An Efficient Multiperiod MINLP Model for Optimal Planning of Offshore Oil and Gas Field Infrastructure

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Abstract

In this paper, we present an efficient strategic/tactical planning model for offshore oilfield development problem that is fairly generic and can be extended to include other complexities. The proposed multiperiod non-convex MINLP model for multi-field site includes three components (oil, water and gas) explicitly in the formulation using 3^{rd} and higher order polynomials avoiding bilinear and other nonlinear terms. With the objective of maximizing total NPV for long-term planning horizon, the model involves decisions related to FPSO (floating production, storage and offloading) installation and expansion schedule and respective oil, liquid and gas capacities, connection between the fields and FPSOs, well drilling schedule and production rates of these three components in each time period. The resulting model can be solved effectively with DICOPT for realistic instances and gives good quality solutions. Furthermore, the model can be reformulated into an MILP after piecewise linearization and exact linearization techniques that can be solved globally in an efficient way. Solutions of realistic instances involving 10 fields, 3 FPSOs, 84 wells and 20 years planning horizon are reported, as well as comparisons between the computational performance of the proposed MINLP and MILP formulations.

Keywords: multiperiod optimization, planning, Offshore Oil and Gas field development, MINLP, Investment and operations planning, FPSO

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1 Introduction

Offshore oil and gas field development represents a very complex problem and involves multi-billion dollar investments and profits (Babusiaux et al., 2004). The complexity comes from the fact that usually there are many alternatives available for installation of the platforms and their sizes, for deciding which fields to develop and what should be the order to develop them, and which and how many wells are to be drilled in those fields and in what order, which field to be connected to which facility, and how much oil and gas to produce from each field. The sequencing of these installations and connections must also be based on physical considerations, e.g. field can only be developed if a corresponding facility is present. The other complexities are the consideration of nonlinear profiles of the reservoir that are critical to predict the actual flowrates of oil, water and gas from each field as there can be significant variations in these flowrates over time, limitation on the number of wells that can be drilled each year due to availability of the drilling rigs, and long-term planning horizon that is the characteristics of the these projects. Moreover, installation and operation decisions in these projects involve very large investments that can lead to large profits, or losses in the worst case if these decisions are not made carefully.

Therefore, based on the above, there is a clear motivation to optimize the investment and operations decisions for oil and gas field development problem to ensure reasonable return on the investments over the time horizon considered. By including all the considerations described above in an optimization model, this leads to a multiperiod MINLP problem. Furthermore, the extension of this model to the cases where we consider the fiscal rules (Van den Heever et al. (2000) and Van den Heever and Grossmann (2001)) and the uncertainties, especially endogenous uncertainty cases (Jonsbraten et al. (1998), Goel and Grossmann (2004, 2006), Goel et al. (2006), Tarhan et al. (2009, 2011) and Gupta and Grossmann (2011)), can lead to a very complex problem to solve. Therefore, an effective model for the deterministic case is proposed that on the one hand captures the realistic reservoir profiles, interaction among various fields and facilities, wells drilling limitations and other practical trade-offs involved in the offshore development planning, and on the other hand can be used as the basis for extensions that include other complexities, especially fiscal rules and uncertainties. This paper focuses on a non-convex MINLP model for the strategic/tactical planning of the offshore oil and gas fields, which includes sufficient details to make it useful for realistic oilfield development projects, as well as for extensions to include fiscal and uncertainty considerations.

The oilfield investment and operation planning is traditionally modeled as separate LP (Lee and Aranofsky (1958), Aronofsky and Williams (1962)) or MILP (Frair, 1973) problems under certain assumptions to make them computationally tractable. Simultaneous optimization of the investment and operation decisions was addressed in Bohannon (1970), Sullivan (1982) and Haugland et al. (1988) using MILP formulations with different levels of details in these models. Behrenbruch (1993) emphasized the need to consider a correct geological model and to incorporate flexibility into the decision process for an oilfield development project.

Iyer et al. (1998) proposed a multiperiod MILP model for optimal planning and scheduling of offshore oilfield infrastructure investment and operations. The model considers the facility allocation, production planning, and scheduling within a single model and incorporates the reservoir performance, surface pressure constraints, and oil rig resource constraints. To solve the resulting large-scale problem, the nonlinear reservoir performance equations are approximated through piecewise linear approximations. As the model considers the performance of each individual well in a reservoir independently, it becomes expensive to solve for realistic multi-field sites. Moreover, the flow rate of water was not considered explicitly for facility capacity calculations.

Van den Heever and Grossmann (2000) extended the work of Iyer et al. (1998) and proposed a multiperiod generalized disjunctive programming model for oil field infrastructure planning for which they developed a bilevel decomposition method. As opposed to Iyer and Grossmann (1998), they explicitly incorporated a nonlinear reservoir model into the formulation. Van den Heever et al. (2000), and Van den Heever and Grossmann (2001) extended their work to handle complex economic objectives including royalties, tariffs, and taxes for the multiple gas fields site. These authors incorporated these complexities into their model through disjunctions as well as big-M formulations. The results were presented for realistic instances involving 16 fields and 15 years. However, the model considers only gas production and the number of wells were used as parameters (fixed well schedule) in the model.

Ortiz-Gomez et al. (2002) presented three mixed integer multiperiod optimization models of varying complexity for the oil production planning. The problem considers fixed topology and is concerned with the decisions involving the oil production profiles and operation/shut in times of the wells in each time period assuming nonlinear reservoir behavior. Based on the continuous time formulation for gas field development with complex economics, Lin and Floudas (2003) presented an MINLP model and solved it with a two stage algorithm. Carvalho and Pinto (2006) considered an MILP formulation for oilfield planning based on the model developed by Tsarbopoulou (2000), and proposed a bilevel decomposition algorithm for solving large scale problems where master problem determines the assignment of platforms to wells and a planning subproblem calculates the timing for the fixed assignments. The work was further extended by Carvalho and Pinto (2006) to consider multiple reservoirs within the model.

In the papers described above, one of the major assumptions is that there is no uncertainty in the parameters. Jonsbraten (1998) addressed the oilfield development planning problem under oil price uncertainty using an MILP formulation which was solved with a progressive hedging algorithm. Aseeri et al. (2004) introduced uncertainty in the oil prices and well productivity indexes, financial risk management, and budgeting constraints into the model proposed by Iyer and Grossmann (1998) and solved the resulting stochastic model using a sampling average approximation algorithm. Goel and Grossmann (2004) considered a gas field development problem under uncertainty in the size and quality of reserves where decisions on the timing of field drilling were assumed to yield an immediate resolution of the uncertainty, i.e. the problem involves decision-dependent uncertainty as discussed in Jonsbraten et al. (1998). Linear reservoir models, which can provide a reasonable approximation for gas fields, were used. In their solution strategy, the authors used a relaxation problem to predict upper bounds, and solved multistage stochastic programs for a fixed scenario tree for finding lower bounds. Goel et al. (2006) later proposed a branch and bound algorithm for solving the corresponding disjunctive/mixed-integer programming model where lower bounds are generated by Lagrangean duality.

Ulstein et al. (2007) addressed the tactical planning of petroleum production that involves regulation of production levels from wells, splitting of production flows into oil and gas products, further processing of gas and transportation in a pipeline network. The model was solved for different cases with demand variations, quality constraints, and system breakdowns. Tarhan et al. (2009) developed a multistage stochastic programming model for planning offshore oil field infrastructure under uncertainty where the uncertainties in initial maximum oil flowrate, recoverable oil volume, and water breakthrough time of the reservoir are revealed gradually as a function of investment and operating decisions. The model is formulated as a disjunctive/mixed-integer nonlinear programming model that consists of individual non-convex MINLP subproblems connected to each other through initial and conditional non-anticipativity constraints. The duality-based branch and bound algorithm was proposed taking advantage of the problem structure and globally optimizing each scenario problem independently. However, it considers either gas/water or oil/water components for single field and single reservoir at a detailed level. Hence, realistic multi-field site instances can be expensive to solve with this model.

Li et al. (2010) presented a stochastic pooling optimization formulation to address the design and operation of natural gas production networks, where the qualities of the flows are described with a pooling model and the uncertainty is handled with a two-stage stochastic approach. The resulting large-scale nonconvex MINLP is solved with a rigorous decomposition method. Elgsæter et al. (2010) proposed a structured approach to optimize offshore oil and gas production with uncertain models which iteratively updates setpoints while documenting the benefits of each proposed setpoint change through excitation planning and result analysis. The approach is able to realize a significant portion of the available profit potential while ensuring feasibility despite large initial model uncertainty.

In this paper, there are six major extensions and differences that are addressed as compared to the previous work:

(1) We consider all three components (oil, water and gas) explicitly in the formulation, which allows to consider realistic problems for facility installation and capacity decisions.

(2) Nonlinear reservoir behavior in the model is approximated by 3rd and higher order polynomials to ensure sufficient accuracy for the predicted reservoir profiles.

(3) Reservoir profiles are modeled as independent polynomials for each field-facility connections for simplicity.

(4) The number of wells is used as a variable for each field to capture the realistic drill rig limitations and the resulting trade-offs among various fields.

(5) We include the possibility of expanding the facility capacities in the future, and including the lead times for construction and expansions for each facility to ensure realistic investments.

(6) Reservoir profiles are also expressed in terms of cumulative water and cumulative gas produced that are derived from WOR and GOR expressions avoiding bilinearities in the model.

The outline of this paper is as follows. First, we present a brief background on the basic structure of an offshore oilfield site and major reservoir features. Next, we introduce the problem statement and the MINLP model for offshore oilfield development problem. The MINLP model is then reformulated as an MILP problem. Furthermore, both models are reformulated with reduced number of binary variables. Numerical results of three realistic cases up to 10 oilfields and 20 years are considered to report the performance of the proposed models.

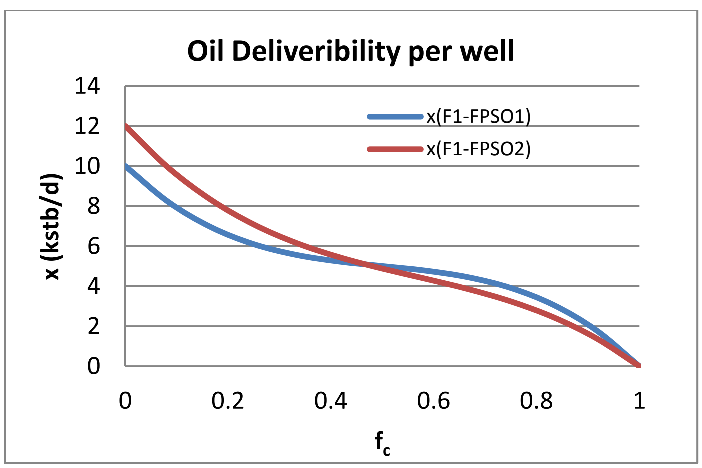
2 Background

An offshore oilfield infrastructure consists of various production facilities such as Floating Production, Storage and Offloading (FPSO), fields, wells and connecting pipelines to produce oil and gas from the reserves. Each oilfield consists of a number of potential wells to be drilled using drilling rigs, which are then connected to the facilities through pipelines to produce oil. There is two-phase flow in these pipelines due to the presence of gas and liquid that comprises oil and water. Therefore, there are three components, and their relative amounts depend on certain parameters like cumulative oil produced. The field to facility connection involves trade-offs associated to the flowrates of oil and gas for a particular field-facility connection, connection costs, and possibility of other fields to connect to that same facility, while the number of wells that can be drilled in a field depends on the availability of the drilling rig that can drill a certain number of wells each year.

