Impact of Reliability Formulations on the Optimal Planning and Operation of Power Systems

Seolhee Cho, Javier Tovar-Facio, Ignacio E. Grossmann

***Abstract*—This study analyzes four expansion planning models with different reliability formulations, such as a reserve margin model, a *N-k* reliability model, and a probabilistic model using the probability of failures, as well as the reference model without reliability. Two reliability criteria, loss of load expectation (LOLE) and expected energy not served (EENS), are used to evaluate the power system reliability. A reliability evaluation strategy is proposed for a valid comparison of the four models with different objective functions and constraints. The effectiveness of the four models is verified with two case studies: an illustrative example and a San Diego County example. The results show that while the models that rigorously estimate reliability (i.e., models using *N-k* reliability or probabilistic constraint) require higher investment and operating costs, they can secure higher reliability than the models using simplified reliability constraints (i.e., reserve margin).**

***Index Terms*—Power systems, Expansion planning, Reliability, Probability of failure**

I. INTRODUCTION

POWER system reliability assessment involves two key aspects, *security* and *adequacy*. Security refers to the ability of power systems to maintain stable operation despite unplanned outages, reflecting their response to disturbances during operation. Adequacy ensures sufficient generation, transmission, and distribution capacity to meet load demand and operational constraints under normal operating conditions [1], [2]. In particular, adequacy can be evaluated using two approaches: a deterministic approach and a probabilistic approach. There are two deterministic approaches, such as *reserve margin* and *N-k reliability*, and two probabilistic approaches, such as *Monte Carlo simulation* and *Analytic method using probability of failure*. The reserve margin method refers to installing a larger capacity than required by a certain level to prevent power shortages. This method has been commonly used in expansion planning models due to its computational simplicity. On the other hand, *N-k* reliability expands the capacity of power systems to withstand *k* unit failures. While *N-2*, *N-3*, ..., and *N-k* reliability methods consider multiple unit failures, the *N-1*, which only focuses on one unit failure, is commonly favored in expansion planning due to its relative simplicity [3]. Several expansion planning studies of power systems using the deterministic approach have been reported. Ochoa-Barragán et al. [4] propose an optimization model for expansion planning of power generation and transmission systems, accounting for the degradation and replacement of energy storage systems. Lara et al. [5] and Li et al. [6] propose mixed-integer linear programming (MILP) models and algorithms that optimize long-term investment and hourly operation decisions. In particular, reserve margin reliability constraints are applied to both models for reliability. Castelli et al. [7] propose a two-stage stochastic MILP model that determines design decisions as first-stage variables and operation strategies as second-stage variables. In particular, the authors evaluate two cases, with and without *N-1* reliability requirement. Despite multiple studies, one limitation of the deterministic approach is that it does not properly capture random failures of generators.

A probabilistic approach is more rigorous and accurate than the deterministic approach because it explicitly includes multiple failure scenarios (or states) to predict power system reliability, and can account for the inherent features of individual components [8]. Monte Carlo simulation generates failure scenarios and assesses the reliability using probability distribution functions associated with the components involved in each scenario [9]. In contrast, analytical methods evaluate the reliability through mathematical models, including state space methods using Markov processes, network reduction methods, conditional probability methods, and cut-set or tie-set methods [10], [11]. Many works integrating probabilistic approaches into expansion planning have been reported. Choi et al. [12] present a new methodology to evaluate the reliability of power generation systems using Monte Carlo simulation. Tekiner et al. [13] propose a two-stage stochastic optimization model for power system expansion planning that minimizes cost and greenhouse gas emissions by using Monte Carlo simulation. Lei et al. [14] propose a novel reliability assessment approach for transmission expansion planning to efficiently estimate reliability enhancement. By categorizing contingencies into three scenarios, the approach reduces computational complexity. Cho et al. [15] develop a *Generalized Disjunctive Programming* model for long-term capacity expansion planning of reliable power generation systems. In particular, the analytical method using the probability

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S. Cho and I. E. Grossmann are with the Department of Chemical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA (e-mail: seolheec@andrew.cmu.edu; grossmann@cmu.edu)

J. Tovar-Facio is with Facultad de Ciencias Químicas, Universidad Autónoma de Chihuahua, Chihuahua, Chih 31125, Mexico (e-mail: jtovarf@uach.mx)

of equipment failures and capacity failure states is used to rigorously estimate the power system reliability. Dehghan et al. [16] propose a reliability-constrained robust expansion planning model and a tri-level decomposition algorithm that addresses uncertainties in electricity demand, wind power generation, and equipment availability. Jooshaki et al. [17] present an MILP model for multistage distribution network expansion planning that effectively incorporates reliability. The proposed model leverages an efficient reliability evaluation technique and accounts for radial operation and reliability-related costs, such as incentive schemes and revenue loss from undelivered energy. Heylen et al. [18] present a modular framework for transmission systems operators to evaluate and compare power system reliability criteria in operational planning and real-time decision-making. By using a probabilistic approach, the framework demonstrates improvements in both cost and reliability. While various studies using different reliability formulations have been performed, to the best of our knowledge, a comprehensive comparison of the various reliability methods in capacity expansion planning has not been conducted. Therefore, the main goal of this paper is to analyze the impact of each reliability assessment model on the optimal design and operation of power systems.

II. Background: reliability evaluation and backup system

A. Probabilistic methods for reliability evaluation

Securing reliability is important, but it is also critical to know how to evaluate it quantitatively. Loss of load expectation (LOLE) and expected energy not served (EENS) are common reliability evaluation criteria [19]. Both criteria are evaluated based on the probability of failure (also known as forced outage rate). While LOLE indicates the expected number of hours or days that the power demand is not satisfied [20], EENS stands for the expected power demand that is not satisfied due to failures of power systems [21]. In particular, $LOLE \leq 0.1$days/year (corresponding to 2.4hours/year) is the most common reliability target used throughout North America's ISOs and RTOs, such as Midcontinent Independent System Operator (MISO), Electric Reliability Council of Texas (ERCOT), and California ISO (CAISO) [22]. Moreover, each ISO and RTO has different costs for the EENS (technically called in the literature as the value of loss load), such as MISO ($\sim$ \$3.5/kWh-\$10.0/kWh) [23], ERCOT ($\sim$ \$5.0/kWh) [24], and CAISO ($\sim$ \$12.0-29.5/kWh) [25]. The detailed method for calculating LOLE and EENS can be found in Appendix A of Supplementary Materials.

B. The role of backup generators for reliability

The analytic method more accurately predicts the required capacity to meet the reliability as it enumerates all possible capacity failure states and estimates the probability of each state using the probability of failures. Since the number of capacity failures increases exponentially as the number of generators (or transmission lines) increases, this approach has been known to be mathematically challenging. One way to improve power system reliability in the analytical method is to add backup generators in parallel, allowing power plants to operate when failures occur [26]. In this paper, backup generators refer to reduced-sized generators installed in power plants, which can be used in emergencies. This approach, adding backup facilities in parallel, is known as a 'reliability-based design optimization (RBDO)' approach in reliability engineering [27]. However, the RBDO approach has not been actively used in the expansion planning of power systems. In some previous studies, it is assumed that backup generators can only be used when the main generators fail [28]. However, in our previous work [15], [29], it is allowed for the backup generators to have a dual role; they can either remain as backups, or else operate to produce additional electric power. Although the amount of electricity produced by power plants increases as the main and backup generators are operated, the power plant in this operation mode is less likely to constantly satisfy the required power output because no available generators can be used in emergencies. Therefore, it is critical to evaluate the impact of the dual role of parallel generators on the optimal design and operation of reliable power systems.

