oil are identified as $xF_{ccr,t}$ and $xL_{cr,t}$ using the binary variable $ZZP_{cr,ccr,t}$ as follows,

$$xF_{ccr,t} \geq \sum_{cr} ZZP_{cr,ccr,t} \qquad \forall\, ccr, t \tag{50}$$

$$\sum_{ccr} xF_{ccr,t} = 1 \qquad \forall\, t \tag{51}$$

$$xL_{cr,t} \geq \sum_{ccr} ZZP_{cr,ccr,t} \qquad \forall\, cr, t \tag{52}$$

$$\sum_{cr} xL_{cr,t} = 1 \qquad \forall\, t \tag{53}$$

Identifying the first and last crude oils also determines any crude oil that crosses over to the next time period. This ensures that the time balance is correctly accounted for. The binary variable $ZZZ_{cr,ccr,t}$ indicates as follows the link that crosses over two time periods,

$$\sum_{cr} ZZZ_{cr,ccr,t} = xF_{ccr,t+1} \qquad \forall\, ccr, t \tag{54}$$

$$\sum_{ccr} ZZZ_{cr,ccr,t} = xL_{cr,t} \qquad \forall\, cr, t \tag{55}$$

3.3.5. Transition time & time balance

Using the given changeover times $\tau_{cr,ccr}$ from crude cr to crude ccr, the total transition time $Trans_t$ in each period t is calculated as follows

$$Trans_t = \sum_{cr}\sum_{ccr} \tau_{cr,ccr} * ZP_{cr,ccr,t} - \sum_{cr}\sum_{ccr} \tau_{cr,ccr} * ZZP_{cr,ccr,t} \qquad \forall\, t \tag{56}$$

The time balance accounts for the total processing time, the transition time and any crossovers to ensure continuous operation and no idle time as follows,

$$\sum_{cr} \theta_{cr,t} + Trans_t + \sum_{cr}\sum_{ccr} tau_{cr,ccr} * ZZZ_{cr,ccr,t} = Ht \qquad \forall\, t \tag{57}$$

3.3.6. Objective function

Finally, the objective function expresses the refinery profits as the product sales minus the costs of product inventory, crude oil, unit operation and net transition times.

$$\begin{aligned} Profit = & \sum_t \sum_p Price_{p,t} * Sl_{p,t} - \sum_t \sum_p ArInv_{p,t} * CInv_{p,t} \\ & - \sum_t \sum_{cr} CFCr_{cr,t} * FCrcr, t * \theta_{cr,t} - \sum_t \sum_{cr} COpC_{cr,t} * FCrcr, t * \theta_{cr,t} \\ & - \sum_t \sum_{cr} \sum_{ccr} CTrans_{cr,ccr} * ZP_{cr,ccr,t} \\ & - \sum_t \sum_{cr} \sum_{ccr} CTrans_{cr,ccr} * (ZZZ_{cr,ccr,t} - ZZP_{cr,ccr,t}) \end{aligned} \tag{58}$$

3.3.7. Subtour elimination constraints

For the cases when the model results yields subcycles in a given week, subtour elimination constraints need to be added. For any resulting subcycle *m* in time period *t* $SC_{m,t}$, the following subtour elimination constraint is added to the model

$$\sum_{cr,ccr \in SC_{m,t}} ZP_{cr,ccr,t} \leq |SC_{m,t}| - 1 \qquad cr \neq ccr, \ \forall SC_{m,t} \tag{59}$$

The model is appended with this constraint until there are no subcycles in resulting crude processing sequence. It should be noted that by simply adding subtour elimination constraints instead of performing a rigorous branch and bound search, the resulting solution is not guaranteed to be optimal. However, as the effect of the transition costs is not usually very large, optimal or near optimal solutions are obtained whose quality can in fact be measured by the % decrease in the profit. This decrease in profit represents an upper bound to the optimality gap since the first solution with subcycles yields an upper bound to the optimum.

4. Example & Discussion

To demonstrate the performance of the proposed MINLP model, several examples are presented in this section. Equations (1)-(9) and (14)-(58) define the MINLP model for the multiperiod refinery planning model, which extends the single-period NLP model by Alattas, Grossmann & Palou-Rivera (2011).

