Selective Tightening Algorithm for the Optimization of Pipeline Network Designs in the Energy Industry

Demian J. Presser [a,b], Diego C. Cafaro[a,b*], Ignacio E. Grossmann[c], R. Cory Allen [d], Yuanyuan Guo [d], Yuzixuan Zhu [d], Yufen Shao[d], Kevin C. Furman [d]

[a] INTEC(UNL-CONICET), Güemes 3450, 3000 Santa Fe, ARGENTINA

[b] Fac. de Ing. Química, Universidad Nacional del Litoral, Santiago del Estero 2829, 3000 Santa Fe, ARGENTINA

[c] Dept. of Chem. Eng., Carnegie Mellon University, 5000 Forbes Ave, 15213 Pittsburgh, PA, US

[d] ExxonMobil Technology and Engineering Company, 22777 Springwoods Village Parkway, 77389 Spring, TX, US

*Corresponding author: dcafaro@fiq.unl.edu.ar - https://orcid.org/0000-0002-3893-5106

Abstract

Energy industries face the challenge on how to design networks to gather flows from unconventional assets due to the relative short lifetime of the wells. The optimal design of the network of pipelines and processing facilities is a challenging problem from both combinatorial and nonlinear viewpoints. To date, real instances of this problem cannot be solved to global optimality unless limited assumptions are adopted. We propose a systematic algorithm to address the optimal planning of gathering networks in a multiperiod horizon. It relies on the strategic selection of links on which fluid-dynamic equations are imposed. By relaxing constraints, the resulting mixed-integer programming models are still complex from the combinatorial standpoint, but they can be solved in reasonable computational times. The algorithm progressively adds constraints to the potential arcs, seeking for the global optimal solution by selectively tightening relaxations. Results show that computational times can be reduced by two orders of magnitude.

1. Introduction

The optimization of networks of pipelines and facilities to gather, process and deliver unconventional oil and gas has received increasing attention from the research community since the shale boom (Drouven et al., 2023). Rapid productivity decline curves of unconventional wells challenge facility planners to make accurate decisions more frequently. Economic returns from unconventional assets are often modest and, as a result, planning facilities and expansions across shale formations is critical for energy companies to stay profitable. Over time, significant advances in horizontal drilling and hydraulic fracturing technologies have facilitated the exploitation of shale resources, but the need for computational tools to aid operational and planning decisions remains high. In response to this need, recent contributions from the mathematical programming field have addressed the optimal design and operation of shale gathering networks, with different levels of detail. Some of them assume tree-like networks carrying multiphase flows, with given pressure drops per segment or echelon (Cafaro and Grossmann, 2014; Montagna et al., 2021). Others present generalized frameworks in which the oil and gas networks are built following no predetermined number of echelons (Montagna et al., 2022; Wen at al., 2022), but liquid and gas streams are separated at source nodes and flow through separate pipelines.

A common assessment in all of these contributions is that the optimization problem is difficult due to two sources of complexity: (a) the combinatorial nature of the network design, and (b) the nonlinear correlations to calculate head losses according to the pipeline diameter (sizing decisions), which are often non-convex. In fact, for rather small problems, in our computational experiments not a single feasible solution has been found after one day of computation using global optimization solvers like BARON (Ryoo and Sahinidis, 1995). Even after applying decomposition strategies, which are not able to converge to the actual global optima, solutions found show optimality gaps over 5% (Montagna et al., 2021). The aim of this work is to build an optimization algorithm to consistently solve the design of generalized gathering networks to global optimality in reasonable CPU times. This algorithm is essential if more realistic and challenging instances of the problem are to be addressed in the future.

Related work

Comprehensive reviews on shale optimization models can be found in Gao and You (2017) and Drouven et al. (2023). In 2014, Cafaro and Grossmann develop a mixed-integer nonlinear programming (MINLP) model for the strategic design and planning of shale gas networks, comprising pipelines, compressors and processing plants. Concave cost functions are used to represent economies of scale to determine the size of facilities and pipelines, which are treated as continuous variables. For simplicity, reference gas pressures at intermediate nodes are given. Through a more detailed representation, Drouven and Grossmann (2017) seek to optimally plan well development tasks explicitly accounting for pressure profiles across shale gas pipeline networks. This work demonstrates that operating costs from gas compression can be reduced significantly by properly handling pressures over time. Although the network design is out of the scope in this case, more complex nonlinear equations are used to calculate pressure drops and compression power. In both contributions, tailored solution algorithms are proposed to address non-convexities and solve real-world problems from the Appalachia. It is interesting to note that when the structure of the pipeline network needs to be optimally determined, research works tend to simplify the modeling of hydraulic equations, usually assuming linear correlations (Wang et al., 2018). In fact, given the challenges posed by MINLP formulations, combinations of metaheuristics have also been tried (He et al., 2019). In the latter work, genetic algorithms are combined with simulated annealing strategies to improve the local optimization ability of the first method. Interconnection of oil wells and manifolds is the main decision to make, yielding an optimization problem of discrete nature and combinatorial complexity. The main drawback of metaheuristics is that the global optimum is only guaranteed for an infinite number of trials, and no reference to the best possible solution is provided.