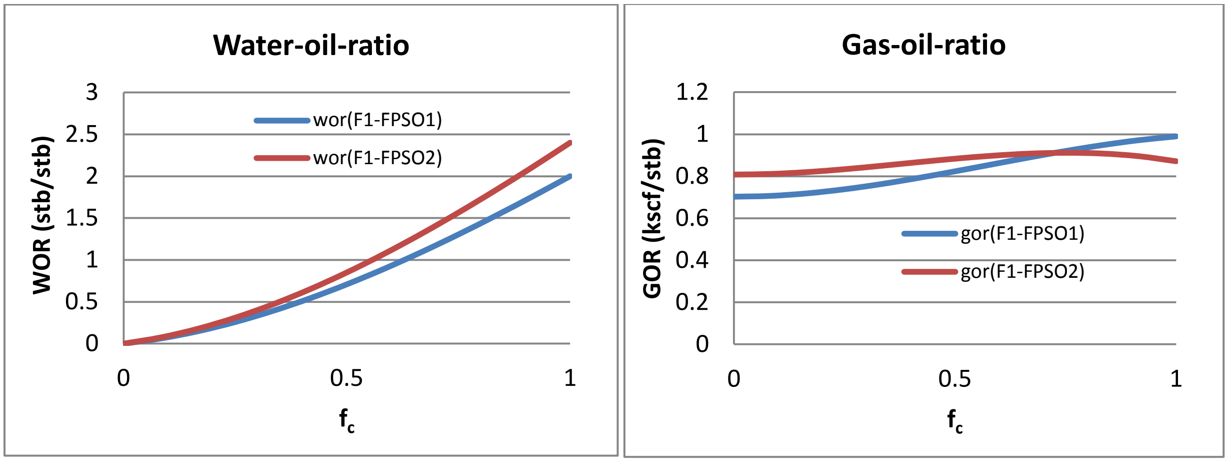
We assume in this paper that the type of offshore facilities connected to fields to produce oil and gas are FPSOs with continuous capacities and ability to expand them in the future. These FPSO facilities costs multi-billion dollars each depending on their sizes and have the capability of operating in remote locations for very deep offshore oilfields (200m-2000m) where seabed pipelines are not cost effective. FPSOs are large ships that can process the produced oil and store until it is shipped to the onshore site or sales terminal. Processing includes the separation of oil, water and gas into individual streams using separators located at these facilities. Each FPSO facility has a lead time between the construction or expansion decision, and the actual availability. The wells are subsea wells in each field that are drilled using drilling ships. Therefore, there is no need to have a facility present to drill a subsea well. The only requirement to recover oil from it is that the well must be connected to a FPSO facility. In this paper, we focus on multi-field site and include sufficient details in the model to account for the various trade-offs involved without going into much detail for each of these fields. However, the proposed model can easily be extended to include various facility types and other details in the oilfield development planning problem.

The location of production facilities and possible field and facility allocation itself is a very complex problem. In this work, we assume that the potential location of facilities and field-facility connections are given. In addition, the potential number of wells in each field is also given. Note that each field can be potentially allocated to more than one FPSO facility, but once the particular field-connection is selected, the other possibilities are not considered. Furthermore, each facility can be used to produce oil from more than one field.

The facilities and connection involved in the offshore planning are often in operation over many years, and it is therefore important to take future conditions into consideration when designing an initial infrastructure or any expansions. This can be incorporated by dividing the planning horizon, for example, 20 years, into a number of time periods with a length of 1 year, and allowing investment and operating decisions in each period, which leads to a multi-period planning problem.



(a) Oil Deliverability per well for field (F1)



(b) Water to oil ratio for field (F1)

(b) Gas to oil ratio for field (F1)

Figure 1: Nonlinear Reservoir Characteristics for field (F1) for 2 FPSO facilities (FPSO 1 and 2)

When oil is extracted from a reservoir oil deliverability, water-to-oil ratio (WOR) and gas-to-oil ratio (GOR) change nonlinearly as a function of the cumulative oil recovered from the reservoir. The initial oil and gas reserves in the reservoirs, as well as the relationships for WOR and GOR in terms of fractional recovery (f_c), are estimated from geologic studies. Figures 1 (a) – (c) represent the oil deliverability from a field per well, WOR and GOR versus fractional oil recovered from that field. We can see from these figures that there are different nonlinear field profiles for different field-FPSO connections to account for the variations in the flows for each of these possible connections.

The maximum oil flowrate (field deliverability) per well can be represented as a 3rd order polynomial equation (a) in terms of the fractional recovery. Furthermore, the actual oil flowrate (x_f) from each of the wells is restricted by both the field deliverability Q_f^d, (b), and facility capacity. We assume that there is no need for enhanced recovery, i.e., no need for injection of gas or water into the reservoir. The oil produced from the wells (x_f) contains water and gas and their relative rates depend on water-to-oil ratio (wor_f) and gas-to-oil ratio (gor_f) that are approximated using 3rd order polynomial functions in terms of fractional oil recovered (eqs. (c)-(d)). The water and gas flow rates can be calculated by multiplying the oil flowrate (x_f) with water-to-oil ratio and gas-to-oil ratio as in eqs. (e) and (f), respectively. Note that the reason for considering fractional oil recovery compared to cumulative amount of oil was to avoid numerical difficulties that could arise due to very small magnitude of the polynomial coefficients in that case.

$$Q_f^d = a_{1,f}(fc_{f,t})^3 + b_{1,f}(fc_f)^2 + c_{1,f}\, fc_f + d_1 \qquad \forall f \qquad \text{(a)}$$

$$x_f \le Q_f^d \qquad \forall f \qquad \text{(b)}$$

$$wor_f = a_{2,f}(fc_f)^3 + b_{2,f}(fc_f)^2 + c_{2,f}\, fc_f + d_{2,f} \qquad \forall f \qquad \text{(c)}$$

$$gor_f = a_{3,f}(fc_f)^3 + b_{3,f}(fc_f)^2 + c_{3,f}\, fc_f + d_{3,f} \qquad \forall f \qquad \text{(d)}$$

$$w_f = wor_f\, x_f \qquad \forall f \qquad \text{(e)}$$

$$g_f = gor_f\, x_f \qquad \forall f \qquad \text{(f)}$$

In Appendix A we derive the polynomial equations for the cumulative water and cumulative gas produced as a function of fractional recovery using equations (c) and (d), respectively, in order to avoid the bilinear terms (e)-(f) that are required in the model based on the above reservoir equations. In the next section, we give a formal description of the oilfield development problem considered in the paper that is formulated as an MINLP problem in the subsequent section.

3 Problem Statement

Given is a typical offshore oilfield infrastructure consisting of a set of oil fields $F = \{1,2,\ldots f\}$ available for producing oil using a set of FPSO (Floating, Production, Storage and Offloading) facilities, $FPSO = \{1,2,\ldots fpso\}$, (see Fig. 2). To produce oil from a field, it must be connected to a FPSO facility that can process the produced oil, store and offload it to the other tankers.

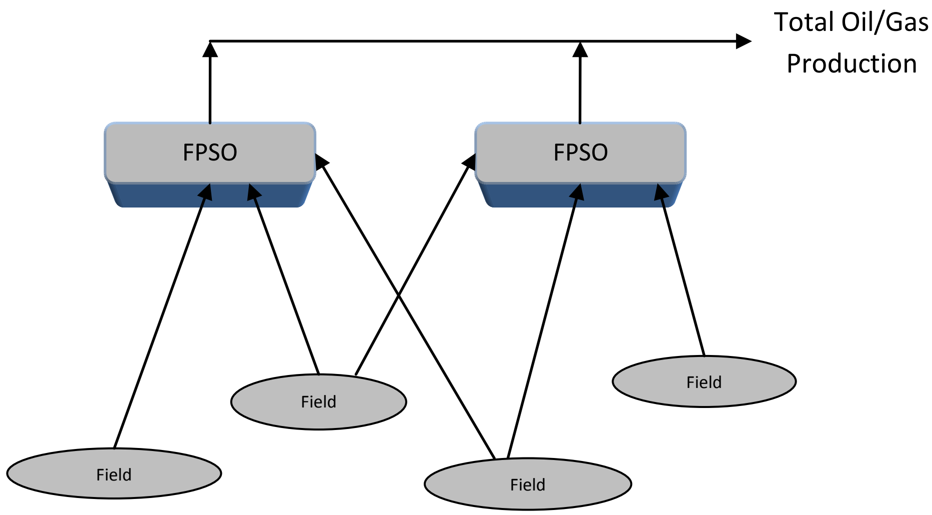


Figure 2: Typical Offshore Oilfield Infrastructure Representation

We assume that the location of each FPSO facility and its possible connections to the given fields are known (Figure 2). Notice that each FPSO facility can be connected to more than one field to produce oil while a field can only be connected to a single FPSO facility. There can be a significant amount of water and gas that comes out with the oil during the production process that needs to be considered while planning for FPSO capacity installations and expansions. The water is usually re-injected after separation from the oil while the gas can be sold in the market. In this case for simplicity we do not consider water or gas re-injection i.e. natural depletion of the reserves.

To develop and operate such a complex and capital intensive offshore oilfield infrastructure, we have to make the optimum investment and operation decisions to maximize the NPV considering a long-term planning horizon. The planning horizon is discretized into a number of time periods t, typically each with 1 year of duration. Investment decisions in each time period t include which FPSO facilities should be installed or expanded, and their respective installation or expansion capacities for oil, liquid and gas, which fields should be connected to which FPSO facility, and the number of wells that should be drilled in a particular field f given the restrictions on the total number of wells that can be drilled in each time period t over all the given fields. Operating decisions include the oil/gas production rates from each field f in each time period t. It is assumed that all the installation and expansion decisions occur at the beginning of each time period t, while operation takes place throughout the time period. There is a lead time of l_1 years for each FPSO facility initial installation and a lead time of l_2 years for the expansion of an earlier installed FPSO facility. Once installed, we assume that the oil, liquid (oil and water) and gas capacities of a FPSO facility can be expanded only once.

Field deliverability, i.e. maximum oil flowrate from a field, WOR and GOR are approximated by a cubic equation, while cumulative water produced and cumulative gas produced from a field are represented by fourth order polynomials in terms of the fractional oil recovered from that field. Notice that these 4^{th} order polynomials correspond to the integration of the cubic equations for WOR and GOR as explained in Appendix A. The motivation for using polynomials for cumulative water produced and cumulative gas produced as compared to WOR and GOR is to avoid bilinear terms in the formulation and to allow converting the resulting model into an MILP formulation. Furthermore, all the wells in a particular field f are assumed to be identical for the sake of simplicity leading to the same reservoir profiles, eqs. (a)-(f), for each of these wells.

4 MINLP Model

We present in this section a multiperiod MINLP model for the offshore oil and gas field infrastructure optimization problem. The objective function (1) is to maximize the total net present value (NPV) of the project. Constraint (2) represents the overall NPV as a function of the difference between total revenue and total cost in each time period t taking the discount factors d_t into account.

$$Max \quad NPV \tag{1}$$

$$NPV = \sum_t d_t (REV_t - COST_t) \tag{2}$$

The total revenues (3) in each time period t are computed based on the total amount of oil and gas produced in that time period and respective selling prices where total oil, water and gas flowrates in each time period t, (x_t^{tot}, w_t^{tot}, g_t^{tot}) are calculated as the sum of the production rate of these components over all the FPSO facilities in equations (4)-(6), respectively.

$$REV_t = \delta_t (\alpha_t x_t^{tot} + \beta_t g_t^{tot}) \qquad \forall t \tag{3}$$

$$x_t^{tot} = \sum_{fpso} x_{fpso,t} \qquad \forall t \tag{4}$$

$$w_t^{tot} = \sum_{fpso} w_{fpso,t} \qquad \forall t \tag{5}$$

$$g_t^{tot} = \sum_{fpso} g_{fpso,t} \qquad \forall t \tag{6}$$

The total cost incurred in (7) is the sum of capital and operating expenses in each time period t. The overall capital expenses (8) consist of the fixed installation costs for FPSO facilities, variable installation and expansion costs corresponding to the FPSOs liquid and gas capacities, connection costs between a field and a FPSO facility and cost of drilling the wells for each field in each time period t. The total operating expenses (9) are the operation cost occurred corresponding to the total amount of liquid and gas produced in each time period t.

$$COST_t = CAP_t + OPER_t \qquad \forall t \qquad (7)$$

$$CAP_t = \sum_{fpso}\left[FC_{fpso,t} b_{fpso,t} + VC^{liq}_{fpso,t}(QI^{liq}_{fpso,t} + QE^{liq}_{fpso,t}) + VC^{gas}_{fpso,t}(QI^{gas}_{fpso,t} + QE^{gas}_{fpso,t})\right]$$

$$+ \sum_{f}\sum_{fpso} FC^{C}_{f,fpso,t} b^{c}_{f,fpso,t} + \sum_{f} FC^{well}_{f,t} I^{well}_{f,t} \qquad \forall t \qquad (8)$$

$$OPER_t = \delta_t\left[OC^{liq}_t(x^{tot}_t + w^{tot}_t) + OC^{gas}_t g^{tot}_t\right] \qquad \forall t \qquad (9)$$

Constraints (10)-(13) predict the reservoir behavior for each field *f* in each time period t. In particular, constraint (10) restricts the oil flow rate from each well for a particular FPSO-field connection in time period t to be less than the deliverability (maximum oil flow rate) of that field per well where equation (11) represents the field deliverability per well at the beginning of time period t+1 for a particular FPSO-field connection as the cubic equation in terms of the fractional oil recovered by the end of time period t from that field. In particular, (11a) corresponds to the oil deliverability in time period 1 while (11b) represents for the rest of time periods in the planning horizon. Constraints (12) and (13) represent the value of water-to-oil and gas-to-oil ratios in time period t for a specific field-FPSO connection as cubic equations in terms of the fractional oil recovery by the end of previous time period, respectively.