III. Problem statement

This paper aims to evaluate expansion planning models with different reliability considerations. In particular, this paper examines three expansion planning models with different reliability constraints: reserve margin, *N-k* reliability, and analytical method using the probability of failure. An expansion planning model without reliability consideration is also used as a reference. Four optimization models are developed using Generalized Disjunctive Programming (GDP) [30]. The objective is to minimize the total cost, including capital expenditure, operating expenses, and penalties. Relevant inputs, such as demand profile, available capacity for generators and transmission lines, the probability of failure of generators and lines, and capital and operating cost parameters, are given. The following design and operation variables are to be determined: number/size of generators and transmission lines, power flow between nodes, and reliability level (LOLE and EENS values). The following assumptions are made: i) Unit commitment is not included in the model, ii) a simple network flow model is used for transmission lines, and iii) only dispatchable generators are considered.

IV. Optimization model

Sets, parameters, and variables defined in this model can be found in the Nomenclature of the Supplementary Materials.

A. Model 1: Expansion planning model without reliability

The expansion planning model without reliability can be formulated as follows:

$$
\begin{aligned}
&min\ \Phi \quad \sum_{t\in\mathcal{T}}\Bigg[\Big(\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}^{\text{PN}}} IC_{i,k,t}+\sum_{l\in\mathcal{L}^{\text{PN}}} IC_{l,t}\Big)+\\
&\Big(\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}\beta_{k,t}C^{\text{av}}_{i,k,t}+\sum_{l\in\mathcal{L}}\beta_{l,t}C^{\text{av}}_{l,t}\Big)+ \qquad (1)\\
&\sum_{n\in\mathcal{N}}\sum_{b\in\mathcal{B}}\eta_n\varrho_b\Big(\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}\gamma_{k,t}P_{i,k,t,n,b}+\sum_{l\in\mathcal{L}}\gamma_{l,t}F_{l,t,n,b}\Big)\Bigg]
\end{aligned}
$$

$$s.t. \begin{bmatrix} Y^{\text{in}}_{i,k,t} \\ \underline{\varphi_k} \le C^{\text{in}}_{i,k,t} \le \overline{\varphi_k} \\ IC_{i,k,t} = \alpha_{k,t} C^{\text{in}}_{i,k,t} \end{bmatrix} \vee \begin{bmatrix} \neg Y^{\text{in}}_{i,k,t} \\ C^{\text{in}}_{i,k,t} = 0 \\ IC_{i,k,t} = 0 \end{bmatrix} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}^{\text{PN}}, t \in \mathcal{T} \tag{2}$$

$$\begin{bmatrix} Y^{\text{in}}_{l,t} \\ \underline{\varphi_l} \le C^{\text{in}}_{l,t} \le \overline{\varphi_l} \\ IC_{l,t} = \alpha_{l,t} C^{\text{in}}_{l,t} \end{bmatrix} \vee \begin{bmatrix} \neg Y^{\text{in}}_{l,t} \\ C^{\text{in}}_{l,t} = 0 \\ IC_{l,t} = 0 \end{bmatrix} \quad \forall l \in \mathcal{L}^{\text{PN}}, t \in \mathcal{T} \tag{3}$$

$$C^{\text{av}}_{i,k,t} = \left\{ \begin{array}{ll} \omega_{i,k} & \forall k \in \mathcal{K}^{\text{EX}} \\ \sum\limits_{tq \in [1,t]} C^{\text{in}}_{i,k,t} & \forall k \in \mathcal{K}^{\text{PN}} \end{array} \right\} \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{4}$$

$$C^{\text{av}}_{l,t} = \left\{ \begin{array}{ll} \omega_{l} & \forall l \in \mathcal{L}^{\text{EX}} \\ \sum\limits_{tq \in [1,t]} C^{\text{in}}_{l,t} & \forall l \in \mathcal{L}^{\text{PN}} \end{array} \right\} \quad \forall t \in \mathcal{T} \tag{5}$$

$$\underline{\theta_k} C^{\text{av}}_{i,k,t} \le P_{i,k,t,n,b} \le \overline{\theta_k} C^{\text{av}}_{i,k,t} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B} \tag{6}$$

$$P_{i,k,t,n,b} - P_{i,k,t,n,b-1} \le \kappa^U_k C^{\text{av}}_{i,k,t} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b > 1, b \in \mathcal{B} \tag{7}$$

$$P_{i,k,t,n,b-1} - P_{i,k,t,n,b} \le \kappa^D_k C^{\text{av}}_{i,k,t} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b > 1, b \in \mathcal{B} \tag{8}$$

$$-C^{\text{av}}_{l,t} \le F_{l,t,n,b} \le C^{\text{av}}_{l,t} \quad \forall l \in \mathcal{L}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B} \tag{9}$$

$$\sum_{k \in \mathcal{K}} P_{i,k,t,n,b} + \sum_{l \in \mathcal{L}^{in}_i} F_{l,t,n,b} \ge \Psi_{i,t,n,b} + \sum_{l \in \mathcal{L}^{out}_i} F_{l,t,n,b} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B} \tag{10}$$

Eqn. (1) represents the objective function that includes investment costs, fixed operating costs, and variable operating costs. Eqns. (2)-(3) ensure that when a generator k and a transmission line l are installed in year t, the corresponding installed capacity and the investment cost are determined. Eqns. (4)-(5) calculate the capacity of the generator k and the transmission line l that are available in year t. Eqn. (6) represents the operation of the dispatchable generator k. Eqns. (7)-(8) represent the ramp-up and ramp-down constraints of the dispatchable generator k. The power flow of the transmission line l is constrained by its capacity, as seen in Eqn. (9), and the power balance is represented by Eqn. (10).

B. Model 2: Expansion planning model with reserve margin constraint

The reserve margin is the simplest method of considering reliability in the expansion planning model. Reserve margin refers to a capacity that is installed but not used under normal operation. This capacity is assumed to be used when demand unexpectedly increases or the operating generators fail. New continuous variables for operation and reserve capacity of generators ($C^{\text{op}}_{i,k,t,n,b}$ and $C^{\text{rv}}_{i,k,t,n,b}$) are introduced. The corresponding GDP model is given as follows:

$$min\ \Phi \quad Eqn. \quad (1)$$

$$s.t. \quad Eqn. \quad (2)-(5), (9)-(10)$$

$$C^{\text{av}}_{i,k,t} = C^{\text{op}}_{i,k,t,n,b} + C^{\text{rv}}_{i,k,t,n,b} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B} \tag{11}$$

$$\underline{\theta_k} C^{\text{op}}_{i,k,t,n,b} \le P_{i,k,t,n,b} \le \overline{\theta_k} C^{\text{op}}_{i,k,t,n,b} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B} \tag{12}$$

$$P_{i,k,t,n,b} - P_{i,k,t,n,b-1} \le \kappa^U_k C^{\text{op}}_{i,k,t,n,b} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b > 1, b \in \mathcal{B} \tag{13}$$

$$P_{i,k,t,n,b-1} - P_{i,k,t,n,b} \le \kappa^D_k C^{\text{op}}_{i,k,t,n,b} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b > 1, b \in \mathcal{B} \tag{14}$$

$$\sum_{k \in \mathcal{K}} C^{\text{rv}}_{i,k,t,n,b} \ge \chi \Psi_{i,t,n,b} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B} \tag{15}$$

Eqn. (11) determines the available capacity from the operation capacity and reserve capacity. The electricity generated by the generator k during subperiod b of the representative day n in year t is constrained by the operation capacity, as shown in Eqn. (12). Eqns. (13)-(14) correspond to the ramp-up and ramp-down constraints, which are constrained by the operation capacity. Eqn. (15) states that the reserve capacity should be greater than a certain level (χ) of the load demand.