In the first example, 5 crude oils are available for processing to produce fuel gas (FG), regular gasoline (RG), premium gasoline (PG), distillate (Dist), fuel oil (FO) and treated residue (HTR). The crude oils, crude1 through crude5, are listed in Appendix A. The planning horizon is 4 weeks or 4 time periods. The other model data are listed in the appendix

The model is solved using GAMS/DICOPT 23.3.3 as the solver with CONOPT and CPLEX for solving the NLP and MILP subproblems, respectively. The MINLP model consists of 13,680 variables and 15,047 equations. The nonlinear variables are 28% of the total number of variables, similar to the ratio in the single-period planning model. There are 900 binary variables, accounting for 6.6% of the total variables. The total time required by DICOPT was 37.5 seconds (94% NLP and 6% MIP), involving 3 major iterations.

The model generates a profitable production plan that is summarized in Table 1, while the crude oil processing rate, time and total feed are listed in Table 2. The product sales and inventory are summarized in Table 3

Table 1 Example 1 Economic results ($1000)

Profit	23994.3
Sales	223684.0
Crude oil cost	162988.0
Other feedstock cost	446.3
Inventory cost	1265.2
Operating cost	32510.8
Transition cost	2480.0

Table 2 Example 1 Feed and processing time information

		Crude	week1	week2	week3	week4
Flow	Processing rate (1000's BPD)	CRUDE1	100.0	100.0	100.0	100.0
		CRUDE2	100.0	100.0	100.0	100.0
		CRUDE3	100.0	100.0	100.0	100.0
		CRUDE6	98.0	98.0	98.0	98.0
		CRUDE8	99.7	99.7	99.7	99.7
	Total Flow (1000's bbl)	CRUDE1	297.5	289.2	289.2	289.2
		CRUDE2	70.0	70.0	70.0	70.0
		CRUDE3	70.0	70.0	70.0	70.0
		CRUDE6	70.0	70.0	70.0	70.0
		CRUDE8	70.0	70.0	70.0	70.0
Time	Processing time (hr)	CRUDE1	71.4	69.4	69.4	69.4
		CRUDE2	16.8	16.8	16.8	16.8
		CRUDE3	16.8	16.8	16.8	16.8
		CRUDE6	17.1	17.1	17.1	17.1
		CRUDE8	16.9	16.9	16.9	16.9

Table 3 Example 1 Product quantities and inventory

	Product	week1	week2	week3	week4
Sales (1000's bbl)	FG	100.1	99.7	102.6	92.5
	PG	140.0	235.7	216.0	119.0
	RG	70.0	245.0	133.3	210.5
	Dist	16.3	13.0	5.7	31.8
	FO	93.9	92.2	92.2	92.9

	HTR	30.5	30.5	30.5	30.5
Inventories (1000's bbl)	FG	100.1	99.7	102.6	92.5
	PG	182.8	235.7	216.0	119.0
	RG	164.0	245.0	133.3	210.5
	Dist	16.3	13.0	5.7	31.8
	FO	93.9	92.2	92.2	92.9
	HTR	30.5	30.5	30.5	30.5

Figure 4 shows the sequence of crude oils and their respective transition times. In this run, the model resulted in subcycles for weeks 2, 3 and 4, but not in week 1. The subcycle is the same in all those time periods. To eliminate the subcycles, we add subtour elimination constraints, Equation (59). This leads to a decrease of the profit from \$23.99MM to \$23.69MM (i.e. 1.3% optimality gap), as shown in Table 4 and Figure 5. Note that the resulting sequence eliminated the subcycles and altered the original sequence of week1.