$$x^{well}_{f,fpso,t} \le Q^{d,well}_{f,fpso,t} \qquad \forall f, fpso, t \qquad (10)$$

$$Q^{d,well}_{f,fpso,1} = d_{1,f,fpso} \qquad \forall f, fpso \qquad (11a)$$

$$Q^{d,well}_{f,fpso,t+1} = a_{1,f,fpso}(fc_{f,t})^3 + b_{1,f,fpso}(fc_{f,t})^2 + c_{1,f,fpso} fc_{f,t} + d_{1,f,fpso}$$

$$\forall f, fpso, t < |T| \qquad (11b)$$

$$wor_{f,fpso,t} = a_{2,f,fpso}(fc_{f,t-1})^3 + b_{2,f,fpso}(fc_{f,t-1})^2 + c_{2,f,fpso} fc_{f,t-1} + d_{2,f,fpso} \qquad \forall f, fpso, t \qquad (12)$$

$$gor_{f,fpso,t} = a_{3,f,fpso}(fc_{f,t-1})^3 + b_{3,f,fpso}(fc_{f,t-1})^2 + c_{3,f,fpso} fc_{f,t-1} + d_{3,f,fpso} \qquad \forall f, fpso, t \qquad (13)$$

The predicted WOR and GOR values in equations (12) and (13) are further used in equations (14) and (15) to calculate the respective water and gas flowrates from field to FPSO in time period t by multiplying it with the corresponding oil flow rate. Notice that these equations give rise to the bilinear terms in the model.

$$w_{f,fpso,t} = wor_{f,fpso,t} x_{f,fpso,t} \qquad \forall f, fpso, t \qquad (14)$$

$$g_{f,fpso,t} = gor_{f,fpso,t} x_{f,fpso,t} \qquad \forall f, fpso, t \qquad (15)$$

The total oil flow rate in (16) from each field *f* in time period t is the sum of the oil flow rates that are directed to FPSO facilities in that time period t, whereas oil that is directed to a particular FPSO facility from a field *f* is calculated as the multiplication of the oil flow rate per well and number of wells available for production in that field, eq. (17).

$$x_{f,t} = \sum_{fpso} x_{f,fpso,t} \qquad \forall f, t \qquad (16)$$

$$x_{f,fpso,t} = N^{well}_{f,t} \cdot x^{well}_{f,fpso,t} \qquad \forall f, fpso, t \qquad (17)$$

Eq. (18) computes the cumulative amount of oil produced from field f by the end of time period *t*, while (19) represents the fractional oil recovery by the end of time period *t*. The cumulative oil produced is also restricted in (20) by the recoverable amount of oil from the field.

$$xc_{f,t} = \sum_{\tau=1}^{t} (x_{f,\tau} \delta_\tau) \qquad \forall f, t \qquad (18)$$

$$fc_{f,t} = \frac{xc_{f,t}}{REC_f} \qquad \forall f, t \qquad (19)$$

$$xc_{f,t} \leq REC_f \qquad \forall f, t \qquad (20)$$

Eqs. (21)-(23) compute total oil, water and gas flow rates into each FPSO facility, respectively, in time period t from all the given fields.

$$x_{fpso,t} = \sum_f x_{f,fpso,t} \qquad \forall fpso, t \qquad (21)$$

$$w_{fpso,t} = \sum_f w_{f,fpso,t} \qquad \forall fpso, t \qquad (22)$$

$$g_{fpso,t} = \sum_f g_{f,fpso,t} \qquad \forall fpso, t \qquad (23)$$

There are three types of capacities i.e. for oil, liquid (oil and water) and gas that are used for modeling the capacity constraints for FPSO facilities. Specifically, Eqs. (24)-(26) restrict the total oil, liquid and gas flow rates into each FPSO facility to be less than its corresponding capacity in each time period t respectively. These three different kinds of capacities of a FPSO facility in time period *t* are computed by equalities (27)-(29) as the sum of the corresponding capacity at the end of previous time period t-1, installation capacity at the beginning of time period t-l_1 and expansion capacity at the beginning of time period t-l_2. Specifically, the term $QI^{oil}_{fpso,t-l_1}$in equation (27) represents the oil capacity of a FPSO facility that started to install l_1 years earlier and is expected to be ready for production in time period *t*, to account for the lead time of l_1 years for a FPSO facility installation. The term $QE^{oil}_{fpso,t-l_2}$ represents the expansion decision in the oil capacity of an already installed FPSO facility that is taken l_2 years before time period t, to consider the lead time of l_2 years for capacity expansion. Similarly, the corresponding terms in equations (28) and (29) represent the lead times for liquid and gas capacity installation or expansion, respectively. Notice that due to one installation and expansion of a FPSO facility, $QI^{oil}_{fpso,t-l_1}$and $QE^{oil}_{fpso,t-l_2}$ can have non-zero values only once in the planning horizon while $Q^{oil}_{fpso,t-1}$ can be non-zero in the multiple time periods.

$$x_{fpso,t} \le Q^{oil}_{fpso,t} \qquad \forall fpso,t \qquad (24)$$

$$x_{fpso,t} + w_{fpso,t} \le Q^{liq}_{fpso,t} \qquad \forall fpso,t \qquad (25)$$

$$g_{fpso,t} \le Q^{gas}_{fpso,t} \qquad \forall fpso,t \qquad (26)$$

$$Q^{oil}_{fpso,t} = Q^{oil}_{fpso,t-1} + QI^{oil}_{fpso,t-l_1} + QE^{oil}_{fpso,t-l_2} \qquad \forall fpso,t \qquad (27)$$

$$Q^{liq}_{fpso,t} = Q^{liq}_{fpso,t-1} + QI^{liq}_{fpso,t-l_1} + QE^{liq}_{fpso,t-l_2} \qquad \forall fpso,t \qquad (28)$$

$$Q^{gas}_{fpso,t} = Q^{gas}_{fpso,t-1} + QI^{gas}_{fpso,t-l_1} + QE^{gas}_{fpso,t-l_2} \qquad \forall fpso,t \qquad (29)$$

Inequalities (30) and (31) restrict the installation and expansion of a FPSO facility to take place only once, respectively, while inequality (32) states that the connection between a FPSO facility and a field can be installed only once during the whole planning horizon. Inequality (33) ensures that a field can be connected to at most one FPSO facility in each time period t, while (34) states that at most one FPSO-field connection is possible for a field *f* during the entire planning horizon T due to engineering considerations. Constraints (35) and (36) state that the expansion in the capacity of a FPSO facility and the connection between a field and a FPSO facility, respectively, in time period t can occur only if that FPSO facility has already been installed by that time period.

$$\sum_{t \in T} b_{fpso,t} \le 1 \qquad \forall fpso \qquad (30)$$

$$\sum_{t \in T} b^{ex}_{fpso,t} \le 1 \qquad \forall fpso \qquad (31)$$

$$\sum_{t \in T} b^{c}_{f,fpso,t} \le 1 \qquad \forall f, fpso \qquad (32)$$

$$\sum_{fpso} b^{c}_{f,fpso,t} \le 1 \qquad \forall f,t \qquad (33)$$

$$\sum_{t \in T} \sum_{fpso} b^{c}_{f,fpso,t} \le 1 \qquad \forall f \qquad (34)$$

$$b^{ex}_{fpso,t} \leq \sum_{\tau=1}^{t} b_{fpso,\tau} \qquad \forall fpso,t \qquad (35)$$

$$b^{c}_{f,fpso,t} \leq \sum_{\tau=1}^{t} b_{fpso,\tau} \qquad \forall f, fpso,t \qquad (36)$$

Inequality (37) states that the oil flow rate per well from a field *f* to a FPSO facility in time period t will be zero if that FPSO-field connection is not available in that time period. Notice that equations (17) and (37) ensure that for production from a field in time period t there must be a field-FPSO connection and at-least one well available in that field at the beginning of time period t. Constraints (38)-(43) are the upper-bounding constraints on the installation and expansion capacities for FPSO facilities in time period *t* corresponding to the three different kinds of capacities mentioned earlier.

$$x^{well}_{f,fpso,t} \leq U^{well,oil}_{f,fpso} \sum_{\tau=1}^{t} b^{c}_{f,fpso,\tau} \qquad \forall f, fpso,t \qquad (37)$$

$$QI^{oil}_{fpso,t} \leq U^{oil}_{fpso} b_{fpso,t} \qquad \forall fpso,t \qquad (38)$$

$$QI^{liq}_{fpso,t} \leq U^{liq}_{fpso} b_{fpso,t} \qquad \forall fpso,t \qquad (39)$$

$$QI^{gas}_{fpso,t} \leq U^{gas}_{fpso} b_{fpso,t} \qquad \forall fpso,t \qquad (40)$$

$$QE^{oil}_{fpso,t} \leq U^{oil}_{fpso} b^{ex}_{fpso,t} \qquad \forall fpso,t \qquad (41)$$

$$QE^{liq}_{fpso,t} \leq U^{liq}_{fpso} b^{ex}_{fpso,t} \qquad \forall fpso,t \qquad (42)$$

$$QE^{gas}_{fpso,t} \leq U^{gas}_{fpso} b^{\exp}_{fpso,t} \qquad \forall fpso,t \qquad (43)$$

The additional restrictions on the oil, liquid and gas expansion capacities of FPSO facilities, (44)-(46), come from the fact that these expansion capacities should be less than a certain fraction (μ) of the initial built capacities, respectively. Notice that available capacitates in the previous time period can be used in the expression instead of initial built FPSO capacities given that only one installation and expansion is allowed for each of these facilities.

$$QE_{fpso,t}^{oil} \leq \mu Q_{fpso,t-1}^{oil} \qquad \forall fpso,t \qquad (44)$$

$$QE_{fpso,t}^{liq} \leq \mu Q_{fpso,t-1}^{liq} \qquad \forall fpso,t \qquad (45)$$

$$QE_{fpso,t}^{gas} \leq \mu Q_{fpso,t-1}^{gas} \qquad \forall fpso,t \qquad (46)$$

The number of wells available for the production from a field is calculated from (47) as the sum of the wells available at the end of previous time period and the number of wells drilled at the beginning of time period t. The maximum number of wells that can be drilled over all the fields during each time period t and in each field *f* during complete planning horizon T are restricted by respective upper bounds in (48) and (49).

$$N_{f,t}^{well} = N_{f,t-1}^{well} + I_{f,t}^{well} \qquad \forall f,t \qquad (47)$$

$$\sum_{f} I_{f,t}^{well} \leq UI_{t}^{well} \qquad \forall t \qquad (48)$$

$$N_{f,t}^{well} \leq UN_{f}^{well} \qquad \forall f,t \qquad (49)$$

The non-convex MINLP model **(Model 1)** for offshore oilfield investment and operations planning involves constraint (1)-(49). In particular, constraints (11b)- (15) and (17) are nonlinear and non-convex constraints in the model that can lead to suboptimal solutions when solved with a method that assumes convexity.

In contrast to Model 1, the proposed MINLP model (**Model 2)** involves all the constraints as in Model 1 except (12)-(15) that are replaced with reservoir profiles based on cumulative water and cumulative gas produced for each field-FPSO connection. The motivation for using polynomials for cumulative water produced and cumulative gas produced as compared to WOR and GOR is to avoid bilinear terms (14)-(15) in the formulation and allow converting the resulting MINLP model into an MILP formulation. In particular, the cumulative water and cumulative gas produced by the end of time period t from a field are represented by 4th order polynomial equations (50) and (51), respectively, in terms of fractional oil recovery by the end of time period t. Notice that these 4th order polynomials (50) and (51) correspond to the cubic equations for WOR and GOR, respectively, that are derived in Appendix A.

$$Q^{wc}_{f,fpso,t} = a_{2,f,fpso}(fc_{f,t})^4 + b_{2,f,fpso}(fc_{f,t})^3 + c_{2,f,fpso}(fc_{f,t})^2 + d_{2,f,fpso}fc_{f,t} \quad \forall f, fpso, t \qquad (50)$$

$$Q^{gc}_{f,fpso,t} = a_{3,f,fpso}(fc_{f,t})^4 + b_{3,f,fpso}(fc_{f,t})^3 + c_{3,f,fpso}(fc_{f,t})^2 + d_{3,f,fpso}fc_{f,t} \quad \forall f, fpso, t \qquad (51)$$

