A NONLINEAR MULTIPERIOD PROCESS OPTIMIZATION MODEL FOR PRODUTION PLANNING IN MULTI-PLANT FACILITIES

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Abstract

In this paper we propose a multiperiod nonlinear programming (NLP) formulation that incorporates empirical process models for the optimal planning of a multi-plant production site. Using as a basis a real world application of a polymer plant that produces 27 products, a model is developed for predicting the detailed production using actual plant data. The empirical process models account for raw material usage, physical product specifications (e.g. viscosity), operation limitations, and production rates. NLP models are proposed for each of the plants, and a multiperiod NLP model is then formulated that determines monthly production and inventory levels of all products for each plant. The NLP model for the polymer plant has several thousand variables and constraints, and a web interface was developed so many users can access the model over the intranet. A graphical input template is linked to the optimization model in which the user can modify key input variables as well as operating and demand parameters. Several numerical examples are presented to illustrate the scope of this model.

Keywords

Production planning, Nonlinear programming, multiperiod.

Introduction

Production planning in a multi-plant facility is a large-scale problem that many industries with complex operations face on a continual basis (Shah et al. 1998). At a single facility there may be several plants operating under multiple sets of conditions to produce a diverse array of intermediate and finished products. Savings can often be realized by introducing more efficient means of planning the operation and production of the different plants at a site. Key issues include operating conditions of individual processes, intermediate product handling, and finished product storage. The main system driver is to meet the forecasted market demands over a specified time horizon.

To produce valid production plans for a multi-plant facility, an accurate representation of each plant process and an appropriate planning framework is required. Many computational models have been proposed that address the production planning of chemical processes. However, most of these models are linear. Only a few authors have incorporated nonlinear process models (e.g. Pinto et al, 2000).

The objective of this paper is to develop a multiperiod nonlinear programming (NLP) optimization model for the planning of production and operation of a real world multi-plant polymer facility. Our formulation accurately reflects and predicts production behavior of each plant by incorporating detailed nonlinear empirical process models. A background of the multi-plant site is given as well as the specific problem statement. The procedure for developing the process models is shown and the multiperiod planning model is described. Our model is then applied to industrial examples under several case scenarios.

Background

The problem considered involves production at an industrial multi-plant facility. In this paper we propose an integrated model, which focuses on the scheduling of production campaigns while meeting both customer orders and inventory targets, and of plant process optimization, which concentrates on operating conditions of the individual processes as well as on maintaining the process

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quality requirements necessary to meet product specifications.

The multi-plant network producing polymers that constitutes the facility is shown in Figure 1. The site contains three Type I plants and two Type II plants. Type I plants produce multiple products; however, only one product can be produced at one time. These plants produce several final products and a few intermediate products including the key raw materials that are fed to the Type II plants. Type II plants are dedicated plants that produce a fixed set of four intermediate products on a continuous basis through a series of separation processes. The relative amounts of each product produced are based on the variable properties of the raw materials. The four intermediates undergo post-processing and blending steps to form 24 of the final products.

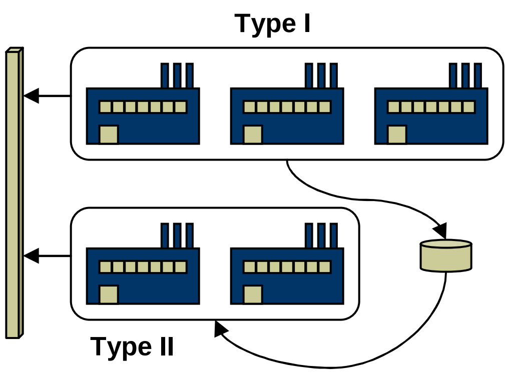

Figure 1. Schematic of Polymer Multi-Plant Network

The model representation of the site must account for interdependencies between the different plants, including intermediate products and shared resources. The planning problem then consists of determining the operating conditions, production rates and key physical properties over a specified number of time periods, typically between three months and up to two years to provide solutions for both short-term and long-term planning problems for which product demands are specified.