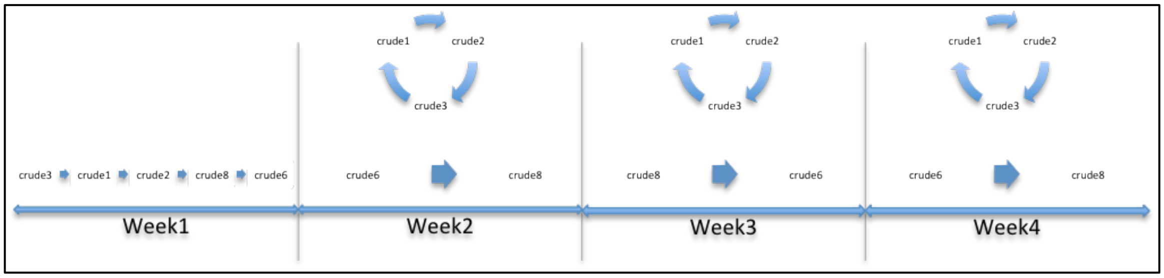

Figure 4 Example 1 Crude processing sequence

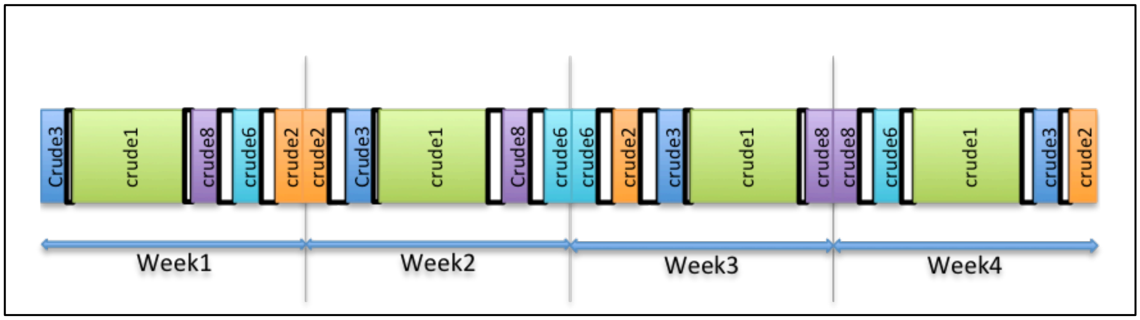

Figure 5 Example 1 Crude processing sequence after subcycle elimination

Table 4 Example 1 Economic results (\$1000's) after subcycle elimination

Profit	23690.0
Sales	223279.0
Crude oil cost	162675.0
Other feedstock cost	446.3
Inventory cost	1263.1

Operating cost	32464.5
Transition cost	2740.0

4.1. Longer time horizon

To test the robustness of the model, two additional examples are considered. The first has a longer time horizon, while the other uses a different set of crude oils.

In example 2 the time horizon is extended to 6 weeks or 6 time periods. The data for the two additional time periods are listed in Appendix A. Table 5 summarizes the economic results of the longer horizon, showing a profitable production plan. Additional results are listed in Table 6 and Table 7 for the crude oil feed, processing time, sales and inventory figures. This example exhibits the same crude oil sequence and subcycles observed in example 1, as seen in Figure 6.

Table 5 Example 2 Economic results ($1000's)

Profit	36845.5
Sales	338346.0
Crude oil cost	244169.0
Other feedstock cost	761.0
Inventory cost	2019.8
Operating cost	50830.4
Transition cost	3720.0

Table 6 Example 2 Feed and processing time information

		Crude	week1	week2	week3	week4	week5	week6
Flow	Processing rate (1000's BPD)	CRUDE1	100.0	100.0	100.0	100.0	100.0	100.0
		CRUDE2	100.0	100.0	100.0	100.0	100.0	100.0
		CRUDE3	100.0	100.0	100.0	100.0	100.0	100.0
		CRUDE6	98.0	98.0	98.0	98.0	98.0	98.0
		CRUDE8	99.7	99.7	99.7	99.7	99.7	99.7
	Total Flow (1000's bbl)	CRUDE1	297.5	289.2	289.2	289.2	289.2	289.2
		CRUDE2	70.0	70.0	70.0	70.0	70.0	70.0
		CRUDE3	70.0	70.0	70.0	70.0	70.0	70.0
		CRUDE6	70.0	70.0	70.0	70.0	70.0	70.0
		CRUDE8	70.0	70.0	70.0	70.0	70.0	70.0
Time	Processing time (hr)	CRUDE1	71.4	69.4	69.4	69.4	69.4	69.4
		CRUDE2	16.8	16.8	16.8	16.8	16.8	16.8
		CRUDE3	16.8	16.8	16.8	16.8	16.8	16.8
		CRUDE6	17.1	17.1	17.1	17.1	17.1	17.1
		CRUDE8	16.9	16.9	16.9	16.9	16.9	16.9