Notice that variables $Q^{wc}_{f,fpso,t}$ and $Q^{gc}_{f,fpso,t}$ will be non-zero in equations (50) and (51) if $fc_{f,t}$ is non-zero even though that particular field-FPSO connection is not present. Therefore, $Q^{wc}_{f,fpso,t}$ and $Q^{gc}_{f,fpso,t}$ represent dummy variables in equations (50) and (51) instead of actual cumulative water ($wc_{f,fpso,t}$) and cumulative gas ($gc_{f,fpso,t}$) recoveries due to the fact that only those cumulative water and cumulative gas produced can be non-zero that has the specific FPSO-field connection present in that time period t. Therefore, we introduce constraints (52)-(55) to equate the actual cumulative water produced, $wc_{f,fpso,t}$, for a field-FPSO connection by the end of time period t to the corresponding dummy variable $Q^{wc}_{f,fpso,t}$ only if that field-FPSO connection is present in time period t else $wc_{f,fpso,t}$ is set to zero. Similarly, constraints (56)-(59) equate the actual cumulative gas produced, $gc_{f,fpso,t}$, to the dummy variable $Q^{gc}_{f,fpso,t}$ only if that field-FPSO connection is present in time period t, otherwise it is set to zero. $M^{wc}_{f,fpso}$ and $M^{gc}_{f,fpso}$ correspond to maximum amount of cumulative water and gas that can be produced for a particular field and FPSO connection during the entire planning horizon, respectively. Note that the motivation for using dummy variables ($Q^{wc}_{f,fpso,t}$ and $Q^{gc}_{f,fpso,t}$) for cumulative water and cumulative gas flows in equations (50)-(51) followed by big-M constraints (52)-(59), instead of using disaggregated variables for the fractional recovery in equations (50)-(51) directly, was to avoid large number of SOS1 variables while MILP reformulation of this model as explained in the next section.

$$wc_{f,fpso,t} \le Q^{wc}_{f,fpso,t} + M^{wc}_{f,fpso}(1-\sum_{\tau=1}^{t} b^{c}_{f,fpso,\tau}) \qquad \forall f, fpso, t \qquad (52)$$

$$wc_{f,fpso,t} \ge Q^{wc}_{f,fpso,t} - M^{wc}_{f,fpso}(1-\sum_{\tau=1}^{t} b^{c}_{f,fpso,\tau}) \qquad \forall f, fpso, t \qquad (53)$$

$$wc_{f,fpso,t} \le M^{wc}_{f,fpso}\sum_{\tau=1}^{t} b^{c}_{f,fpso,\tau} \qquad \forall f, fpso, t \qquad (54)$$

$$wc_{f,fpso,t} \ge -M^{wc}_{f,fpso}\sum_{\tau=1}^{t} b^{c}_{f,fpso,\tau} \qquad \forall f, fpso, t \qquad (55)$$

$$gc_{f,fpso,t} \le Q^{gc}_{f,fpso,t} + M^{gc}_{f,fpso}(1-\sum_{\tau=1}^{t} b^{c}_{f,fpso,\tau}) \qquad \forall f, fpso, t \qquad (56)$$

$$gc_{f,fpso,t} \ge Q^{gc}_{f,fpso,t} - M^{gc}_{f,fpso}(1-\sum_{\tau=1}^{t} b^{c}_{f,fpso,\tau}) \qquad \forall f, fpso, t \qquad (57)$$

$$gc_{f,fpso,t} \le M^{gc}_{f,fpso}\sum_{\tau=1}^{t} b^{c}_{f,fpso,\tau} \qquad \forall f, fpso, t \qquad (58)$$

$$gc_{f,fpso,t} \ge -M^{gc}_{f,fpso}\sum_{\tau=1}^{t} b^{c}_{f,fpso,\tau} \qquad \forall f, fpso, t \qquad (59)$$

Eq. (60) and (61) compute the water and gas flow rates in time period t from a field to FPSO facility as the difference of cumulative amounts produced by the end of current time period t and previous time period t-1 divided by the time duration of that period.

$$w_{f,fpso,t} = (wc_{f,fpso,t} - wc_{f,fpso,t-1}) / \delta_t \qquad \forall f, fpso, t \qquad (60)$$

$$g_{f,fpso,t} = (gc_{f,fpso,t} - gc_{f,fpso,t-1}) / \delta_t \qquad \forall f, fpso, t \qquad (61)$$

The non-convex MINLP model **(Model 2)** involves constraint (1)-(11) and (16)-(61) where constraints (11b), (50) and (51) are univariate polynomials while constraint (17) involves bilinear terms with integer variables. The correspondence between reservoir profiles for both the MINLP models and their comparison is presented in Appendixes A and B, respectively. In the following section, we reformulate MINLP Model 2 into an MILP problem that can be solved to global optimality in an effective way. Notice that due to the presence of bilinear terms in equations (14) and (15), Model 1 cannot be reformulated into an MILP problem.

5 MILP Reformulation

The nonlinearites involved in **Model 2** include univariate polynomials (11b), (50), (51) and bilinear equations (17). In this section, we reformulate this model into an MILP model, **Model 3** using piecewise linearization and exact linearization techniques that can give the global solution of the resulting approximate problem.

To approximate the 3rd and 4th order univariate polynomials (11b), (50) and (51) SOS1 variables $b_{f,t}^{l}$ are introduced to select the adjacent points *l-1* and *l* for interpolation over an interval *l*. Constraints (62)-(65) represent the piecewise linear approximation for the fractional recovery and corresponding oil deliverability, cumulative water and cumulative gas produced for a field in each time period t, respectively, where $\widetilde{fc}^{l}$, $\tilde{Q}_{f,fpso}^{d,well,l}$, $\tilde{Q}_{f,fpso}^{wc,l}$ and $\tilde{Q}_{f,fpso}^{gc,l}$ are the values of the corresponding variables at point *l* used in linear interpolation based on the reservoir profiles (11b), (50) and (51). Note that only $b_{f,t}^{l}$ variables are sufficient to approximate the constraints (11b), (50) and (51) by selecting a specific value of the fractional recovery for each field in each time period t that applies to all possible field-FPSO connections for that field. This avoids the requirement of a large number of SOS1 variables and resulting increase in the solution times that would have been required in the case if constraints (50) and (51) were represented in terms of the disaggregated variables for fractional recovery in Model 2.

$$fc_{f,t} = \sum_{l=1}^{n} \lambda_{f,t}^{l} \widetilde{fc}^{l} \qquad \forall f,t \qquad (62)$$

$$Q_{f,fpso,t+1}^{d,well} = \sum_{l=1}^{n} \lambda_{f,t}^{l} \tilde{Q}_{f,fpso}^{d,well,l} \qquad \forall f,\ fpso, t < |T| \qquad (63)$$

$$Q_{f,fpso,t}^{wc} = \sum_{l=1}^{n} \lambda_{f,t}^{l} \tilde{Q}_{f,fpso}^{wc,l} \qquad \forall f,\ fpso, t \qquad (64)$$

$$Q_{f,fpso,t}^{gc} = \sum_{l=1}^{n} \lambda_{f,t}^{l} \tilde{Q}_{f,fpso}^{gc,l} \qquad \forall f,\ fpso, t \qquad (65)$$

Equation (66) allows only one of the point *l* to be selected for which $b_{f,t}^{l}$ equals *1* while equation (67) states that $\lambda_{f,t}^{l}$ can be non-zero for only two consecutive points *l* and *l-1* that are used for convex combination during interpolation, eq. (68). Thus, the corresponding *l*th piece is used for linear interpolation as all other $\lambda_{f,t}^{l}$ are zero for a field in time period t and determines the value of the interpolated variable as a convex combination of their values at both the end of this piece *l* in equations (62)-(65).

$$\sum_{l=1}^{n-1} b_{f,t}^{l} = 1 \qquad \forall f,t \qquad (66)$$

$$\lambda_{f,t}^{l} \leq b_{f,t}^{l-1} + b_{f,t}^{l} \qquad \forall f,t,l \qquad (67)$$

$$\sum_{l=1}^{n} \lambda_{f,t}^{l} = 1 \qquad \forall f,t \qquad (68)$$

The other nonlinear constraints (17) in Model 2 contain bilinear terms that can be linearized using exact linearization (Glover, 1975). To linearize constraint (17) we first express the integer variable, $N_{f,t}^{well}$, for the number of wells in terms of the binary variables $Z_{f,k,t}^{well}$ using eq. (69) where $Z_{f,k,t}^{well}$ determines the value of the *k*th term of the binary expansion.

$$N_{f,t}^{well} = \sum_{k} 2^{|k|-1} \cdot Z_{f,k,t}^{well} \qquad \forall f,t \qquad (69)$$

The bilinear term in constraint (17) can then be rewritten as follows,

$$x_{f,fpso,t} = \sum_{k} 2^{|k|-1} \cdot Z_{f,k,t}^{well} \cdot x_{f,fpso,t}^{well} \qquad \forall f, fpso, t \qquad (70)$$

Constraint (70) can be reformulated as a linear constraint (71) by introducing a nonnegative continuous variable $ZX_{f,fpso,k,t}^{well} = Z_{f,k,t}^{well} \cdot x_{f,fpso,t}^{well}$ which is further defined by constraints (72)-(75) by introducing an auxiliary variable $ZX1_{f,fpso,k,t}^{well}$.

$$x_{f,fpso,t} = \sum_{k} 2^{|k|-1} \cdot ZX_{f,fpso,k,t}^{well} \qquad \forall f, fpso, t \qquad (71)$$

$$ZX^{well}_{f,fpso,k,t} + ZX1^{well}_{f,fpso,k,t} = x^{well}_{f,fpso,t} \qquad \forall f, fpso, k, t \qquad (72)$$

$$ZX^{well}_{f,fpso,k,t} \le U^{well}_{f,fpso} Z^{well}_{f,k,t} \qquad \forall f, fpso, k, t \qquad (73)$$

$$ZX1^{well}_{f,fpso,k,t} \le U^{well}_{f,fpso}(1 - Z^{well}_{f,k,t}) \qquad \forall f, fpso, k, t \qquad (74)$$

$$ZX^{well}_{f,fpso,k,t} \ge 0, ZX1^{well}_{f,fpso,k,t} \ge 0 \qquad \forall f, fpso, k, t \qquad (75)$$

The reformulated MILP Model 3 involves constraints (1)-(10), (11a), (16), (18)-(69) and (71)-(75) which are linear and mixed-integer linear constraints and allow to solve this approximate problem to global optimality using standard mixed-integer linear programming solvers.

Remarks

The previous two sections present a multiperiod MINLP model for the oilfield investment and operations planning problem for long-term planning horizon and its reformulation as an MILP model using linearization techniques. The MINLP models involve non-convexities and can yield suboptimal solutions when using an MINLP solver that relies on convexity assumptions, while the reformulated MILP model is guaranteed to be solved to global optimality using linear programming based branch and cut methods. However, given the difficulties involved in solving large scale instances of the MINLP and MILP models, especially due to the large number of binary variables, we extend these formulations by reducing the number of the binary variables. The next section describes the proposed procedure for binary reduction for MINLP and MILP formulations.

6 Reduced MINLP and MILP models

Due to the potential computational expense of solving the large scale MINLP and MILP models presented in the previous sections, we further reformulate them by removing many binary variables, namely $b^{c}_{f,fpso,t}$. These binary variables represent the timing of the connections between fields and FPSOs and are used for discounting the connection cost in the objective function along with some logic constraints in the proposed models. The motivation for binary reduction comes from the fact that in the solution of these models the connection cost is only ~2-3% of the total cost, and hence, this cost can be removed from the objective function as its exact discounting does not has a significant impact on the optimal solution. In particular, we propose to drop the index t from $b^{c}_{f,fpso,t}$, which results in a significant decrease in the number of binary variables (~33% reduction) and the solution time can be improved significantly for both the MINLP and MILP formulations.