C. Model 3: Expansion planning model with N-k reliability constraint

N-k reliability ensures that the power systems have sufficient capacity to satisfy the load demand even if k number of components fail. A new set, failure scenario $sc \in \mathcal{SC}$, is introduced. Operation constraints such as nodal power balance and power flow must be satisfied in every failure scenario sc. All parameters and variables related to the operation constraints are updated by adding the set of failure scenarios sc. The corresponding GDP model is given as follows:

$$min\ \Phi \sum_{t \in \mathcal{T}} \Bigg\{ \Big(\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}^{\text{PN}}} IC_{i,k,t} + \sum_{l \in \mathcal{L}^{\text{PN}}} IC_{l,t} \Big) + \sum_{n \in \mathcal{N}} \sum_{sc \in \mathcal{SC}} \eta_n \varepsilon_{sc} \Bigg[\Big(\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} \beta_{k,t} C^{\text{sv}}_{i,k,t,n,sc} + \sum_{l \in \mathcal{L}} \beta_{l,t} C^{\text{sv}}_{l,t,n,sc} \Big) + \sum_{b \in \mathcal{B}} \varrho_b \Big(\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} \gamma_{k,t} P_{i,k,t,n,b,sc} + \sum_{l \in \mathcal{L}} \gamma_{l,t} F_{l,t,n,b,sc} \Big) \Bigg] \Bigg\} \tag{16}$$

$$s.t. \quad Eqn. \quad (2)-(5)$$

$$C^{\text{sv}}_{i,k,t,n,sc} = \xi_{i,k,t,n,sc} C^{\text{av}}_{i,k,t} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, sc \in \mathcal{SC} \tag{17}$$

$$C^{\text{sv}}_{l,t,n,sc} = \xi_{l,t,n,sc} C^{\text{av}}_{l,t} \quad \forall l \in \mathcal{L}, t \in \mathcal{T}, n \in \mathcal{N}, sc \in \mathcal{SC} \tag{18}$$

$$\underline{\theta_k} C^{\text{sv}}_{i,k,t,n,sc} \le P_{i,k,t,n,b,sc} \le \overline{\theta_k} C^{\text{sv}}_{i,k,t,n,sc} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B}, sc \in \mathcal{SC} \tag{19}$$

$$P_{i,k,t,n,b,sc} - P_{i,k,t,n,b-1,sc} \le \kappa^U_k C^{\text{sv}}_{i,k,t,n,sc} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b > 1, sc \in \mathcal{SC} \tag{20}$$

$$P_{i,k,t,n,b-1,sc} - P_{i,k,t,n,b,sc} \le \kappa^D_k C^{\text{sv}}_{i,k,t,n,sc} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b > 1, sc \in \mathcal{SC} \tag{21}$$

$$-\,C^{\text{sv}}_{l,t,n,sc} \le F_{l,t,n,b,sc} \le C^{\text{sv}}_{l,t,n,sc} \quad \forall l \in \mathcal{L}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B}, sc \in \mathcal{SC} \tag{22}$$

$$\sum_{k\in\mathcal{K}} P_{i,k,t,n,b,sc} + \sum_{l\in\mathcal{L}_i^{in}} F_{l,t,n,b,sc} \ge \Psi_{i,t,n,b} + \sum_{l\in\mathcal{L}_i^{out}} F_{l,t,n,b,sc} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B}, sc \in \mathcal{SC} \tag{23}$$

Eqn. (16) represents the modified objective function, including investment costs, fixed operating costs, and variable operating costs. The fixed and variable operating costs are calculated for each failure scenario sc. Eqns. (17)-(18) determine the available capacity of the generator and the transmission line at each failure scenario sc ($C^{\text{sv}}_{i,k,t,n,sc}$ and $C^{\text{sv}}_{l,t,n,sc}$), in which $\xi_{i,k,t,n,sc}$ and $\xi_{l,t,n,sc}$ indicate whether or not the generator k and the transmission line l operate in the failure scenario sc. It is assumed that the failed generator or transmission line on a particular day n at a certain failure scenario sc will be fully repaired within one day. As stated in Eqns. (19)-(21), the power output from the generator is determined by the capacity available in each scenario sc. Likewise, the power flow of the transmission line is also constrained by the capacity available in each scenario sc (Eqn. (22)). Eqn. (23) requires that the demand always be satisfied in each failure scenario sc.

D. Model 4: Expansion planning model using the probability of failure

As explained in Appendix A of Supplementary Materials, all possible capacity failure states should be enumerated to use the analytical method and the corresponding probability of failure states should be estimated using the failure rates. A new set, capacity failure state $st \in \mathcal{ST}$, is introduced. As opposed to Models 1-3, Model 4 has another investment option, adding backup generators to the main generator to improve reliability. It should be noted that the *N-k* reliability considers the failure of k unit failures at each failure scenario, and the load demand should be satisfied in all scenarios (i.e., load shedding is not allowed). On the other hand, the analytical method using the probability of failure calculates the optimal capacity based on the reliability penalty (such as LOLE and EENS penalties), while considering from a single failure up to all failures, and allowing for load shedding. The corresponding GDP model is given as follows:

$$\begin{aligned} min\ \Phi \sum_{t\in\mathcal{T}} \Bigg\{ & \sum_{i\in\mathcal{I}} \sum_{k\in\mathcal{K}^{\text{PN}}} IC_{i,k,t} + \sum_{i\in\mathcal{I}} \sum_{k\in\mathcal{K}} IC^B_{i,k,t} + \\ & \sum_{l\in\mathcal{L}^{\text{PN}}} IC_{l,t} + \sum_{i\in\mathcal{I}} \sum_{k\in\mathcal{K}} \beta^B_{k,t} C^B_{i,k,t} + \\ & \sum_{st\in\mathcal{ST}} \delta_{st} \Bigg[\sum_{i\in\mathcal{I}} \sum_{k\in\mathcal{K}} \beta_{k,t} C^{\text{sv}}_{i,k,t,st} + \sum_{l\in\mathcal{L}} \beta_{l,t} C^{\text{sv}}_{l,t,st} + \\ & \sum_{n\in\mathcal{N}} \sum_{b\in\mathcal{B}} \eta_n \varrho_b \Bigg(\sum_{i\in\mathcal{I}} \sum_{k\in\mathcal{K}} \gamma_{k,t} P_{i,k,t,n,b,st} + \gamma_{k,t} P^B_{i,k,t,n,b,st} + \\ & \sum_{l\in\mathcal{L}} \gamma_{l,t} F_{l,t,n,b,st} + \sum_{i\in\mathcal{I}} \psi \mathrm{O}_{i,t,n,b,st} + \sum_{i\in\mathcal{I}} \sigma EENS_{i,t,n,b,st} \Bigg) \Bigg] \Bigg\} \end{aligned} \tag{24}$$