Problem Statement

Given is a site containing a set of plants $p \in PLANTS$ whose operation and production are to be optimized over T time periods. Also given is a set of products $k \in FINAL$ for which there is market demand and their current prices. A set of raw materials $r \in RM$ and their costs are given. Monthly demand forecasts are specified for each product k over the time horizon. The number of days d each plant operates per month is required to account for scheduled and unscheduled plant downtimes. Additional production costs and production rate data are given, as well as inventory targets, and storage capacities and costs.

We assume that production rates, raw material consumption rates and manipulated physical parameters are constant within a given time period. There are no direct costs associated with storage or storage vessels, costs are assigned only when product stocks are above or below inventory targets. Also, we assume that we cannot sell more of a product than given by the demand forecast (i.e. demand forecast acts as an upper bound on the product sales).

The problem then consists of determining the optimal monthly production of each product, month-end inventory levels of each product, number of days dedicated to each product in each plant, and key operating parameters. The objective is either to minimize the deviation from the demand (match forecast), or to maximize the profit. The solution with either objective yields both the profit and the deviation of sales from the demand forecast.

Process Models

We consider empirical models for our planning problem. These are based on original models provided by The Dow Chemical Company. They were developed from experimental results and actual plant operation data to locally capture an accurate representation of plant behavior. The original model of the plant was developed in an Excel spreadsheet.

The specific process models for each section of the site are presented below. $PLANTS$ is the set of all plants p at the site. PI is the subset of Type I plants, and PII is the subset of Type II plants so that $PLANTS$ is given by $PLANTS = PI \cup PII$. C is the set of all components c in the system. RM is the subset of raw materials r. INT is the subset of intermediate products i. $FINAL$ is the subset of finished products k. Therefore, the set of all components is given by $C = RM \cup INT \cup FINAL$. We define the variables F_p^c to denote the mass flow rate of component c in or out of plant p. FN^k are the mass flow rates of the finished products k.

The constraints for the Type I plants are given by the following generalized correlations,

$$F_p^i = eff_p^i * \left[cap_p^i - \alpha \left\{ \left(\beta - visc_p^i \right) + \sqrt{\left(\beta - visc_p^i \right)^2} \right\} \right] \quad (1)$$

$$\forall i \in INT,\ p \in PI$$

$$F_p^r = \sum_{i \in INT} \gamma_p^r visc_p^i + \gamma_p^{'r} conf_p^i + \gamma_p^{''r} F_p^i + \gamma_p^{'''} \quad (2)$$

$$\forall r \in RM,\ p \in PI$$

$$\sum_{p \in PI} F_p^r \left[\sum_{i \in INT} DAYS_p^i \right] = \sum_{p \in PI} TDAY_p * RMAV^r \quad (3)$$

$$\forall r \in RM$$

Equation (1) predicts the production rates as a function of plant efficiency eff_p^i, plant capacity cap_p^i and product viscosity $visc_p^i$. The raw materials rates shown in Eq. (2) are a function of viscosity, molecular configuration data $conf_p^i$ and product flows. The values for parameters α, β, and γ are obtained by regressing actual plant operation data. Equation (3) ensures that raw materials used for a given number of days $DAYS_p^i$ making all products i does not exceed the available raw material amounts $RMAV^r$ for the total number of days that plant operates $TDAY_p$.

There is blending and storage of intermediate products from Type I plants that are used for processes in Type II plants. Equation (4) is the mass balance for this transitional step where certain intermediates from *PI* are combined into flow *FTOT* before entering plants *PII*.

$$\sum_{p \in PI} F_p^i = FTOT \quad \forall i = INT_{PII} \tag{4}$$

$$F_p^i = f\left(visc_p^i, conf_p^i, FTOT, etc.\right) \tag{5}$$

$$F_p^i = \sum_{k \in FINAL} BLEND^{i,k} * FN^k \qquad \forall i \in INT, p \in PII \tag{6}$$

The separation processes that describe the Type II plants are modeled by the following constraints. The production rates from *PII* plants in Eq. (5) are represented by functions of several production factors. We only show general form due to confidentiality reasons. Equation (6) calculates the necessary amount of intermediate products to produce the final products based on blending ratios $BLEND^{i,k}$ to mix product *i* into product *k*.