Table 7 Example 2 Product quantities and inventory

	Product	week1	week2	week3	week4	Week5	Week6
Sales (1000's bbl)	FG	100.1	99.7	102.6	102.6	102.6	102.6
	PG	140.0	235.7	216.0	119.0	280.0	249.0
	RG	70.0	245.0	70.9	84.0	245.0	133.3
	Dist	16.3	13.0	5.7	5.7	5.7	5.7
	FO	93.9	92.2	92.2	92.2	92.2	92.2
	HTR	30.5	30.5	30.5	30.5	30.5	30.5
Inventories (1000's bbl)	FG	100.1	99.7	102.6	102.6	102.6	102.6
	PG	182.8	235.7	216.0	216.0	313.0	249.0
	RG	164.0	245.0	133.3	195.7	245.0	133.3
	Dist	16.3	13.0	5.7	5.7	5.7	5.7
	FO	93.9	92.2	92.2	92.2	92.2	92.2
	HTR	30.5	30.5	30.5	30.5	30.5	30.5

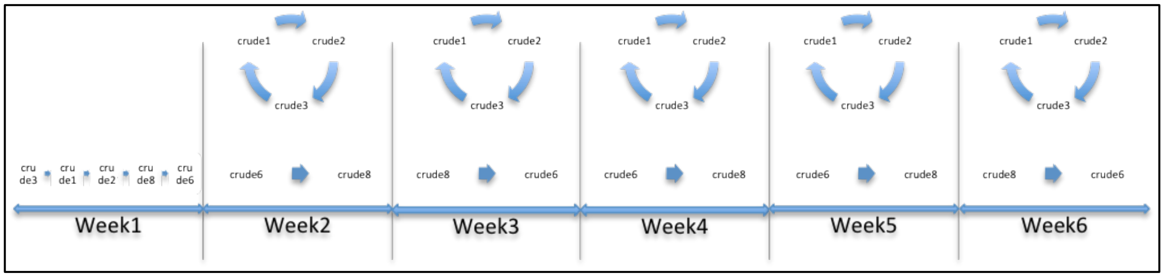

Figure 6 Example 2 Crude processing sequence

In terms of models statistics, the model increased to 22,757 equations and 20,522 variables, with 1350 of them being binary. The ratio of the nonlinear and discrete variables stayed at 28% and 6.6% respectively. The total time by DICOPT increased to 113.13 seconds (95% NLP and 5% MIP), requiring 3 major iterations.

Similar to example 1, the subcycle elimination constraints of Equation (59) are applied to this example reducing the profit from \$36.84MM to \$36.41MM (i.e. 1.2% optimality gap), and yielding a new crude processing sequence, as shown in Table 8 and Figure 7.

Table 8 Example 2 Economic results (\$1000's) after subcycle elimination

Profit	36413.4
Sales	337908.0
Crude oil cost	243857.0
Other feedstock cost	760.0
Inventory cost	2016.3
Operating cost	50761.5
Transition cost	4100.0