$$s.t. \quad Eqn. \quad (2)-(5)$$

$$\begin{bmatrix} Y^B_{i,k,t} \\ \underline{\varphi^B_k} \le C^{Bn}_{i,k,t} \le \overline{\varphi^B_k} \\ IC^B_{i,k,t} = \alpha^B_{k,t} C^{Bn}_{k,t} \end{bmatrix} \vee \begin{bmatrix} \neg Y^B_{i,k,t} \\ C^{Bn}_{i,k,t} = 0 \\ IC^B_{i,k,t} = 0 \end{bmatrix} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}^{\text{PN}}, t \in \mathcal{T} \tag{25}$$

$$C^B_{i,k,t} = \left\{ \begin{array}{ll} \omega^B_{i,k} & \forall k \in \mathcal{K}^{\text{EX}} \\ \sum\limits_{tq\in[1,t]} C^{Bn}_{i,k,t} & \forall k \in \mathcal{K}^{\text{PN}} \end{array} \right\} \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{26}$$

$$C^B_{i,k,t} \le C^{\text{av}}_{i,k,t} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T} \tag{27}$$

$$C^{\text{sv}}_{i,k,t,st} = \upsilon^M_{i,k,st} C^{\text{av}}_{i,k,t} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, st \in \mathcal{ST} \tag{28}$$

$$C^{B^{\text{sv}}}_{i,k,t,st} = \upsilon^B_{i,k,st} C^B_{i,k,t} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, st \in \mathcal{ST} \tag{29}$$

$$C^{\text{sv}}_{l,t,st} = \upsilon^L_{l,st} C^{\text{av}}_{l,t} \quad \forall l \in \mathcal{L}, t \in \mathcal{T}, st \in \mathcal{ST} \tag{30}$$

$$\underline{\theta_k} C^{\text{sv}}_{i,k,t,st} \le P_{i,k,t,n,b,st} \le \overline{\theta_k} C^{\text{sv}}_{i,k,t,st} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B}, st \in \mathcal{ST} \tag{31}$$

$$P_{i,k,t,n,b,st} - P_{i,k,t,n,b-1,st} \le \kappa^U_k C^{\text{sv}}_{i,k,t,st} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b > 1, st \in \mathcal{ST} \tag{32}$$

$$P_{i,k,t,n,b-1,st} - P_{i,k,t,n,b,st} \le \kappa^D_k C^{\text{sv}}_{i,k,t,st} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b > 1, st \in \mathcal{ST} \tag{33}$$

$$\underline{\theta_k} C^{B^{\text{sv}}}_{i,k,t,st} \le P^B_{i,k,t,n,b,st} \le \overline{\theta_k} C^{B^{\text{sv}}}_{i,k,t,st} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B}, st \in \mathcal{ST} \tag{34}$$

$$P^B_{i,k,t,n,b,st} - P^B_{i,k,t,n,b-1,st} \le \kappa^U_k C^{B^{\text{sv}}}_{i,k,t,st} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b > 1, st \in \mathcal{ST} \tag{35}$$

$$P^B_{i,k,t,n,b-1,st} - P^B_{i,k,t,n,b,st} \le \kappa^D_k C^{B^{\text{sv}}}_{i,k,t,st} \quad \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, n \in \mathcal{N}, b > 1, st \in \mathcal{ST} \tag{36}$$

$$-\,C^{\text{sv}}_{l,t,st} \le F_{l,t,n,b,st} \le C^{\text{sv}}_{l,t,st} \quad \forall l \in \mathcal{L}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B}, st \in \mathcal{ST} \tag{37}$$

$$\begin{aligned} & \sum_{k\in\mathcal{K}} \Big(P_{i,k,t,n,b,st} + P^B_{i,k,t,n,b,st} \Big) + \sum_{l\in\mathcal{L}_i^{in}} F_{l,t,n,b,st} + \\ & LS_{i,t,n,b,st} = \Psi_{i,t,n,b} + \sum_{l\in\mathcal{L}_i^{out}} F_{l,t,n,b,st} + \mathrm{O}_{i,t,n,b,st} \\ & \forall i \in \mathcal{I}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B}, st \in \mathcal{ST} \end{aligned} \tag{38}$$

$$\begin{bmatrix} Z_{i,t,n,b,st} \\ LS_{i,t,n,b,st} \le 0 \\ LOLE_{i,t,n,b,st} = 0 \\ EENS_{i,t,n,b,st} = 0 \end{bmatrix} \vee \begin{bmatrix} \neg Z_{i,t,n,b,st} \\ LS_{i,t,n,b,st} \ge 0 \\ LOLE_{i,t,n,b,st} = \varrho_b \\ EENS_{i,t,n,b,st} = LS_{i,t,n,b,st} \end{bmatrix} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, n \in \mathcal{N}, b \in \mathcal{B}, st \in \mathcal{ST} \tag{39}$$

$$\sum_{i\in\mathcal{I}} \sum_{n\in\mathcal{N}} \sum_{b\in\mathcal{B}} \sum_{st\in\mathcal{ST}} \eta_n \delta_{st} LOLE_{i,t,n,b,st} \le \nu_t \quad \forall t \in \mathcal{T} \tag{40}$$

$$\delta_{st} = \prod_{k\in\mathcal{K}^O_{st}} (1-\lambda_k) \prod_{l\in\mathcal{L}^O_{st}} (1-\lambda_l) \prod_{k\in\mathcal{K}^F_{st}} \lambda_k \prod_{l\in\mathcal{L}^F_{st}} \lambda_l \quad \forall st \in \mathcal{ST} \tag{41}$$

Eqn. (24) represents the modified objective function, which includes investment costs, fixed and variable operating costs, over-generation penalty, and the EENS penalty. Investment constraints such as the installation of main generators and transmission lines are applied using Eqns. (2)-(5). Another investment constraint related to the installation of backup generators is shown in Eqns. (25)-(26). It should be noted that as the backup generators are assumed to be auxiliary generators for reliability, the capacity of backup generators should be less or equal than that of the main generators, as shown in Eqn. (27). The available capacity of the main generator, the backup generator, and the transmission line at each capacity failure state st ($C^{\text{sv}}_{i,k,t,st}$, $C^{B^{\text{sv}}}_{i,k,t,st}$, and $C^{\text{sv}}_{l,t,st}$) is estimated using the parameters indicating whether the generator k and the transmission line l are active in the failure state st, (i.e., $\upsilon^{M}_{i,k,st}$, $\upsilon^{B}_{i,k,st}$ and $\upsilon^{L}_{l,st}$), as stated in Eqns. (28)-(30). Eqns. (31)-(36) constrain the power outputs from the main generator k and the backup generator parallel to the main generator k. Eqn. (37) states the power flow of the transmission line l is constrained by the capacity available at the failure state st. As mentioned before, the power balance (Eqn. (38)) is modified as load shedding ($LS_{i,t,n,b,st}$) is allowed. However, as shown in Eqn. (39), there will be LOLE and EENS if there is load-shedding at the capacity failure state st; otherwise, there will be zero LOLE and EENS. While EENS is penalized in the objective function as seen in Eqn. (24), LOLE is specified as a constraint as shown in Eqn. (40), in which ν_t is a target LOLE (i.e., 2.4 hours/year). The probability of each failure state (δ_{st}) is calculated as shown in Eqn. (41), where $\mathcal{K}^{O}_{st}$ and $\mathcal{L}^{O}_{st}$ are the sets of active generator k and transmission line l at each capacity failure state st. $\mathcal{K}^{F}_{st}$ and $\mathcal{L}^{F}_{st}$ are the sets of failed generator k and transmission line l at each capacity failure state st, and λ_k and λ_l are the probability of failure of generator k and transmission line l, respectively.