Therefore, to formulate the reduced models that correspond to Model 2 and 3 we use the binary variables $b^{R}_{f,fpso}$ to represent the connection between field and FPSOs instead of using $b^{c}_{f,fpso,t}$ which results in a significant decrease in the number of binary variables in the model. As an example for a field with 5 possible FPSO connections and 20 years planning horizon the number of binary variables required can be reduced from 100 to 5. The connection cost term in the objective function (8) is also removed as explained above yielding constraint (76). Moreover, some of the constraints in the previous MINLP and MILP models that involve binary variables $b^{c}_{f,fpso,t}$ are reformulated to be valid for $b^{R}_{f,fpso}$ based reduced model, i.e. constraints (77)-(87). Notice that constraints (87) and (17) ensure that the oil flow rate from a field to FPSO facility in time period t, $x_{f,fpso,t}$, will be non-zero only if that particular field-FPSO connection is installed and there is atleast one well available in that field for production in time period t, i.e. $b^{R}_{f,fpso}$ equals 1 and $N^{well}_{f,t}$ is non-zero, otherwise $x_{f,fpso,t}$ is set to zero. Moreover, it may be possible that variable $x^{well}_{f,fpso,t}$ can take non-zero value in equation (87) if $b^{R}_{f,fpso}$ equals 1 even though there is no well available in that field in time period t, but this will not have any effect on the solution given that the fractional recovery from a field and other calculations/constraints in the model are based on the actual amount of oil produced from the field, i.e. variable $x_{f,fpso,t}$ which is still zero in this case. Therefore, variable $x^{well}_{f,fpso,t}$ can be considered as a dummy variable in the reduced model.

$$CAP_t = \sum_{fpso}\left[FC_{fpso,t} b_{fpso,t} + VC^{liq}_{fpso,t}(QI^{liq}_{fpso,t} + QE^{liq}_{fpso,t}) + VC^{gas}_{fpso,t}(QI^{gas}_{fpso,t} + QE^{gas}_{fpso,t})\right]$$

$$+ \sum_{f} FC^{well}_{f,t} I^{well}_{f,t} \qquad \forall\, t \qquad (76)$$

$$wc_{f,fpso,t} \le Q^{wc}_{f,fpso,t} + M^{wc}_{f,fpso}(1 - b^{R}_{f,fpso}) \qquad \forall f,\, fpso, t \qquad (77)$$

$$wc_{f,fpso,t} \ge Q^{wc}_{f,fpso,t} - M^{wc}_{f,fpso}(1 - b^{R}_{f,fpso}) \qquad \forall f,\, fpso, t \qquad (78)$$

$$wc_{f,fpso,t} \leq M^{wc}_{f,fpso} b^{R}_{f,fpso} \qquad \forall f, fpso, t \qquad (79)$$

$$wc_{f,fpso,t} \geq -M^{wc}_{f,fpso} b^{R}_{f,fpso} \qquad \forall f, fpso, t \qquad (80)$$

$$gc_{f,fpso,t} \leq Q^{gc}_{f,fpso,t} + M^{gc}_{f,fpso}(1 - b^{R}_{f,fpso}) \qquad \forall f, fpso, t \qquad (81)$$

$$gc_{f,fpso,t} \geq Q^{gc}_{f,fpso,t} - M^{gc}_{f,fpso}(1 - b^{R}_{f,fpso}) \qquad \forall f, fpso, t \qquad (82)$$

$$gc_{f,fpso,t} \leq M^{gc}_{f,fpso} b^{R}_{f,fpso} \qquad \forall f, fpso, t \qquad (83)$$

$$gc_{f,fpso,t} \geq -M^{gc}_{f,fpso} b^{R}_{f,fpso} \qquad \forall f, fpso, t \qquad (84)$$

$$\sum_{fpso} b^{R}_{f,fpso} \leq 1 \qquad \forall f \qquad (85)$$

$$b^{R}_{f,fpso} \leq \sum_{\tau=1}^{t} b_{fpso,\tau} \qquad \forall f, fpso, t \qquad (86)$$

$$x^{well}_{f,fpso,t} \leq U^{well,oil}_{f,fpso} b^{R}_{f,fpso} \qquad \forall f, fpso, t \qquad (87)$$

The non-convex MINLP **Model 2-R** for offshore oilfield investment and operations planning after binary reduction involves constraints (1)-(7), (9)-(11), (16)-(31), (35), (38)-(51), (60)-(61) and (76)-(87). The reformulated MILP **Model 3-R** after binary reduction involves constraints (1)-(7), (9)-(10), (11a), (16), (18)-(31), (35), (38)-(51), (60)-(69) and (71)-(87) which are linear and mixed-integer linear constraints. Similarly, **Model 1-R** corresponds to the non-convex MINLP model, which is based on WOR and GOR expression after binary reduction as described above.

The resulting reduced models with fewer binaries can be solved much more efficiently as compared to the original models. To calculate the discounted cost of connections between field and FPSOs that corresponds to the reduced model solution, we use the well installation schedule $N^{well}_{f,t}$ from the optimal solution of reduced models to find the Field-FPSO connection timing and subtract the corresponding discounted connection cost from the optimal NPV of the reduced model. The resulting NPV represents the optimal NPV of the original models in case connection costs are relatively small.

7 Numerical Results

In this section we present 3 instances of the oilfield planning problem where we consider from 3 to 10 fields while the time horizon ranges from 10 to 20 years. The maximum number of possible FPSOs is taken 3 in all the instances. We compare the computational results of the various MINLP and MILP models proposed in the previous sections for these 3 instances. Table 1 summarizes the main features of these MINLP and reformulated MILP models. In particular, the reservoir profiles and respective nonlinearities involved in the models are compared in the table.

Table 1: Comparison of the nonlinearities involved in 3 model types

	Model 1	**Model 2**	**Model 3**
Model Type	MINLP	MINLP	MILP
Oil Deliverability	3rd order polynomial	3rd order polynomial	Piecewise Linear
WOR	3rd order polynomial	-	-
GOR	3rd order polynomial	-	-
wc	-	4th order polynomial	Piecewise Linear
gc	-	4th order polynomial	Piecewise Linear
Bilinear Terms	N*x N*x*WOR N*x*GOR	N*x	None
MILP Reformulation	Not Possible	Possible	Reformulated MILP

7.1 Instance 1

In this instance (Figure 3) we consider 3 oil fields that can be connected to 3 FPSOs with 7 possible connections among these fields and FPSOs. There are a total of 25 wells that can be drilled, and the planning horizon considered is 10 years, which is discretized into 10 periods of each 1 year of duration. We need to determine which of the FPSO facilities is to be installed or expanded, in what time period, and what should be its capacity of oil, liquid and gas, to which fields it should be connected and at what time, and the number of wells to be drilled in each field during each time period. Other than these installation decisions, there are operating decisions involving the flowrate of oil, water and gas from each field in each time period. The objective function is to maximize total NPV over the given planning horizon.

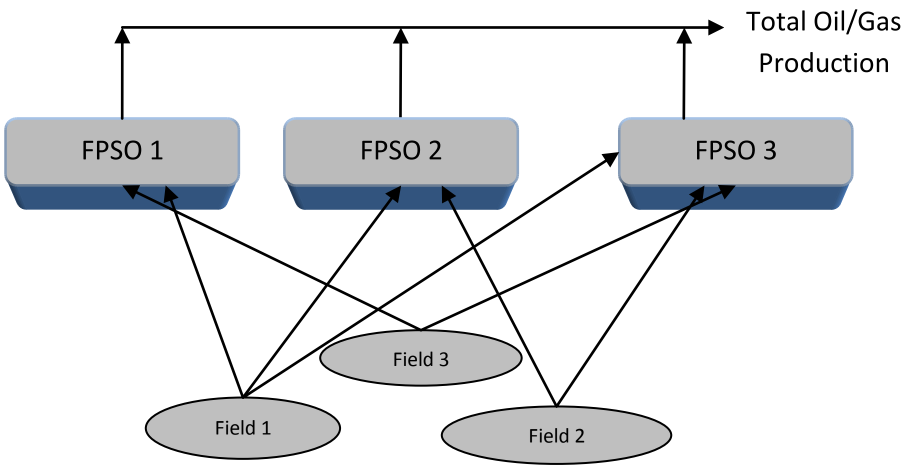


Figure 3: Instance 1 (3 Fields, 3 FPSO, 10 years) for oilfield problem

The problem is solved using DICOPT 2x-C solver for Models 1 and 2, and CPLEX 12.2 for Model 3. These models were implemented in GAMS 23.6.3 and run on Intel Core i7 machine. The optimal solution of this problem that corresponds to Model 2, suggests installing only FPSO 3 with a capacity of 300 kstb/d, 420.01 kstb/d and 212.09 MMSCF/d for oil, liquid and gas, respectively, at the beginning of time period 1. All the three fields are connected to this FPSO facility at time period 4 when installation of the FPSO facility is completed and a total of 20 wells are drilled in these 3 fields in that time period to start production. One additional well is also drilled in field 3 in time period 5 and there are no expansions in the capacity of FPSO facility. The total NPV of this project is $6912.04 M.

Table 2: Performance of various solvers with Model 1 and 2 for Instance 1

	Model 1		**Model 2**	
Constraints	1,357		1,997	
Continuous Var.	1,051		1,271	
Discrete Var.	151		151	
Solver	Optimal NPV (million$)	Time (s)	Optimal NPV (million$)	Time (s)
DICOPT	6980.92	3.56	6912.04	3.07
SBB	7038.26	211.53	6959.06	500.64
BARON	6983.65	>36,000	6919.28	>36,000

Table 2 compares the computational results of Model 1 and 2 for this instance with various MINLP solvers. We can observe from these results that DICOPT performs best among all the MINLP solvers in terms of computational time, while solving directly both Models 1 and 2. The number of OA iterations required is approximately 3-4 in both cases, and solving Model 2 is slightly easier than solving Model 1 directly with this solver. However, the solutions obtained are not guaranteed to be the global solution. SBB is also reasonable in terms of solution quality but it takes much longer time to solve. BARON can in principle find the global optimum solution to models 1 and 2, but it is very slow and takes more than 36,000s to be within ~23% and ~10% of optimality gap for these models, respectively. Note that we use the DICOPT solution to initialize in this case, but BARON could only provide a slightly better solution (6983.65 vs. 6980.92 and 6919.28 vs. 6912.04) than DICOPT in more than 10 hours for both the cases.

The performance of Models 1 and 2 are compared before and after reducing the binary variables for connection, i.e. Models 1-R and 2-R, in Table 3. There is one third reduction in the number of binary variables for both models. It can also be seen that there is a significant decrease in the solution time after binary reduction (for e.g. 1.55s vs. 3.56s for Model 1). Moreover, the reduced models also yield better local solutions too for both the MINLP formulations. Notice that these MINLP Models are solved with DICOPT here for comparison as it is much faster as compared to other solvers as seen from the previous results.

The MILP Model 3 and its binary reduction Model 3-R that are formulated from Model 2 and Model 2-R, respectively, solved with CPLEX 12.2 and results in Table 3 show the significant reduction in the solution time after binary reduction (6.55s vs. 37.03s) while both the models give same optimal NPV i.e. $7030.90M. Notice that these approximate MILP models are solved upto global optimality in few seconds while global solution of the original MINLP formulations is much expensive to obtain. Although the higher the number of points for the approximate MILP model the better will be the solution quality, but we found that beyond 5 points for the piecewise approximation there was not much significant change in the optimal solution, while it led to large increases in the solution time due to increase in the SOS1 variables in the model. Therefore we use 5 point estimates for piecewise linearization to formulate Model 3 and 3-R for all the instances.

Table 3: Comparison of models 1, 2 and 3 with and w/o binary reduction.

	Model 1	**Model 1-R**	**Model 2**	**Model 2-R**	**Model 3**	**Model 3-R**
Constraints	1,357	1,320	1,997	1,960	3,094	3,057
Continuous Var.	1,051	988	1,271	1,208	2,228	2,165
Discrete Var.	151	109	151	109	219	177
SOS1 Var.	0	0	0	0	120	120
NPV(million$)	6980.92	7049.54	6912.04	6919.28	7030.90	7030.90
Time(s)	3.56	1.55	3.07	2.85	37.03	6.55

*Model 1 and 2 solved with DICOPT 2x-C, Model 3 solved with CPLEX 12.2

The global solution from the MILP approximation Model 3-R gives a higher NPV for this example as compared to solving Model 2 directly (7030.90 vs. 6912.04). Therefore, this model can potentially be used for finding global or near optimal solution to the MINLP formulation. Table 4 compares the solution of the original models 1 and 2 (MINLPs) with the one where we fix the discrete variables coming from Model 3-R in these two models and solve the resulting NLPs. We can see that the local solutions are significantly improved. Notice also that no other solver could find the solutions that are better than these solutions in reasonable computational time as can be seen from Table 1. Moreover, it is interesting to note that the discrete decisions that come from the MILPs that corresponds to Model 2 seems to be optimal for Model 1 too which ensures the close correspondence between Models 1 and 2 and its reformulations.

Table 4: Improved solutions (NPV in million$) for the Models 1 and 2 using Model 3 solution

Model 1	**Model 1 (fixed binaries from Model 3-R)**	**Model 2**	**Model 2 (fixed binaries from Model 3-R)**
6980.92	7076.6177	6912.04	7004.0783

7.2 Instance 2

This is a slightly larger instance for oilfield planning problem than the previous one where we consider 5 oil fields that can be connected to 3 FPSO's with 11 possible connections. There are a total of 31 wells that can be drilled in all of these 5 fields and the planning horizon considered is 20 years. Table 5 compares the results of Model 1 and 2 with various MINLP solvers for this example. DICOPT still performs best even for this larger instance in terms of solution quality and time. SBB, which relies on a branch and bound based scheme, becomes very slow due to the increase in the number of binary variables and problem size. BARON also becomes expensive to solve this larger instance and could not improve the DICOPT solution that is used for its initialization for both the cases in more than 10 hours.