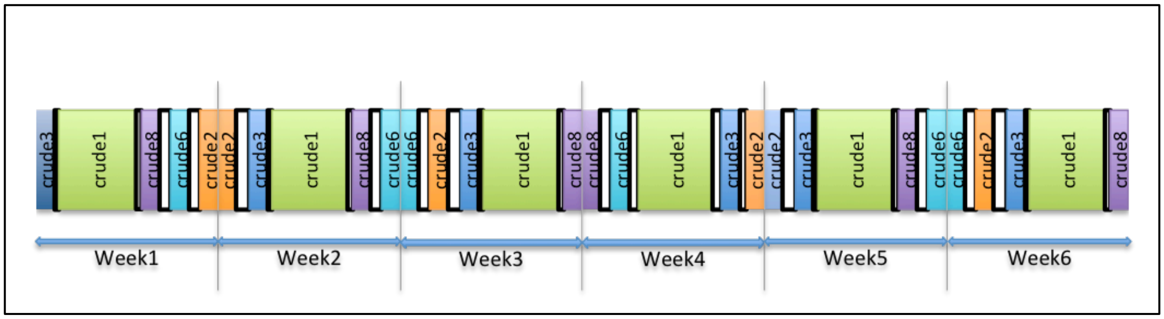

Figure 7 Example 2 Crude processing sequence after subcycle elimination

4.2. Modified crude oils run

The other criterion used to test robustness is changing the available crude oils. In example 3, the base case of 4 weeks/4 time periods horizon is used, but with 4 different crude oils. The economics of the profitable scenario calculated by the model is listed in Table 9. The feed rate, processing rate, sales and inventory results are listed in Table 10 and Table 11.

Table 9 Example 3 Economic results ($1000's)

Profit	28855.1
Sales	236323.0
Crude oil cost	172212.0
Other feedstock cost	425.9
Inventory cost	1309.2
Operating cost	31700.2
Transition cost	1820.0

Table 10 Example 3 Feed and processing time information

		Crude	week1	week2	week3	week4
Flow	Processing rate (1000's BPD)	CRUDE1	100.0	100.0	100.0	100.0
		CRUDE2	100.0	100.0	100.0	100.0
		CRUDE4	100.0	100.0	100.0	100.0
		CRUDE5	100.0	100.0	100.0	100.0
	Total Flow (1000's bbl)	CRUDE1	70.0	70.0	70.0	70.0
		CRUDE2	70.0	70.0	70.0	70.0
		CRUDE4	70.0	70.0	70.0	70.0
		CRUDE5	373.3	406.7	344.2	406.7
Time	Processing time (hr)	CRUDE1	16.8	16.8	16.8	16.8
		CRUDE2	16.8	16.8	16.8	16.8
		CRUDE4	16.8	16.8	16.8	16.8
		CRUDE5	89.6	97.6	82.6	97.6

Table 11 Example 3 Product quantities and inventory

	Product	week1	week2	week3	week4
Sales (1000's bbl)	FG	97.2	103.3	95.8	94.2
	PG	140.0	261.7	208.8	119.0
	RG	70.0	245.0	129.4	232.0
	Dist	26.8	27.0	15.2	50.1
	FO	102.3	109.1	96.4	110.1
	HTR	18.9	18.9	18.9	18.9
Inventories (1000's bbl)	FG	97.2	103.3	95.8	94.2
	PG	192.0	261.7	208.8	119.0
	RG	155.8	245.0	129.4	232.0
	Dist	26.8	27.0	15.2	50.1
	FO	102.3	109.1	96.4	110.1
	HTR	18.9	18.9	18.9	18.9

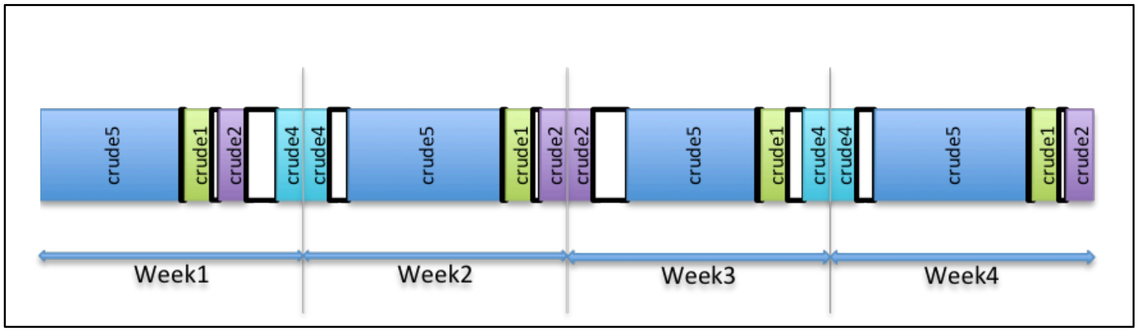

Figure 8 Example 3 Crude processing sequence

The fewer, but different, crude oils translated into a smaller model, with 12,005 equations and 10,937 variables, with 704 of them being binary. The same ratio of nonlinear variables is generated here (28%), but slightly smaller ratio of discrete variables (6.4%). The solver DICOPT required shorter time at 27.69 seconds (94% NLP and 6% MIP), requiring 3 major iterations.