V. Evaluation strategy

As the four optimization models (Models 1-4) in section 4 have different objective functions and constraints, the objective function values cannot be directly compared. Therefore, in this paper, we propose the following evaluation strategy to effectively compare the four models. Models 1-3 involve a two-step procedure, as seen in Figure 1. In the first step, Models 1-3 determine the optimal design, such as the number and capacity of generators and transmission lines, and the optimal operation solutions, such as power production from each generator and power flow of transmission lines, without considering over-generation, LOLE, and EENS. After obtaining the optimal design results, in the second step, investment variables in Model 4 are fixed to the optimal design results in Models 1-3. The capacity failure states from one facility failure up to all facility failures are enumerated for the fixed design results, and the probability of each failure state is calculated using the probability of failure of the selected generators and transmission lines. By running Model 4 with the fixed design variables from Models 1-3, LOLE and EENS for the fixed design variables are calculated. The impact of different formulations on the reliability of power systems can then be evaluated by comparing the LOLE and EENS of each model.

TABLE I
Method generation

Label	Model used	Feature
M1	Model 1	No reliability
M2	Model 2	Reserve margin (i.e., 25%)
M3-a	Model 3	One generator failure scenarios (*N-1*)
M3-b		Two generators failure scenarios (*N-2*)
M4-a	Model 4	Dual role of backup generators is **not** allowed
M4-b		Dual role of backup generators is allowed

VI. Case studies

Pyomo.GDP is used as the optimization modeling software [31], and Gurobi is selected as the MILP solver [32]. All cases are implemented on a Linux server running Ubuntu with 1TB of RAM and 4 Intel(R) Xeon(R) Gold 6234 CPUs (3.30 GHz) with 8 cores each. Python 3.10.12, Pyomo 6.6.2, and Gurobi 10.0.2 are used; the Big-M reformulation is chosen to reformulate the GDP model into the MILP model [33]. This work proposes six methods using the four models, as seen in Table I. **M1** uses Model 1 without reliability consideration. **M2** uses Model 2 with the reserve margin constraint; in particular, the reserve margin rate is assumed to be 25%. **M3-a** and **M3-b** use Model 3, but **M3-a** only considers single generator failure scenarios (i.e., *N-1*), whereas **M3-b** considers two generator failure scenarios (i.e., *N-2*). **M4-a** and **M4-b** employ the analytical method using the probability of failure and can add backup generators for reliability. However, **M4-a** assumes that the added backup generator can only be used when the main generator fails (i.e., the dual role of backups is not allowed). In contrast, **M4-b** assumes the backup can be used with the main generator to increase power production (i.e., the dual role of backups is allowed).

A. Case 1: Illustrative example

As shown in Figure B.1 in Supplementary Materials, the illustrative example has three nodes, each with one natural gas combined cycle (NGCC) power plant. The proposed example is a 3-year planning problem with four representative days (i.e., average demand day from each season) and 24 operating hours. While Nodes 1 and 2 are assumed to be supply and demand nodes (marked in orange), Node 3 is assumed to be a supply-only node (marked in green). The NGCC and transmission lines (i.e., the lines that connect the nodes) can be installed and expanded during the planning horizon to meet the demand. All technical and economic data used for this case study can be found in Table B.1 of Supplementary Materials.

37 *N-1* and 37 *N-2* failure scenarios are generated for M3-a and M3-b, respectively. A normal operation scenario where all three generators are available is included in both *N-1* and *N-2* failure scenarios. For *N-1* method, 36 scenarios are generated, assuming one generator fails on a representative day in the planning year. Likewise, 36 two generator failure scenarios are generated for *N-2* method. The failures of transmission lines are not considered when generating failure scenarios. Similarly, 8 capacity failure states are generated for M4-a and M4-b from all available to all failures. The failure

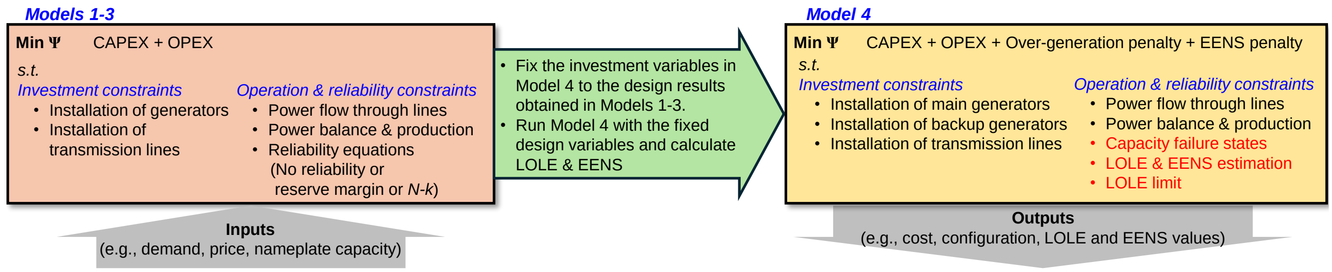

Fig. 1. Reliability evaluation strategy (two-step scheme for Models 1-3, and one-step for Model 4)