Table 5: Comparison of various models and solvers for Instance 2

	Model 1		**Model 2**	
Constraints	3,543		5,543	
Continuous Var.	2,781		3,461	
Discrete Var.	477		477	
Solver	Optimal NPV (million$)	Time (s)	Optimal NPV (million$)	Time (s)
DICOPT	11412.48	58.53	11204.86	18.43
SBB	11376.57	1057.68	11222.34	3309.73
BARON	11412.48	>36,000	11204.86	>36,000

There are significant improvements in computational times for Model 1 and 2 after binary reduction as can be seen in Table 6 (5.69s vs. 58.53s and 9.92s vs. 18.43s). Moreover, there are possibilities to find even better local solution too from the reduced model as in the case of Model 2. The reduced models (Model 1-R and 2-R) should yield the same optimal solutions as the original models (Model 1 and 2), respectively, for small connection costs but there are slight differences in the NPV values reported in Table 6 as these models are solved here with DICOPT that gives the local solutions. The reformulated MILP after binary reduction Model 3-R becomes slightly expensive to solve as compared to finding local solutions for the original MINLP models, but the solution obtained in this case is the global one (within 2% optimality tolerance). Notice that the MILP solutions can be either lower (instance 1) or higher (instance 2) than the global optimal for MINLP models as these involves approximations of oil deliverability, cumulative water and cumulative gas produced all three functions and the resulting MILP could over or underestimate the original NPV function. We do not present the result of Model 3 here as it gives the same NPV as Model 3-R but at much higher computational expense since a larger number of binary variables is involved in the model. Note that some of the binary variables are pre-fixed in all of the models considered based on the earliest installation time of the FPSO facilities and corresponding limitations on the FPSO expansions, field-FPSO connections and drilling of the wells in the fields that improves the computational performance of these models.

Table 6: Comparison of models 1, 2 and 3 with and w/o binary reduction

	Model 1	Model 1-R	Model 2	Model 2-R	Model 3-R
Constraints	3,543	3,432	5,543	5,432	8,663
Continuous Var.	2,781	2,572	3,461	3,252	6,103
Discrete Var.	477	301	477	301	451
SOS1 Var.	0	0	0	0	400
NPV(million$)	11412.48	11335.01	11204.86	11294.82	11259.61
Time(s)	58.53	5.69	18.43	9.92	871.80

*Model 1 and 2 solved with DICOPT 2x-C, Model 3 solved with CPLEX 12.2

The solution of Model 3-R can also be used to fix discrete variables in the MINLPs to obtain near optimal solutions to the original problem as done for instance 1. Table 7 presents the solutions of the NLPs obtained after fixing binary decisions and show that none of the solver in Table 5 could provide better NPV values than this case. Overall, we can say that the results for this larger instance also show similar trends as what is observed for instance 1.

Table 7: Improved solutions (NPV in million$) for Models 1 and 2 using Model 3-R solution

Model 1	Model 1 (fixed binaries from Model 3-R)	Model 2	Model 2 (fixed binaries from Model 3-R)
11412.48	11412.48	11204.86	11356.31

7.3 Instance 3

In this instance we consider 10 oil fields (Figure 4) that can be connected to 3 FPSOs with 23 possible connections. There are a total of 84 wells that can be drilled in all of these 10 fields and the planning horizon considered is 20 years.

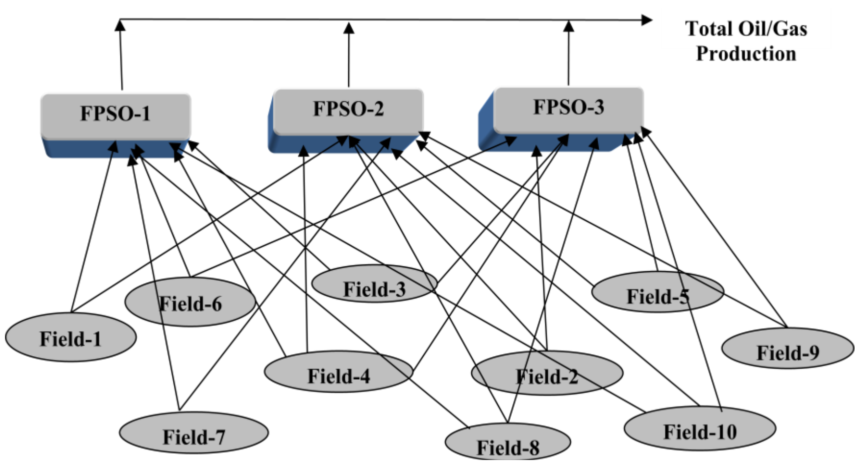
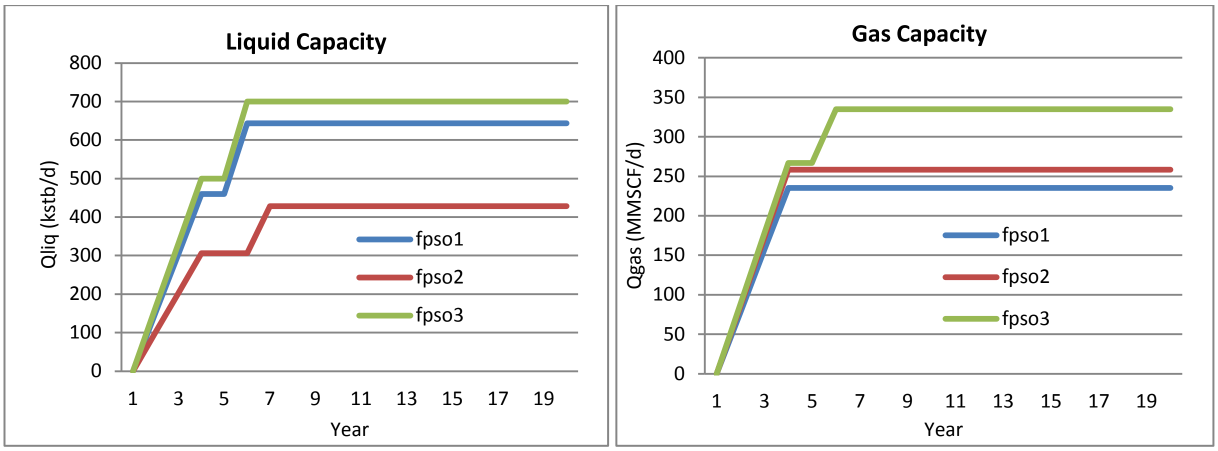


Figure 4: Instance 3 (10 Fields, 3 FPSO, 20 years) for oilfield problem

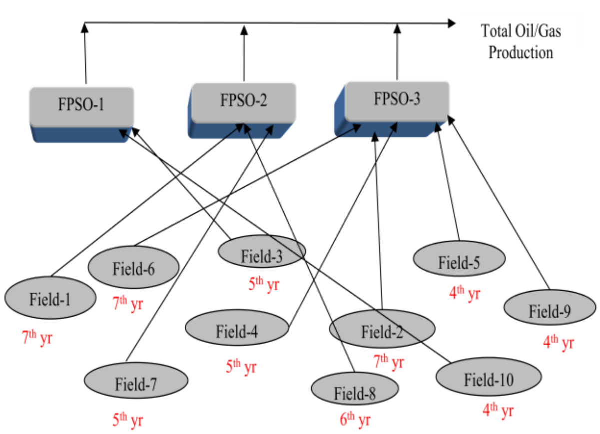
The optimal solution of this problem that corresponds to Model 2-R solved with DICOPT 2x-C, suggests to install all the 3 FPSO facilities in the first time period with their respective liquid (Figure 5-a) and gas (Figure 5-b) capacities. These FPSO facilities are further expanded in future when more fields come online or liquid/gas flow rates increases as can be seen from these figures.



(a) Liquid capacities of FPSO facilities (b) Gas capacities of FPSO facilities

Figure 5: FPSO installation and expansion schedule

After initial installation of the FPSO facilities by the end of time period 3, these are connected to the various fields to produce oil in their respective time periods for coming online as indicated in Figure 6. The well installation schedule for these fields Figure 7 ensures that the maximum number of wells drilling limit and maximum potential wells in a field are not violated in each time period t. We can observe from these results that most of the installation and expansions are in the first few time periods of the planning horizon.



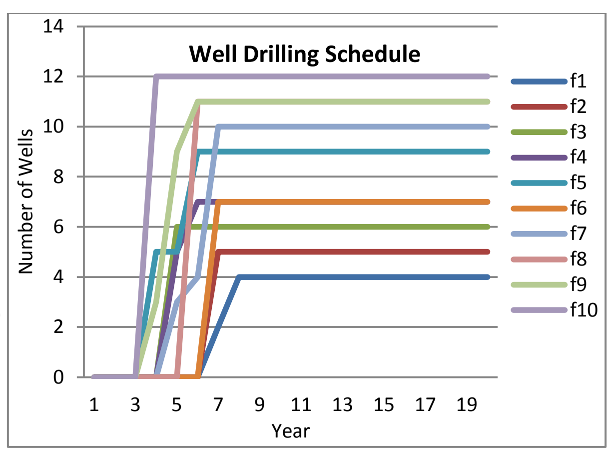
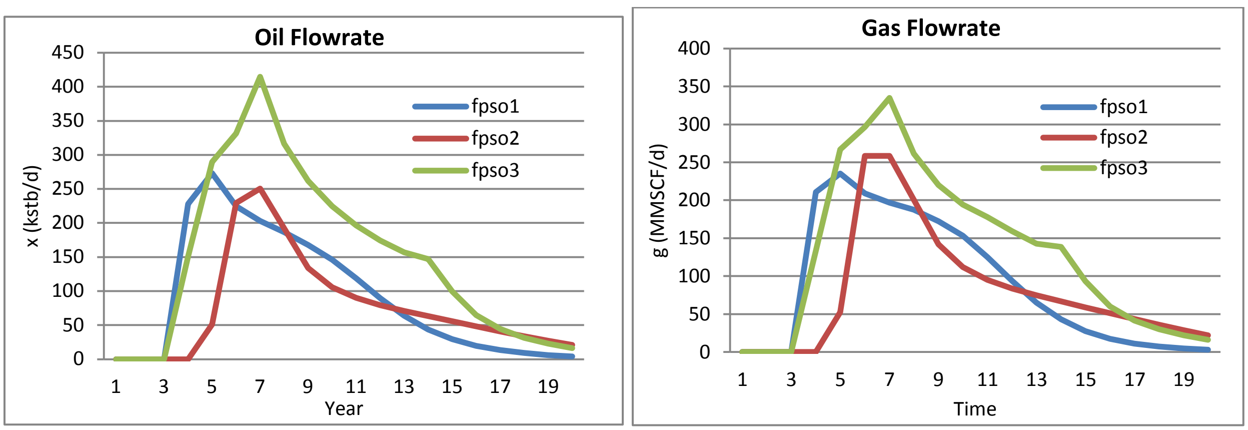


Figure 6: FPSO-field connection schedule Figure 7: Well drilling schedule for fields

Other than these investment decisions, the operations decisions are the production rates of oil and gas from each of the fields, and hence, the total flow rates for the installed FPSO facilities that are connected to these fields as can be seen from Figures 8 (a)-(b). Notice that the oil flow rates increases initially until all the fields come online and then they start to decrease as the oil deliverability decreases when time progresses. Gas flow rate, which depends on the amount of oil produced, also follows a similar trend. The total NPV of the project is $30946.39M.



(a) Total oil flowrates from FPSO's (b) Total gas flowrates from FPSO's

Figure 8: Total flowrates from each FPSO facility

Tables 8-10 represent the results for the various model types considered for this instance. We can draw similar conclusions as discussed for instances 1 and 2 based on these results. DICOPT performs best in terms of solution time and quality, even for the largest instance compared to other solvers as can be seen from Table 8.

Table 8: Comparison of various models and solvers for Instance 3

	Model 1		Model 2	
Constraints	5,900		10,100	
Continuous Var.	4,681		6,121	
Discrete Var.	851		851	
Solver	Optimal NPV (million$)	Time (s)	Optimal NPV (million$)	Time (s)
DICOPT	31297.94	132.34	30562.95	114.51
SBB	30466.36	4973.94	30005.33	18152.03
BARON	31297.94	>72,000	30562.95	>72,000

There are significant computational savings with the reduced models as compared to the original ones for all the model types in Table 9. Even after binary reduction of the reformulated

MILP, Model 3-R becomes expensive to solve, but yields global solutions, and provides a good discrete solution to be fixed/initialized in the MINLPs for finding better solutions.

Table 9: Comparison of models 1, 2 and 3 with and w/o binary reduction

	Model 1	**Model 1-R**	**Model 2**	**Model 2-R**	**Model 3-R**
Constraints	5,900	5,677	10,100	9,877	17,140
Continuous Var.	4,681	4,244	6,121	5,684	12,007
Discrete Var.	851	483	851	483	863
SOS1 Var.	0	0	0	0	800
NPV(million$)	31297.94	30982.42	30562.95	30946.39	30986.22
Time(s)	132.34	53.08	114.51	67.66	16295.26

*Model 1 and 2 solved with DICOPT 2x-C, Model 3 with CPLEX 12.2

We can see from Table 10 that the solutions that come from the Models 1 and 2 after fixing discrete variables based on MILP solution (even though it was solved within 10% of optimality tolerance) are the best among all other solutions obtained in Table 8. Therefore, the MILP approximation is an effective way to obtain near optimal solution for the original problem. Notice also that the optimal discrete decisions for Models 1 and 2 are very similar even though they are formulated in a different way. However, only Model 2 can be reformulated into an MILP problem that gives a good estimate of the near optimal decisions to be used for these MINLPs.