4.3. Impact of the FI disjunctive formulation

As noted earlier, the previous work (Alattas et al, 2011) utilized the Heaviside function in Equation (12) instead of the current mixed-integer formulation (14)-(16) for the FI value choice. This resulted in an NLP model versus an MINLP model. To evaluate the impact on the solution and solver time by reducing the number of 0-1 variables in the multiperiod MINLP model, example 1 with the 5 crude oils and 4 week time horizon was solved using the Heaviside function formulation. The results are the same as in Table 1 through Table 4 and Figure 4. Thus, both formulations give the same results.

On the other hand, the two models differed in terms of the size of the MINLP model. The MINLP model using the Heaviside function for the FI disjunction included 12,347 constraints and 12,520 variables (320 binary). In contrast, the MINLP model using the mixed-integer constraints for the FI disjunction included 15,047 constraints and 13,680 variables (900 binary variables). However, it should be noted that despite the reduction of size with the Heaviside function, it actually includes a larger number of nonlinear terms, which in turn tend to be ill-conditioned. Therefore, despite the reduction in size and its relatively robust performance in the single-period NLP model, the Heaviside function formulation either failed to converge or required much longer solution time using the default options of solver DICOPT/CPLEX since many NLP subproblems were found to be infeasible, presumably due the difficulties in handling this function. Setting the option for DICOPT to linearize infeasible NLP subproblems gave the solution in 18.23 seconds. Though shorter than example 1 with the default DICOPT/CPLEX options, the Heaviside formulation in the multiperiod model is less robust than the proposed MINLP model in which linear mixed-integer constraints replace ill-conditioned nonlinearities.

5. Conclusion

In this paper we have addressed the multiperiod refinery planning problem by extending our previous single period nonlinear planning model that uses the FI model for the CDU. In this work, we replaced the simple Heaviside function formulation for the FI value choice by mixed-integer constraints. This improvement was more natural due the inherent MINLP nature of the multiperiod model making it also more robust.

For the multiperiod extension, the traveling salesman constraints were used to generate the sequence of selected crude oils in each time period. The combination of the FI model with the traveling salesman constraints produced an MINLP multiperiod refinery planning model that proved robust in terms of different planning horizon and different crude oils. However, the model for some crude oil combination produced subcycles. These subcycles were eliminated by adding appropriate subtour elimination constraints, yielding a near optimal solution. Althought the solution time increased from the single period model, the multiperiod MINLP model required reasonable solution times.

Finally, the multiperiod extension of the Heaviside formulation was less robust due to the ill-conditioning of the Heaviside function. The new MINLP formulation is more robust and more general in its application to both single-period and multiperiod problems.

Acknowledgment

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6. Nomenclature

Variables

$ArInv_{p,t}$	The area under the curve of the inventory of product p in time period t
$FCr_{cr,t}$	The processing rate of crude cr in time period t
$F_{j,i,cr,t}$	Feed stream of component i of crude cr to crude cut j in time period t
$Invi_{p,t}$	The initial inventory of product p at the start of time period t
$Inv_{p,t}$	The inventory of product p in time period t
$\alpha_{j,i,cr,t}$	Relative volatility as $K_{j,i,cr,t}/K_{j,ref,cr,t}$ in crude cut j for component i from crude cr at time period t with component ref
$K_{j,i,cr,t}$	Equilibrium constant in crude cut j for component i from crude cr at time period t
$PB_{j,I,cr,t}$	Bottom product stream of crude cut j of component i from crude cr in time period t
$PD_{j,i,cr,t}$	Top product stream of crude cut j of component i from crude cr in time period t
Profit	The total refinery profits
$Pr_{r,l',k,cr,t}$	Property r of the outlet stream l' of refinery unit k for crude feed cr in time period t
$Pv_{j,i,cr,t}$	Vapor pressure in crude cut j for component i from crude cr at time period t
$Sl_{p,t}$	The sales of product p in time period t
$ST_{l,k,cr,t}$	Stream l from refinery unit k for crude feed cr in time period t
$ST_{p,cr,t}$	Refinery product stream p for crude cr in time period t
$TI_{j,cr,t}$, $TE_{j,cr,t}$	Initial and end boiling point temperature values for crude cut *j* in time period t for crude feed cr
$T_{j,cr,t}$	Separation temperature of crude cut j for crude cr at time period t (cut point