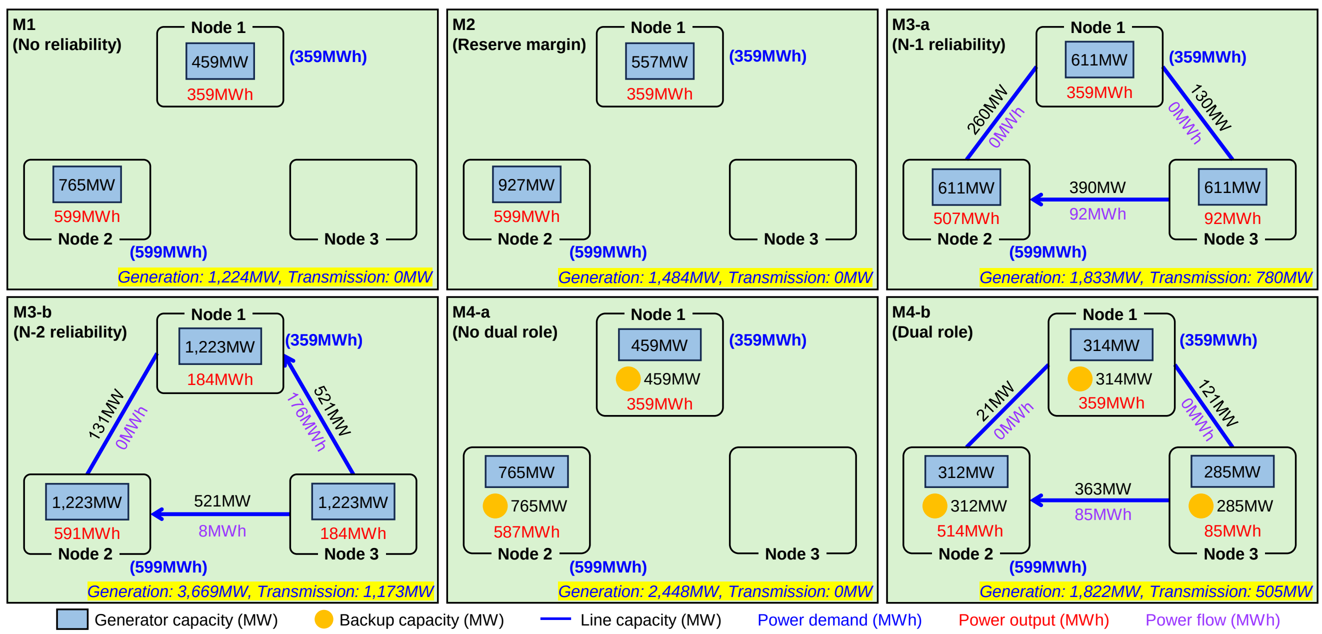

Fig. 2. The optimal configuration and a snapshot operation result during the peak demand hour on the last representative day in the last year of Case 1

of transmission lines is not considered either, but the load shedding is allowed.

1) Optimal design, operation, and reliability results: Figure 2 depicts the optimal configuration of the generators and lines in the last year, and a snapshot operation result during the peak demand hour. M1 and M2 install two generators in Nodes 1 and 2. Transmission lines are not installed, and Node 3 is not used. Contrary to M1 and M2, the results of M3-a and M3-b show that generators are distributed throughout all nodes rather than centralizing two nodes to avoid the entire failure of the power systems. Three transmission lines are added to connect all nodes. As M3-b considers more extreme failure situations, such as a simultaneous failure of two generators, the generation capacity of M3-b is 2 times larger than that of M3-a (i.e., M3-a: 1,833MW; M3-b: 3,669MW). M4-a has the same main generator capacity as M1; however, in M4-a, backup generators are added to the main generators rather than expanding the capacity of the main generators to avoid the failure of the power systems. On the other hand, the generation capacity of M4-b is smaller than M4-a due to the dual role of backup generators, but the transmission capacity is larger than M4-a. Due to the flexible role of backup generators in M4-b, more transmission lines are required to transmit electricity between nodes.

Figure 3 depicts the average EENS and LOLE values for all six methods. As explained earlier, the LOLE and EENS of four methods (from M1 to M3-b) are calculated after fixing design variables in Model 4 with the optimal design results obtained from the original Models 1-3. First, it is observed that M1 has the highest LOLE (1,340 hours/year), whereas M4-a has the lowest LOLE (0.5 hours/year). In M1 and M2, it is difficult to respond flexibly to the generator failure due to a lack of generation and transmission capacity. As a result, both methods have very large LOLE and EENS values (LOLE: 1,226 $\sim$ 1,340hours/year; EENS: 394,010MWh/year), meaning that 7% of the annual load demand cannot be fulfilled for approximately 51 to 56 days due to the generator failure. As shown in the results of M3-a and M3-b, both LOLE and EENS are significantly improved when more advanced reliability models are used. However, the target LOLE value is still not satisfied because only limited failure scenarios (one generator or two generator failures) are considered. M4-a does not consider the dual role of the backup generators; it means the backup generators can only be used when the main generators fail. Because of this limited role, M4-a requires more backup generators than M4-b (i.e., M4-a: 1,224MW; M4-

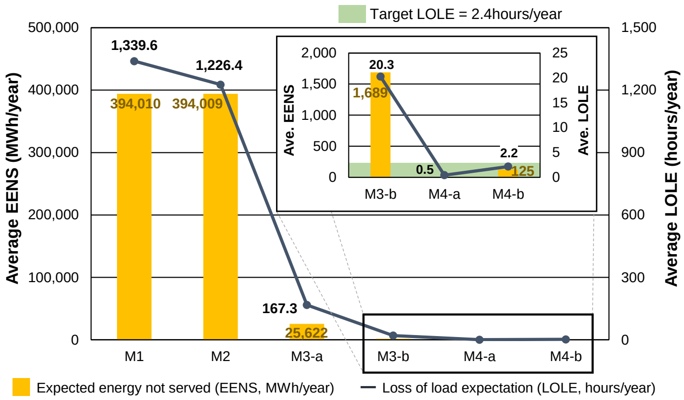

Fig. 3. LOLE and EENS values of all methods in Case 1

b: 958MW), resulting in the highest reliability of M4-a. On the other hand, in M4-b, over-installation of backup generators is prevented as backup generators can be used along with the main generators; however, this results in a small decrease in reliability. It should be mentioned that even if the use of backup generators decreases reliability, the target LOLE can still be achieved.

2) Cost and computational results: Table II shows the cost summary of all six methods, including capital expenditure (CAPEX), operating expenses (OPEX), and penalties. The detailed cost results are in Table C.1 in Supplementary Materials. As explained earlier, the over-generation and EENS penalties of the four methods (from M1 to M3-a) are calculated after fixing design variables in Model 4 with the optimal design results obtained from the original Models 1-3. While M1 requires the least CAPEX and OPEX ($436.4M), M3-b requires the highest CAPEX and OPEX ($780.5M). Among the methods using the probabilistic approach (from M3-a to M4-b), M3-a has the least CAPEX and OPEX ($537.6M), whereas M3-b has the highest CAPEX and OPEX. As previously analyzed, since M1 and M2 do not significantly reflect reliability in the design and operation stages, they require less CAPEX and OPEX due to their smaller system capacity. However, this means that in the event of a generator failure, it cannot be effectively managed, resulting in a very large EENS penalty. M3-a and M3-b have higher CAPEX and OPEX due to their increased capacity than M1 and M2. However, this increases reliability, significantly reducing the EENS penalty (M1 and M2: $10,638M; M3-a: $692M; M3-b: $46M). M3-b, which considers scenarios involving the failure of two generators, has a lower EENS penalty than M3-a, which only considers the failure of one generator. Interestingly, M4-a and M4-b exhibit much lower EENS penalties despite having less generation and transmission capacity than M3-b. This result shows that increasing capacity and the deployment of the expanded capacity, the use of backup generators, and operation strategies can lead to reliability improvements. The total cost results confirm that the expansion planning model using the probability of failure, which also allows backup generators to have dual roles (i.e., M4-b), can simultaneously improve the system's cost and reliability.

Table C.2 in Supplementary Materials shows the computational results of Case 1. Due to the LOLE and EENS constraints and capacity state enumeration, M4-a and M4-b have more binary variables (i.e., # binary variables of M1 to M3-b: 36; # binary variables of M4-a and M4-b: 13,878). While M1 to M3-b could find the optimal solution at the root node, M4-a and M4-b require more branching to find the optimal solution, which increases the computational time. In particular, when considering the dual role of backup generators, a longer computational time is needed compared to the other methods.