Multiperiod Optimization Model

The process models described in the previous section for each plant are integrated to form the multiperiod optimization model of the entire site. The generalized form of the multiperiod model with time periods *t* is shown below, where *x* are variables and θ are parameters.

$$\begin{aligned} \max \quad & \sum_t f_t\left(x_t, x_{t-1}, \theta_t\right) \\ s.t. \quad & g_t\left(x_t, x_{t-1}, \theta_t\right) \le 0 \quad t = 1...T \\ & h_t\left(x_t, x_{t-1}, \theta_t\right) = 0 \quad t = 1...T \\ & x_t \in X_t \end{aligned} \tag{P}$$

Note that the coupling of the time periods occurs through the inventory variables x_{t-1}. The entire multiperiod model includes the process model equations (Eqs. 1-6) for each time period as well as the following constraints.

Final production of a given product FN_t^k plus the existing inventory from the previous time period INV_{t-1}^k is equal to the sales of that product S_t^k plus any additional product which becomes the inventory in the current time period INV_t^k (Eq. (7)). Equation (8) represents an economic penalty when the inventory levels do not meet the storage targets. This is calculated as a percentage of total production costs for inventory levels above or below the specified inventory targets IT^k. Since Eq. (8) is nondifferentiable, we use an alternative representation for as shown in Eqs. (9) and (10).

$$FN_t^k + INV_{t-1}^k = S_t^k + INV_t^k \qquad \forall k \in FINAL, t = 1...T \tag{7}$$

$$IC_t = \sum_{k \in FINAL} pen * PC^k \left| INV_t^k - IT^k \right|, t = 1...T \tag{8}$$

$$IC_t \ge \sum_{k \in FINAL} pen * PC^k \left(INV_t^k - IT^k \right), t = 1...T \tag{9}$$

$$IC_t \ge \sum_{k \in FINAL} pen * PC^k \left(IT^k - INV_t^k \right), t = 1...T \tag{10}$$

The sales of product *k* cannot exceed the demand forecast DEM_t^k (Eq. (11)). Equation (12) monitors the difference Δ_t^k between the forecast demand and the actual sales of product *k*.

$$S_t^k \le DEM_t^k \quad \forall k \in FINAL, t = 1...T \tag{11}$$

$$S_t^k + \Delta_t^k = DEM_t^k \quad \forall k \in FINAL, t = 1...T \tag{12}$$

Two objective functions are considered. The first one in Eq. (13) is for matching the forecast where the objective minimizes the sum of the deviations Δ_t^k of sales of all products *k* in all time periods *t* from the demand forecast. The second objective is to maximize profit. Here the product orders are disregarded and the sales and costs are optimized. Profit is calculated in Eq. (14) as the revenues from all product sales minus the costs associated with raw materials purchases, plant operation, additional production and storage. Note that values for both the *MATCH* and *PROFIT* variables are calculated in either objective mode of the optimization.

$$\min MATCH = \sum_t \sum_k \Delta_t^k \tag{13}$$

$$\max PROFIT = \sum_{t=1}^{T} REV_t - COST_t \tag{14}$$

Solution of Multiperiod NLP

The resulting multiperiod optimization model corresponds to an NLP problem, which is generally nonconvex. In this work we use the reduced gradient method implemented in the CONOPT2 code (Drud 1985). Although a global optimum cannot be guaranteed because of the nonconvexities, in our experience we did not detect multiple local solutions. CONOPT2 proved to be a robust solver for our problem as solutions readily converged under varied initial conditions.