	temperature)
$Trans_t$	The total transition time of time period t
$Tr_{j,i,cr,t}$	Reduced temperature of component i from crude cr in crude cut j in time period t
$x_{PB,j,icr,t}$	Component i composition fraction in bottom product stream PB of crude cut j from crude cr in time period t
$x_{PD,j,icr,t}$	Component i composition fraction in top product stream PD of crude cut j from crude cr in time period t
$XP_{p,cr,t}$	The production rate of produce p from crude feed cr in time period t
$\gamma_{i,j,cr,t}$	Place holder for the selected fractionation index (FI) value for component i and crude cr at the crude cut j in time period t
$\theta_{cr,t}$	Processing time of crude cr in time period t

Binary Variables

$xF_{cr,t}$	0-1 variable whether crude cr is processed first time period t
$xL_{cr,t}$	0-1 variable whether crude cr is processed last in time period t
$Y_{i,j,cr,trp}$	0-1 variable for stripping (FIs) or rectifying (FIr) value of the fractionation index associated with component i for crude cut j in time period t for crude feed cr
$YP_{cr,t}$	0-1 variable whether crude cr is processed in time period t
$ZP_{cr,ccr,t}$	0-1 variable whether crude cr is followed by crude ccr in time period t
$ZZP_{cr,ccr,t}$	0-1 variable whether the link between crude cr and ccr is broken in time period t
$ZZZ_{cr,ccr,t}$	0-1 variable whether crude cr followed by crude ccr in time period t crosses over to the next time period

Parameters

$a_{k,l,l',cr}$	Yield equation coefficient for refinery unit k for feed l and outlet feed l' for crude feed cr
$ALFCr_{cr,t}$	The lower limit on availability of crude cr in time period t
$AUFCr_{cr,t}$	The upper limit on availability of crude cr in time period t
$zt_{i,cr,t}$	The composition of component i in the feed crude oil cr in time period t
$capacity_{k}$	Capacity of refinery unit k
$CFCr_{cr,t}$	The cost of crude cr in time period t
$CInv_{p,t}$	The cost of inventory of product p in time period t
$COpC_{cr,t}$	The operating cost for crude feed cr in time period t
$CTrans_{cr,ccr}$	The transition cost from crude cr to crude ccr
$DemandL_{p,t}$	The lower limit of the demand for product p in time period t
$DemandU_{p,t}$	The upper limit of the demand for product p in time period t
FIr_{j}	Fractionation index value for the rectifying section of crude cut j
FIs_{j}	Fractionation index value for the stripping section of crude cut j
Ht	The length or duration of the time period
M_{L}	Big M value when the separation temperature is greater than the component boiling point
M_{U}	Big M value when the separation temperature is greater than the component boiling point
Pc_{i}	Critical pressure of component i
$Price_{p,t}$	The price of product p in time period t
$PR_{r,p,t}$	Property r Specification of refinery product p in time period t
PVA_{i}, PVB_{i}, PVC_{i},	Parameters for the vapor pressure equation
$T_{b,i}$	Boiling point temperature of component i
$\tau_{cr,ccr}$	The transition time from crude cr to

	crude ccr
ω_i	Eccentric factor of component i

Sets

HC_i	Set of hydrocarbon components in crude oil feed to the CDU
PsC_i	Set of pseudo-components in crude oil feed to the CDU
$SC_{m,t}$	Subset of crude oils in subcycle *m* in time period *t*

Subscripts

cr,ccr	Crude oil feed
i	Crude component i
j	Crude cut or separation unit j
k	Refinery process unit
l,l'	Refinery stream
LK_j	Light key of crude cut j
HK_j	Heavy key of crude cut j
p	Product
r	Property r of a refinery stream or product
t	Time period
m	Subcycle of processed crude oils

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Appendix A Data For the Example Problems