Table 10: Improved solutions (NPV in million$) for Models 1 and 2 using Model 3-R solution

Model 1	**Model 1 (fixed binaries from Model 3-R)**	**Model 2**	**Model 2 (fixed binaries from Model 3-R)**
31297.94	31329.8136	30562.95	31022.4813

Remarks

(a) The optimal NPV of both models 1 and 2 are very close (within ~1-3%) for all the instances. Moreover, the difference is even smaller when we compare the global solutions and they tend to have same discrete decisions at the optimal solution. Hence, in principle we can use either of these models for the oilfield problem directly or with some other method. However, since Model 1 involves a large number of non-convexities because of the extra bilinear terms, it is more prone to converging to local solutions, and needs good initializations as compared to Model 2. Moreover, as opposed to Model 2, it is not possible to convert Model 1 to an MILP model that can be solved to global optimality. However, the nonlinearities and non-convexities perform reasonably well for both of these models as seen from the computational results, and few trials with DICOPT can give good quality local solutions within few seconds for these models.

(b) Model 2 is more accurate in terms of physical representation of water and gas flow profiles than Model 1 as explained in Appendix B, especially when the length of each time period is large. Model 1 usually overestimates the NPV as it assumes constant GOR and WOR for a time period t while extracting the oil from a field during that time period, where WOR and GOR are calculated based on the fractional recovery by the end of time period t-1, i.e. point estimates are used for WOR and GOR. On the other hand, Model 2 estimates the cumulative water and gas flow rates at the end of time period t taking into account the amount of oil produced in that time period and variability of WOR and GOR during current time period t i.e. average values of WOR and GOR over the time period. Because of the general trend of increasing WOR and GOR as time progresses and hence underestimating the actual water and gas flow rates in Model 1 during each time period t due to point estimates for WOR and GOR at the end of time t-1, it gives slightly higher NPV's as can be seen from the solutions obtained. In contrast, if WOR and GOR are estimated at the end of time period t instead t-1, the solutions from Model 1 should give lower NPV values as compared to Model 2.

(c) The solutions from MILP model are of good quality if we use a reasonable number of point estimates (5 or more) for the piecewise linear approximation. Due to the increase in the solution time for the model as the problem size or number of point estimates increases, specialized decomposition strategies could be investigated to solve these MILPs in a fast and reliable way.

(d) It can be seen from the results that the approximate MILPs are a good way to find discrete decisions that lead to global or near optimal solution for the original MINLP when fixing these decisions. None of the MINLP solvers could find better solutions than the ones obtained using the MILP solution. Furthermore, these MILP's also give a way to estimate the quality of local solutions obtained from the fast MINLP local solvers either by solving these models till optimality if it is easier to solve or by its LP relaxation for large instances.

8 Conclusions

In this paper, we have proposed a new generic MINLP model for offshore oilfield infrastructure planning considering multiple fields, three components (oil, water and gas) explicitly in the formulation, facility expansions decisions and nonlinear reservoir profiles. The model can determine the installation and expansion schedule of facilities and respective oil, liquid and gas capacities, connection between the fields and FPSO's, well drilling schedule and production rates of oil, water and gas simultaneously in a multiperiod setting. The resulting model yields good solutions to realistic instances when solving with DICOPT directly. Furthermore, the model can be reformulated into an MILP using piecewise linearization and exact linearization techniques with which the problem can be solved to global optimality. The proposed MINLP and MILP formulations are further improved by using a binary reduction scheme resulting in the improved local solutions and more than an order of magnitude reduction in the solution times. Realistic instances involving 10 fields, 3 FPSO's and 20 years planning horizon have been solved and comparisons of the computational performance of the proposed MINLP and MILP formulations are presented. Moreover, the models presented here are very generic and can either be used for simplified cases (e.g. linear profiles for reservoir, fixed well schedule etc.) or extended to include other complexities. There are various trade-offs involve in selecting a particular model for oilfield problem. In case that we are concerned with the solution time, especially for the large instances, it would be better to use DICOPT on Model 2R directly that gives good quality solution very fast. If fast computing times are of no much concern one may want to use MILP approximation model that can yield better solutions but at higher computational cost. Furthermore, these MILP solutions also provide a way to access the quality of suboptimal solutions from the MINLPs or finding better once using its solution for the original problem.

9 Acknowledgements

The authors acknowledge to ExxonMobil Upstream Research Company for the financial support of this work.

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Nomenclature

Indices

t, τ	time periods, $t \epsilon \{1,2,3,\dots,T\}$
f	field
fpso	FPSO facility

Binary Variables

$b_{fpso,t}$	whether or not FPSO facility *fpso* is installed at the beginning of time period *t*
$b^{ex}_{fpso,t}$	whether or not FPSO facility *fpso* is expanded at the beginning of time period *t*
$b^{c}_{f,fpso,t}$	whether or not a connection between field *f* and FPSO facility *fpso* is installed at the beginning of time period *t*
$b^{R}_{f,fpso}$	whether or not a connection between field *f* and FPSO facility *fpso* is installed

Integer Variables

$I^{well}_{f,t}$	Number of wells drilled in field *f* at the beginning of time period t
$N^{well}_{f,t}$	Number of wells available in field *f* for production in time period t

Continuous Variables

NPV	net present value
REV_t	total revenues in time period *t*
$COST_t$	total costs in time period *t*
CAP_t	total capital costs in time period *t*
$OPER_t$	total operating costs in time period *t*
x^{tot}_t	total oil flow-rate in time period *t*
w^{tot}_t	total water flow-rate in time period *t*
g^{tot}_t	total gas flow-rate in time period *t*
$x_{f,t}$	oil production rate from field *f* in time period *t*
$w_{f,t}$	water production rate from field *f* in time period *t*
$g_{f,t}$	gas production rate from field *f* in time period *t*
$xc_{f,t}$	cumulative oil produced from field *f* by the end of time period *t*

$wc_{f,fpso,t}$ cumulative water produced from field *f* to FPSO facility *fpso* by the end of time period *t*

$gc_{f,fpso,t}$ cumulative gas produced from field *f* to FPSO facility *fpso* by the end of time period *t*

$fc_{f,t}$ fraction of oil recovered from field *f* by the end of time period *t*

$x_{f,fpso,t}^{well}$ oil flow rate per well from field *f* to FPSO facility *fpso* in time period *t*

$Q_{f,fpso,t}^{d,well}$ field deliverability (maximum oil flow rate) per well for field *f* and FPSO facility *fpso* combination in time period *t*

$Q_{f,fpso,t}^{wc}$ dummy variable for cumulative water produced from field *f* to FPSO facility *fpso* by the end of time period *t*

$Q_{f,fpso,t}^{gc}$ dummy variable for cumulative gas produced from field *f* to FPSO facility *fpso* by the end of time period *t*

$x_{fpso,t}$ total oil flow rate into FPSO facility *fpso* in time period *t*

$w_{fpso,t}$ total water flow rate into FPSO facility *fpso* in time period *t*

$g_{fpso,t}$ total gas flow rate into FPSO facility *fpso* in time period *t*

$x_{f,fpso,t}$ total oil flow rate from field *f* to FPSO facility *fpso* in time period *t*

$w_{f,fpso,t}$ total water flow rate from field *f* to FPSO facility *fpso* in time period *t*

$g_{f,fpso,t}$ total gas flow rate from field *f* to FPSO facility *fpso* in time period *t*

$Q_{fpso,t}^{oil}$ oil processing capacity of FPSO facility *fpso* in time period *t*

$Q_{fpso,t}^{liq}$ liquid (oil and water) capacity of FPSO facility *fpso* in time period *t*

$Q_{fpso,t}^{gas}$ gas capacity of FPSO facility *fpso* in time period *t*

$QI_{fpso,t}^{oil}$ oil installation capacity of FPSO facility *fpso* at the beginning of time period *t*

$QI_{fpso,t}^{liq}$ liquid installation capacity of FPSO facility *fpso* at the beginning of time period *t*

$QI_{fpso,t}^{gas}$ gas installation capacity of FPSO facility *fpso* at the beginning of time period *t*

$QE_{fpso,t}^{oil}$ oil expansion capacity of FPSO facility *fpso* at the beginning of time period *t*

$QE_{fpso,t}^{liq}$ liquid expansion capacity of FPSO facility *fpso* at the beginning of time period *t*

$QE_{fpso,t}^{gas}$ gas expansion capacity of FPSO facility *fpso* at the beginning of time period *t*

Parameters

$FC_{fpso,t}$	fixed capital cost for installing FPSO facility *fpso* at the beginning of time period t
$FC_{f,fpso,t}$	fixed cost for installing the connection between field *f* and FPSO facility *fpso* at the beginning of time period t
$FC_{f,t}^{well}$	fixed cost for drilling a well in field *f* at the beginning of time period t
$VC_{fpso,t}^{liq}$	variable capital cost for installing or expanding the liquid (oil and water) capacity of FPSO facility *fpso* at the beginning of time period t
$VC_{fpso,t}^{gas}$	variable capital cost for installing or expanding the gas capacity of FPSO facility *fpso* at the beginning of time period t
OC_t^{liq}	operating cost for per unit of liquid (oil and water) produced in time period t
OC_t^{gas}	operating cost for per unit of gas produced in time period t
REC_f	total amount of recoverable oil from field *f*
$U_{f,fpso}^{well,oil}$	Upper bound on the oil flow rate per well from field *f* to FPSO facility *fpso*
U_{fpso}^{oil}	Upper bound on the installation or expansion of oil capacity of a FPSO facility
U_{fpso}^{liq}	Upper bound on the installation or expansion of liquid capacity of a FPSO facility
U_{fpso}^{gas}	Upper bound on the installation or expansion of gas capacity of a FPSO facility
UN_f^{well}	Maximum number of wells that can be drilled in field *f* during planning horizon *T*
UI_t^{well}	Maximum number of wells that can be drilled during each time period t
$M_{f,fpso}^{wc}$	Maximum cumulative water that can be produced for a field-FPSO connection
$M_{f,fpso}^{gc}$	Maximum cumulative gas that can be produced for a field-FPSO connection
l_1	lead time for initial installation of a FPSO facility
l_2	lead time for expansion of an earlier installed FPSO facility
μ	Maximum fraction of the initial built FPSO capacities that can be expanded
α_t	price of oil in time period *t*
β_t	price of gas in time period *t*
d_t	discounting factor for time period *t*
δ_t	number of days in time period *t*
$a_{()}, b_{()}, c_{()}, d_{()}$	coefficients for polynomials used for reservoir models

Appendix A: Derivation of the Reservoir Profiles for Model 2 from Model 1

Model 1 involves nonlinearities in the form of three polynomials for oil deliverability, GOR and WOR, (A1)-(A3), and 2 bilinear equations for water and gas flow rates, (A4)-(A5).

$$Q_f^d = a_{1,f}(fc_f)^3 + b_{1,f}(fc_f)^2 + c_{1,f}\, fc_f + d_{1,f} \qquad \forall f \qquad \text{(A1)}$$

$$wor_f = a_{2,f}(fc_f)^3 + b_{2,f}(fc_f)^2 + c_{2,f}\, fc_f + d_{2,f} \qquad \forall f \qquad \text{(A2)}$$

$$gor_f = a_{3,f}(fc_f)^3 + b_{3,f}(fc_f)^2 + c_{3,f}\, fc_f + d_{3,f} \qquad \forall f \qquad \text{(A3)}$$

$$w_f = wor_f\, x_f \qquad \forall f \qquad \text{(A4)}$$

$$g_f = gor_f\, x_f \qquad \forall f \qquad \text{(A5)}$$

To derive the reservoir profile for Model 2 from the above equations of Model 1 we consider the following two properties:

1. ***The area under the curve GOR vs. cumulative oil produced for a field yields the cumulative amount of gas produced.***
2. ***The area under the curve WOR vs. cumulative oil produced for a field yields the cumulative amount of water produced.***

Explanation of Property 1

From equation (A3) we have GOR for a field as a cubic function in terms of fractional recovery (or cumulative oil produced xc_f and recoverable oil REC_f) as follows (Model 1):

$$gor_f = a_{3,f}\, fc_f^3 + b_{3,f}\, fc_f^2 + c_{3,f}\, fc_f + d_{3,f} \qquad \text{(A6)}$$

$$gor_f = a_{3,f}\left(\frac{xc_f}{REC_f}\right)^3 + b_{3,f}\left(\frac{xc_f}{REC_f}\right)^2 + c_{3,f}\left(\frac{xc_f}{REC_f}\right) + d_{3,f} \qquad \text{(A7)}$$

A differential change in the cumulative oil produced multiplied by the GOR yields the corresponding fractional change in the cumulative amount of gas produced, gc, as seen in Figure 9 and corresponding equation (A8).