TABLE II
COST SUMMARY OF CASE 1

	M1	M2	M3-a	M3-b	M4-a	M4-b
CAPEX (M$)	122	148	203	396	269	213
OPEX (M$)	314	321	335	384	315	337
PEN (M$)	10,638	10,638	692	46	0	3
Total cost (M$)	11,075	11,107	1,227	941	584	554

B. Case 2: San Diego County example

Figure B.2 in Supplementary Materials represents the status of existing and potential generators and transmission lines in San Diego County [34], [35]. This example is a 10-year planning problem with 2-year intervals. Four representative days are selected, and 24 operating hours with a 2-hour interval (a total of 12 operation periods) are used. As seen in Figure B.2, there are four nodes and five lines. Nodes 1 and 4 have existing power plants with different types and capacities. Potential power plants could be installed in all nodes, and parallel backup generators can be added to these potential power plants. Nodes 1 and 4 are already connected through an existing line (solid line), and potential lines can be added to connect the other nodes (dotted lines). The assumptions used in this case study are as follows: i) New NGCC can be installed in all nodes during the planning horizon to meet the demand; ii) backup generator can be added to potential power plants when they are installed; iii) existing power plants cannot expand its capacity or be retired; iv) only potential power plants are considered when generating failure scenarios (for M3-a and M3-b), and capacity failure states (for M4-a and M4-b); v) the distance between nodes is estimated by measuring the distance between centers of each node. San Diego County is actually interconnected to other counties in California as a part of California ISO; therefore, in actual emergencies, other areas can function as backups. However, in this case study, we do not consider these interconnections. 81 *N-1* operating scenarios and 121 *N-2* operating scenarios are generated for M3-a and M3-b, respectively. The method of generating scenarios is the same as in Case 1. Technical and economic data for this case can be found in Table B.2 of Supplementary Materials.

1) Optimal design and reliability results: Figure 4 shows the optimal configurations of the generators and transmission lines in the last year of the planning horizon for the six methods. It is observed that M3-b and M4-a have the largest generation capacity (i.e., 4,692MW), whereas M1 has the

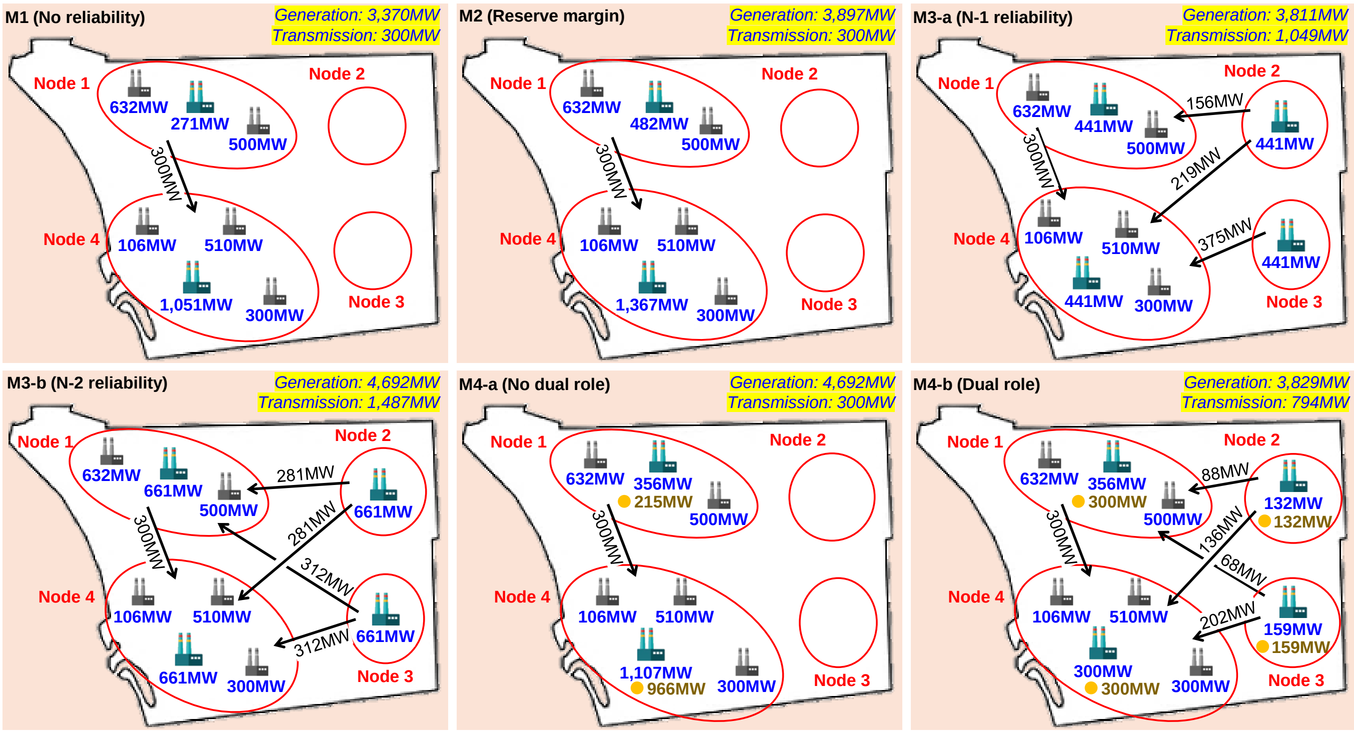

Fig. 4. The optimal configuration in the last year of the planning horizon of Case 2

smallest generation capacity (i.e., 3,370MW). Likewise, M3-b has the largest transmission capacity (i.e., 1,487MW), whereas M1, M2, and M4-a have the smallest transmission capacity (i.e., 300MW). As seen in Figure 4, M1 and M2 install two power plants in Nodes 1 and 4 and expand their capacity to satisfy the local electricity demand. Transmission lines are not installed, and Nodes 2 and 3 are not used. The total generation capacity of M2 is 16% larger than that of M1 due to the reserve margin rate (i.e., M1: 3,370MW; M2: 3,897MW). Contrary to M1 and M2, M3-a and M3-b have a distributed configuration; power plants are distributed throughout all nodes rather than centralizing two nodes to avoid the entire failure of the power systems. The difference between M3-a and M3-b is that M3-b has one more transmission line to connect Nodes 1 and 3. As M3-b considers more extreme failure situations, such as a simultaneous failure of two generators, the generation capacity of M3-b is 1.2 times larger than that of M3-a (i.e., M3-a: 3,811MW; M3-b: 4,692MW). The transmission capacity of M3-b is also 1.4 times larger than that of M3-a to satisfy the demand even if two generators fail (i.e., M3-a: 1,049MW; M3-b: 1,487MW). It is interesting to note that M3-b and M4-a have the same generation capacity, including both the main and backup generators (i.e., 4,692MW); however, in M4-a, backup generators are added to the main power plants rather than installing new main power plants and connecting other nodes through transmission lines. This indicates that when the dual role of backup generators is not allowed, installing additional parallel generators in the same nodes is a more reliable operation strategy than transmitting electricity from other nodes. On the other hand, the generation capacity of M4-b is smaller than M4-a due to the dual role of backup

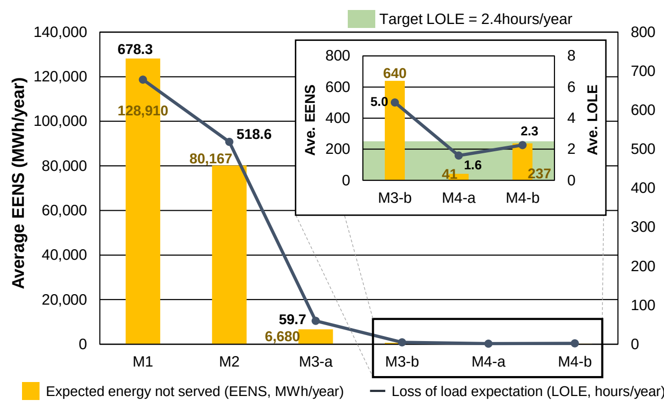

Fig. 5. LOLE and EENS values of all methods in Case 2

generators. When backup generators perform a dual role, they can produce power alongside the main power plants, reducing the need for over-designed power generation systems. Therefore, M4-b has smaller capacity of both main and backup generators than M4-a (i.e., main - M4-a: 3,511MW; M4-b: 2,938MW, backup - M4-a: 1,181MW; M4-b: 890MW). However, due to the flexible role of backup generators in M4-b, more transmission lines are needed to transmit electricity between nodes, resulting in a larger transmission capacity in M4-b (i.e., transmission lines - M4-a: 300MW; M4-b: 794MW).