Table A-1 Example 1 Transition time in hours

cr	CRUDE1	CRUDE2	CRUDE3	CRUDE4	CRUDE5	CRUDE6	CRUDE7	CRUDE8
CRUDE1	0	5	8	10	15	10	15	5
CRUDE2	10	0	12	20	22	20	22	10
CRUDE3	4	6	0	12	16	12	16	20
CRUDE4	15	20	10	0	12	13	8	9
CRUDE5	3	8	9	17	0	20	17	8
CRUDE6	8	10	15	9	13	0	12	10
CRUDE7	12	20	22	5	8	10	0	20
CRUDE8	22	20	22	15	20	10	22	0

Table A-2 Example 1 Transition cost

cr	CRUDE1	CRUDE2	CRUDE3	CRUDE4	CRUDE5	CRUDE6	CRUDE7	CRUDE8
CRUDE1	0	100	160	200	300	240	40	180
CRUDE2	200	0	240	400	200	400	128	240
CRUDE3	80	120	0	240	320	160	156	400
CRUDE4	300	400	200	0	240	60	240	160
CRUDE5	60	160	180	340	0	100	400	60
CRUDE6	180	60	240	100	20	0	60	100
CRUDE7	240	100	20	60	160	180	0	240
CRUDE8	120	160	200	300	40	300	188	0

Table A-3 Product demands

Product	Limit type	week1	week2	week3	week4	week5	week6
RG	Minimum	10	35	10	12	35	12
PG	Maximum	20	80	40	17	40	80

Table A-4 Crude oil availability applied to each time period

	Maximum	Minimum
All crude oils	200	10

Table A-5 Crude oil prices ($/bbl) for all time periods

crude	Price
CRUDE1	75
CRUDE2	65
CRUDE3	75
CRUDE4	65
CRUDE5	75

CRUDE6	65
CRUDE7	75
CRUDE8	65

Table A-6 Unit capacities (1000's BPD)

unit	Maximum
CDU	100
Cat Ref	20
Cat Crack	30

Table A-7 Example 1 Additional Information ($/bbl)

Data	Value
Inventory Cost	0.00306
CDU Operating Cost	5
Reformer Operating cost	7.5
Catalytic Cracker Operating Cost – Light distillate feed	40
Catalytic Cracker Operating Cost – Gas oil feed	4
Hydrotreater Operating Cost	5

Table A-8 Summary of crude oil assays[23]

Crude oil	API	SG	LV% Distilled	TBP (K)
Crude1	37	0.8398	0	258.4
			5	315.7
			10	345.7
			30	445.1
			50	548.2
			70	669.4
			90	836.1
			95	910.3
			100	984.9
Crude2	33.1	0.8597	0	266.3
			5	327.4
			10	359.9
			30	465.8
			50	576.4
			70	712.2
			90	899.2
			95	988.8
			100	1078.4
Crude3	36.4	0.8428	0	263.7
			5	321.4

			10	351.4
			30	455.1
			50	559.4
			70	686.1
			90	853.4
			95	930.4
			100	1007.3
Crude4	33.2	0.8591	0	269.9
			5	330.5
			10	362.9
			30	466.3
			50	574.4
			70	705.4
			90	888.3
			95	975.4
			100	1062.6
Crude5	35.4	0.8478	0	267.7
			5	324.1
			10	353.1
			30	450.9
			50	554.5
			70	671.6
			90	847.1
			95	926.3
			100	1005.5
Crude6	30.8	0.8718	0	296.3
			5	349.7
			10	377
			30	486.4
			50	596.6
			70	734
			90	913.6
			95	999.6
			100	1085.5
Crude7	34.6	0.8519	0	264.5
			5	327.9
			10	362.3
			30	477.6
			50	583
			70	697.4
			90	855.2

			95	928.7
			100	1002.2
Crude8	32.4	0.8633	0	271.5
			5	332.9
			10	366.3
			30	472.7
			50	579.3
			70	717.5
			90	905.2
			95	995.2
			100	1085.2

Table A-9 Refinery Product Prices ($/bbl) for time periods

Product	Price
FG	35
PG	135
RG	121
Dist	87
FO	76.5