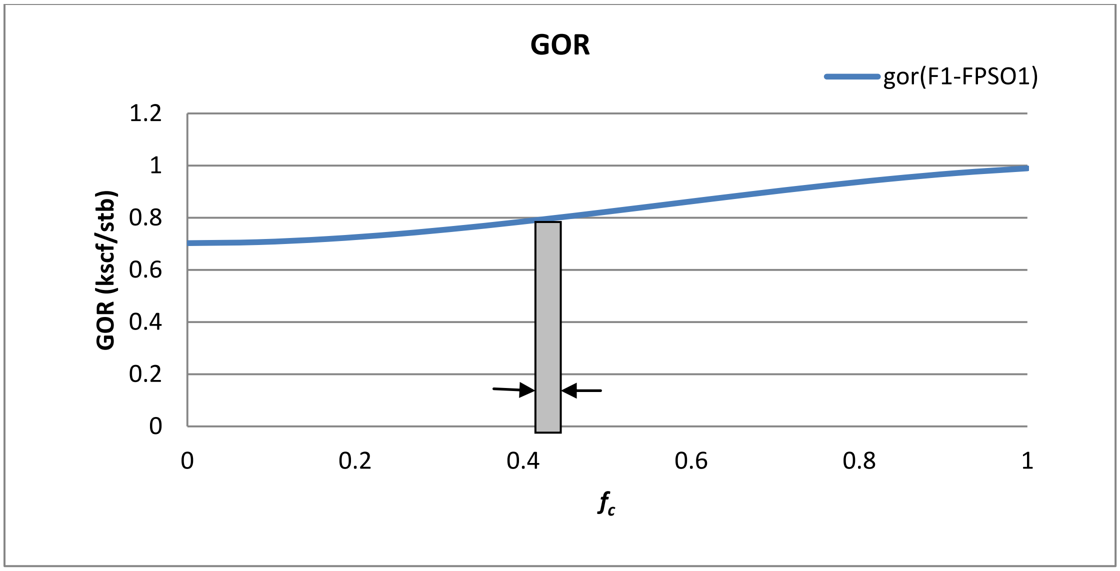

Figure 9: GOR profile for field (F1) and FPSO (FPSO 1) connection

$$d(gc_f) = gor_f . d(xc_f) \quad \text{(A8)}$$

We should note that Figure 9 corresponds to gor vs f_c but it is easy to convert it to gor vs xc given that the reservoir size (REC_f) is known. Integrating in (A8) both sides from zero (i.e. area under the curve between GOR and xc), yields,

$$\int_0^{gc_f} d(gc_f) = \int_0^{xc_f} gor_f . d(xc_f) \quad \text{(A9)}$$

$$\int_0^{gc_f} d(gc_f) = \int_0^{xc_f} \left\{ a_{3,f} \left(\frac{xc_f}{REC_f}\right)^3 + b_{3,f} \left(\frac{xc_f}{REC_f}\right)^2 + c_{3,f} \left(\frac{xc_f}{REC_f}\right) + d_{3,f} \right\} . d(xc_f) \quad \text{(A10)}$$

$$gc_f = \frac{a_{3,f}}{4} \left(\frac{xc_f^4}{REC_f^3}\right) + \frac{b_{3,f}}{3} \left(\frac{xc_f^3}{REC_f^2}\right) + \frac{c_{3,f}}{2} \left(\frac{xc_f^2}{REC_f}\right) + d_{3,f} (xc_f) \quad \text{(A11)}$$

$$gc_f = \frac{a_{3,f}.REC_f}{4} \left(\frac{xc_f}{REC_f}\right)^4 + \frac{b_{3,f}.REC_f}{3} \left(\frac{xc_f}{REC_f}\right)^3 + \frac{c_{3,f}.REC_f}{2} \left(\frac{xc_f}{REC_f}\right)^2 + d_{3,f}.REC_f \left(\frac{xc_f}{REC_f}\right) \quad \text{(A12)}$$

$$gc_f = \frac{a_{3,f}.REC_f}{4} (fc_f)^4 + \frac{b_{3,f}.REC_f}{3} (fc_f)^3 + \frac{c_{3,f}.REC_f}{2} (fc_f)^2 + d_{3,f}.REC_f (fc_f) \quad \text{(A13)}$$

$$gc_f = a'_{3,f}(fc_f)^4 + b'_{3,f}(fc_f)^3 + c'_{3,f}(fc_f)^2 + d'_{3,f}(fc_f) \quad \text{(A14)}$$

(A14) is the desired expression for the cumulative gas produced as a function of fractional recovery (or cumulative oil produced), i.e. area under the curve GOR vs. fractional recovery (or cumulative oil produced) that is used in Model 2. We can see that the order of the polynomial for gc expression (4th order) is 1 more than the order of the polynomial corresponding to the GOR expression in (A6). Also, there is a direct correspondence between the coefficients of the both of these polynomials. The gc vs f_c curve (4th order polynomial) corresponding to the Figure 4 (GOR vs f_c) that represents expression (A14) is shown in Figure 10.

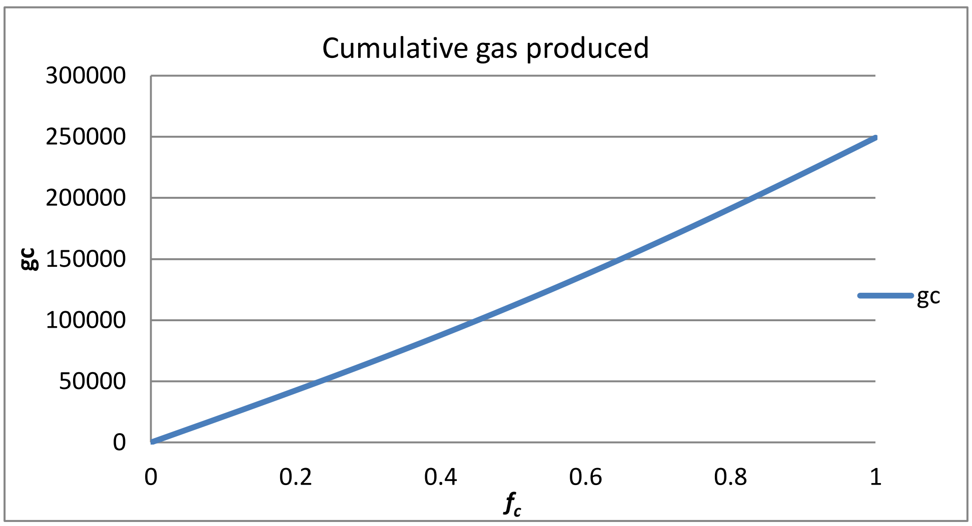

Figure 10: gc profile for field (F1) and FPSO (FPSO 1) connection

Similarly, we can derive the following expression (A15) for cumulative water produced as a function of fractional recovery (or cumulative oil produced) using WOR expression, (i.e. statement 2).

$$wc_f = a'_{2,f}(fc_f)^4 + b'_{2,f}(fc_f)^3 + c'_{2,f}(fc_f)^2 + d'_{2,f}(fc_f) \quad \text{(A15)}$$

Notice that we can derive the expressions (polynomial or any other functions) from the existing model of GOR and WOR to gc and wc in terms of fractional recovery (or cumulative oil produced) respectively and vice-versa.

Appendix B: Comparisons of the Models based on (gor, wor) and (gc, wc) functions

1. Model 1 (GOR and WOR as a function of cumulative oil produced) requires the bilinear equations (B1) and (B2) for water and gas flow rates while Model 2 does not need these equations as these flowrates can be expressed as equations (B3) and (B4) given that we have polynomials for gc and wc. Hence, Model 2 involving only univariate polynomials should computationally perform better.

$$g_{f,t} = gor_{f,t} x_{f,t} \qquad \forall f,t \qquad \text{(B1)}$$

$$w_{f,t} = wor_{f,t} x_{f,t} \qquad \forall f,t \qquad \text{(B2)}$$

$$g_{f,t} = (gc_{f,t} - gc_{f,t-1}) / \delta_t \qquad \forall f,t \qquad \text{(B3)}$$

$$w_{f,t} = (wc_{f,t} - wc_{f,t-1}) / \delta_t \qquad \forall f,t \qquad \text{(B4)}$$

2. The WOR and GOR functions in (A2) and (A3) introduce a large number of non-convexities in Model 1 as compared to the gc and wc functions in (A14) and (A15) that are univariate monotonically increasing functions. Hence, these functions will be better for approximating them by piecewise linearization. As an example the GOR and corresponding gc functions for a field are shown in Figure 11.

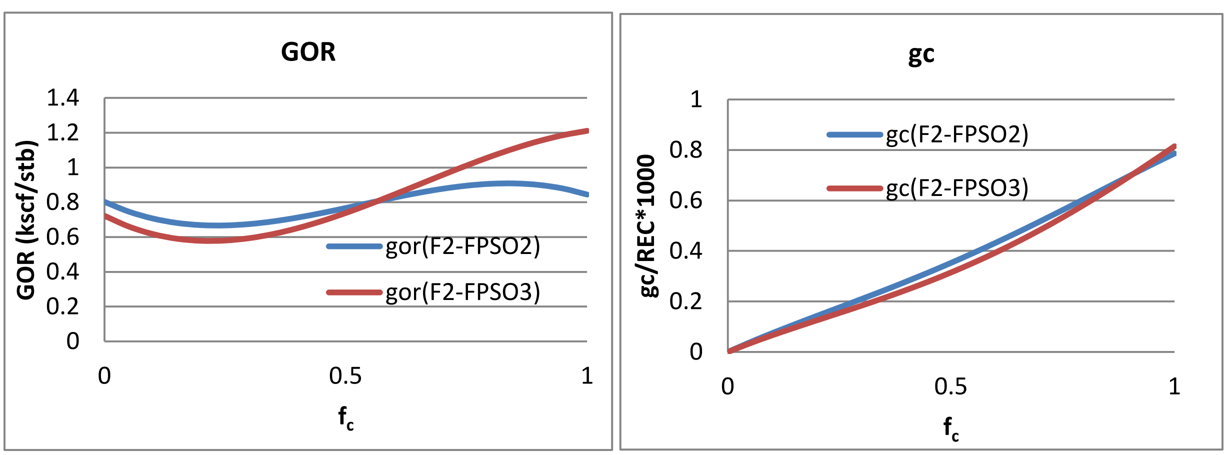

Figure 11: GOR and gc profiles for 1 field and 2 FPSO connections

3. In Model 1 we assume that the WOR and GOR equations (B5) and (B6) used in time period t are calculated in terms of the fractional oil recovery by the end of previous time period t-1,

i.e. uses point estimate values. Therefore, GOR and WOR essentially perform as constants in current time period t, and the oil flowrate does not account for the variability in GOR and WOR values during that time period.

$$wor_{f,t} = a_{2,f}(fc_{f,t-1})^3 + b_{2,f}(fc_{f,t-1})^2 + c_{2,f}\, fc_{f,t-1} + d_{2,f} \qquad \forall f,t \tag{B5}$$

$$gor_{f,t} = a_{3,f}(fc_{f,t-1})^3 + b_{3,f}(fc_{f,t-1})^2 + c_{3,f}\, fc_{f,t-1} + d_{3,f} \qquad \forall f,t \tag{B6}$$

$$w_{f,t} = wor_{f,t} x_{f,t} \qquad \forall f,t \tag{B7}$$

$$g_{f,t} = gor_{f,t} x_{f,t} \qquad \forall f,t \tag{B8}$$

However, equations (B9) and (B10) for wc and gc explicitly predicts the cumulative amount of water and gas produced, respectively, by the end of period t as a function of cumulative oil produced by the end of period t, and hence also accounts for the variability of the GOR and WOR values during current period t i.e. considers average values of WOR and GOR over the time period t. Therefore, Model 2 is also better in terms of representing the physical reservoir characteristics.

$$wc_{f,t} = a_{2,f}(fc_{f,t})^3 + b_{2,f}(fc_{f,t})^2 + c_{2,f}\, fc_{f,t} + d_{2,f} \qquad \forall f,t \tag{B9}$$

$$gc_{f,t} = a_{3,f}(fc_{f,t})^3 + b_{3,f}(fc_{f,t})^2 + c_{3,f}\, fc_{f,t} + d_{3,f} \qquad \forall f,t \tag{B10}$$

Notice that equations (B5) and (B6) for Model 1 could also be represented as a function of fractional recovery by the end of time period t instead of time period t-1, however, the model will still consider the WOR and GOR values based on the point estimate instead average values over the time period t as used in Model 2.