Figure 5 depicts the average EENS and LOLE values for all six methods. As explained earlier, the LOLE and EENS of four methods (from M1 to M3-b) are calculated after fixing design variables in Model 4 with the optimal design results obtained from the original Models 1-3. Similar to the results of Case 1, it is observed that M1 has the highest LOLE (678.3 hours/year), whereas M4-a has the lowest LOLE (1.6 hours/year). When reliability is not considered (i.e., M1), responding flexibly to generator failures is challenging due to insufficient generation and transmission capacity. Consequently, it exhibits high LOLE and EENS values (LOLE: 678.3 hours/year; EENS: 128,910 MWh/year). This indicates that approximately 0.86% of the annual load demand remains unmet for about 28.3 days due to generator failures. As shown in the results of M2, using the reserve margins model improves LOLE and EENS compared to M1, although it fails to meet the target LOLE values. As shown in the results of M3-a and M3-b, both LOLE and EENS are significantly improved when using more advanced reliability models. However, the target LOLE value is still not satisfied because only limited failure scenarios (one generator or two generator failures) are considered. Since the backup generators can only be used in emergencies, M4-a has the highest reliability (correspondingly to the lowest EENS and LOLE) compared to other models. On the other hand, as shown in the results of M4-b, simultaneous use of backup generators reduces reliability as it reduces flexibility to respond to emergencies. It should be mentioned that even if the use of backup generators decreases reliability, the target LOLE can still be achieved.

2) Cost and computational results: Table III shows the cost summary of all six methods, including capital expenditure (CAPEX), operating expenses (OPEX), and penalties. The detailed cost results are in Table C.3 in Supplementary Materials. As explained earlier, the over-generation and EENS penalties from M1 to M3-b are calculated after fixing design variables in Model 4 with the optimal design results obtained from the original Models 1-3. While M1 requires the least CAPEX and OPEX ($1,471M), M3-b requires the highest CAPEX and OPEX ($2,189M). Among the methods using the probabilistic approach (from M3-a to M4-b), M3-a has the least CAPEX and OPEX ($1,758M), whereas M3-b has the highest CAPEX and OPEX. As previously analyzed, M1 does not account for reliability during the capacity planning and operation stages, leading to lower CAPEX and OPEX. However, this also means that in the event of a generator failure, the power demand cannot be constantly supplied, resulting in a significantly large EENS penalty (approximately 5 times the CAPEX and OPEX). M3-a and M3-b have higher CAPEX and OPEX due to their increased capacity than M1 and M2. However, this increases reliability, significantly reducing the EENS penalty (M1: $5,777M; M2: $5,317M; M3-a: $301M; M3-b: $29M). M3-b, which considers scenarios involving the failure of two generators, has a lower EENS penalty than M3-a, which only considers the failure of one generator. This indicates that M3-b has higher flexibility in handling failures than M3-a. Interestingly, M4-a and M4-b exhibit much lower EENS penalties than M3-b. This result shows that not only increasing capacity but also the deployment of the expanded capacity, the use of backup generators, and effective operation strategies can lead to reliability improvements. As seen in Table D.1, the results of the total cost, including CAPEX, OPEX, and penalties, confirm that the expansion planning model using the probability of failure, which also allows backup generators to have dual roles (i.e., M4-b), can simultaneously improve the system's cost and reliability.

TABLE III
COST SUMMARY OF CASE 2

	M1	M2	M3-a	M3-b	M4-a	M4-b
CAPEX (M$)	446	628	678	1,025	973	716
OPEX (M$)	1,025	1,082	1,080	1,164	1,029	1,085
PEN (M$)	5,778	3,608	301	29	2	11
Total cost (M$)	7,247	5,317	2,059	2,218	2,004	1,812

Table C.4 in Supplementary Materials shows the computational results of all methods for Case 2. Due to the LOLE and EENS constraints and capacity state enumeration, M4-a and M4-b have a larger number of binary variables than other methods, which increases the computational time (i.e., # binary variables of M1 to M3-b: 80; # binary variables of M4-a and M4-b: 30,840). Using Model 4 allows for more accurate and robust reliability analysis, but it has the drawback of requiring longer computational times. To apply this method to larger examples, it will be necessary to develop effective decomposition methods in the future.

VII. CONCLUSIONS

This paper has compared the impact of different reliability formulations on the optimal planning and operation of power systems. In particular, four expansion planning models with different reliability constraints are analyzed: i) a model without reliability, ii) a model with reserve margin constraint, iii) a model with *N-k* reliability constraint, and iv) a model with a probabilistic constraint using the probability of failures. Two reliability criteria, loss of load expectation (LOLE) and expected energy not served (EENS), are used to evaluate the reliability of the optimal solution of all proposed models. A reliability evaluation strategy is proposed to enable a valid comparison among four models with different objective functions and constraints. The four proposed models have been applied to two case studies, the illustrative and the San Diego County cases. Major findings of this paper are as follows:

- The reserve margin reliability model tends to install generators in each area to satisfy the local demand. On the other hand, *N-1* and *N-2* reliability models tend to select decentralized power generation facilities throughout all available areas and add transmission lines rather than installing generators in certain areas to avoid the entire failure of power systems. Probabilistic methods that allow adding backup generators tend to select designs that ensure sufficient backup facilities to enhance reliability.
- When reliability is not considered or a simplified reliability model is used, such as reserve margin, lower CAPEX and OPEX are required due to their smaller system capacity. However, these methods cannot effectively manage generator failures, resulting in high reliability-related penalties. Advanced reliability models, such as probabilistic methods, require larger CAPEX and OPEX because they expand the capacity considering reliability.

However, such a proactive design plan allows the power systems to promptly react to failures, resulting in low reliability-related penalties.

- While *N-k* reliability or probabilistic models that rigorously estimate reliability are computationally challenging, they can secure higher reliability than the models that use simplified reliability constraints (i.e., reserve margin).
- The reliability of the power system is influenced not only by the expansion of generation capacity, but also by the deployment of the expanded transmission capacity, the use of backup generators, and the operation strategies. Therefore, a decision-making tool must be developed that considers all these aspects simultaneously.
- The performances of the four proposed expansion planning models vary depending on the characteristics of examples being solved, such as the dataset and assumptions. Therefore, it is important to choose a proper reliability formulation for a valid capacity expansion planning.

Although probabilistic methods can provide more reliable design and operation results than other methods, they require longer computational time. In future work, effective decomposition methods that can efficiently solve the probabilistic models must be developed to solve these large-scale problems.

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