In order to facilitate the use of the multiperiod NLP model, a web interface was developed so that many users could access the model over the intranet. A key feature of the implementation is that the model is easily accessible by multiple business units including marketing, planning, and manufacturing. The match forecast objective can be used for planning and manufacturing decisions, while the profitability objective can be used to drive marketing decisions.

Numerical Examples

To illustrate the model performance, we consider several cases with varied time periods and objective functions. Recall that the site contains five plants that produce, store, and sell 27 finished polymer products. Our first example involves twelve time periods, which represents one year of plant operation. Case 1 uses the match forecast case, while Case 2 maximizes the profit objective function. We consider varied horizons of 3, 6 and 24 time periods to show the effects of scale on the model size and computation times for each case.

The multiperiod NLP model was coded in the GAMS modeling environment (Brooke et al. 1997) and solved with CONOPT2 Version 2.071G. Examples were solved on a 700 MHz Pentium III PC, with 256 Mbytes of RAM.

Table 1.Computional Results for Varied Time Horizons

Time Periods	No. of Variables	No. of Constraints	Case 1 CPUs	Case 2 CPUs
3	1073	1051	2.9	4.4
6	2171	2155	19.4	24.8
12	4367	4363	61.5	119
24	8759	8779	209	554

First, the demand forecast for one year is given. Computational results for the NLP are shown in Table 1. The optimal solution for Case 1 has a profit of $117.6 million. No deviation from the demand forecast was observed ($MATCH = 0$), and all orders were met. This case which minimizes the deviation from forecast is helpful in selecting the operating and planning modes to best meet the customer sales demands.

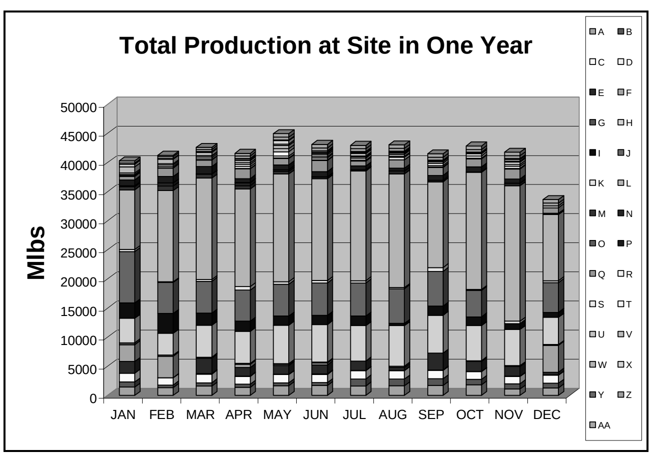

Figure 2. Production Scheme for One Year Horizon

The problem was then run for Case 2 for profit maximization. The optimal solution is profit equal to $136 million. The profit is $18.4 million more than in Case 1, but the deviation from the demand forecast is now 7%. This result occurred because the production and sales of two products, namely products G and P, were reduced so that the forecast was not matched. However, the related operation and production costs of these products were reduced as well, leading to the increase in profit. The profit maximization case can be useful in situations where it is necessary to predict the most profitable product mix, or the best sales scenarios for the site. A snapshot of the site production can be seen in Figure 2.

The model is next run for varying time horizons. Problem size is proportional to the number of time periods considered, while computation times grow at a faster rate (Table 1). The difference in computation times between the two objective function cases is simply because the match forecast case is essentially a feasibility problem, while the profit maximization objective requires not only satisfying the constraints, but also to establish the optimal trade-offs between all costs and revenues.

Conclusions

A new multiperiod model has been presented for optimal production planning in a multi-plant polymer facility. The model uses nonlinear process models to optimize production under either a match forecast or a maximize profit objective. A user-friendly web interface connected with the model provides a tool that has linked decision making for marketing, planning, and production. The large-scale nature of our problem grows in size as more time periods are considered. Therefore, to expand the our model for multiple sites we need to develop a solution procedure to reduce the resulting long solution times. Specific decomposition schemes are explored in a future paper.

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