With a focus on facilities rather than pipelines, Tan and Barton (2016) develop a multiperiod MILP model to dynamically allocate mobile plants to wellpads, aiming to process shale gas production through more flexible and robust designs. A two-stage stochastic programming approach is proposed to address uncertainty in future gas production, demand and price. Facilities to install at any node are basically two: gas-to-liquids plants converting methane into heavier hydrocarbons, and liquefaction plants to obtain liquefied natural gas (LNG). Like in previous models, the optimal time to buy, sell and allocate plants is determined through 0-1 variables that permit to maximize the expected NPV of the shale gas production. A real case study from the Bakken shale formation (U.S.) is solved with the proposed model. Similarly, Allen et al. (2019) address the sizing and allocation of modular facilities that can be relocated according to realizations of uncertain productivity scenarios. Many skids or modules can work in parallel at every node. A novel multistage stochastic programming model is developed, which is based on a tree of multidimensional nodes accounting for size, technology and location of facilities, as well as production forecast scenarios. More recently, Hong et al. (2020a) propose an extension of the previous model that includes wellpad production start times as decisions variables, such as the model by Cafaro and Grossmann (2014). In contrast to Allen et al. (2019), productivity curves are treated as deterministic data. This work shows the benefits of integrating well development plan with facilities sizing and expansion decisions.

Back to pipeline networks, Hong et al. (2020b) seek to solve the optimal design of gathering systems over natural gas reservoirs with a greater level of detail, i.e. using a 3D representation of the surface of the field. Nonlinear correlations for pipeline pressure drops along irregular terrain profiles are approximated by piece-wise linear functions, yielding an MILP approach. Pipeline routes are selected from a reduced set of alternatives that are generated by an ant-colony algorithm, which is run before the MILP. The optimal location of a centralized processing facility is also a model decision, seeking to minimize the net present cost of pipeline network. Later on, Zhou et al. (2022) consider a variety of pressurization methods to optimize shale gas transportation, seeking to minimize purchase and operating costs of compressor units across a given gathering network. The latter models are applied for validation to real-world gas fields in China.

One of the first approaches to consider shale oil gathering networks has been developed by Montagna et al. (2021). A comprehensive mixed-integer nonlinear programming (MINLP) formulation is proposed to design a pipeline network connecting shale oil wells to tank batteries for the separation of gas and water from the production stream. Starting production times and productivity profiles from all wellpads are given data. The model accounts for detailed multiphase pressure drop calculations to size the pipeline diameters according to the product flow to handle over time (Lockhart and Martinelli, 1949). Like most contributions addressing the optimal design of pipeline networks, reference values for inlet and outlet pressures at the segments are given beforehand to simplify the pipeline sizing equations. Besides that, the formulation assumes that facilities and pipeline interconnections should configure a treelike network with a fixed number of echelons (Cafaro et al., 2022). Due to production traceability, multiphase flows cannot be split until reaching the battery assigned for phase separation. The model proves to be computationally intractable even for rather small instances. To overcome that limitation, the authors propose a bi-level decomposition strategy comprising: (1) a nonconvex NLP formulation that includes all the fluid dynamic equations to estimate the multiphase transportation capacity of the pipelines, and (2) an MILP formulation for the network design and pipeline sizing. A real case study with a challenging superstructure of alternatives is addressed in this work to build a gathering network that comprises a total of 40 wellpads with an average of 6 wells per pad, over a 4-year horizon. The model proposes 20 potential nodes for junctions and 5 potential locations for tank batteries. Moreover, there are 3 alternative diameters for the pipelines and 3 tank battery capacities to choose from. After executing the solution algorithm, a near optimal solution is obtained after 12 hours of computation (5.2% optimality gap). Despite being much better than the one reported by global optimizers, the algorithm does not guarantee that there are no better alternatives, and lacks a strategy for finding better solutions and bounds.

Finally, Montagna et al. (2022) present a generalized framework in which the oil and gas gathering networks are built following no predetermined number of echelons. There is a set of generic nodes to be connected to reach the final destinations. In any of these nodes, facilities for merging, storing, purifying, processing and/or delivering flows are installed to make the oil and gas flows be ready for delivery or use. One of the major differences with regards to the models with a fixed number of echelons is that the flow direction may be reversed in any pipeline segment over the time horizon. Besides that, given that the number of segments connecting a source node to a delivery node is optimally determined by the model, an additional challenge is to track pressures along the paths. Together with the network design one should accurately define the inlet and outlet pressures at every segment for every time period. By including intermediate pressures as decisions variables, flowrates and flow directions can be optimally handled along the time horizon to make a better use of the pipeline transportation capacity. However, nonlinear constraints are required, thus posing further challenges on the modelling and computation. In this case, only single-phase pipelines segments are addressed. MILP (mixed-integer linear) and MIQCP (mixed-integer quadratically constrained) models are proposed for oil and gas gathering networks, respectively. A real-world problem comprising 150 wellpads arranged in 15 rows is tackled in this work. There are 4 alternative locations to deliver processed shale oil and gas flows, 3 alternative pipeline diameters, and any row is a candidate location to install (and expand) processing and/or delivery facilities for both fluids. However, the superstructure of potential interconnections is significantly reduced by the authors to cope with combinatorial complexity. Even after simplification, the natural gas model comprises roughly 70,000 constraints (of which 18,000 are quadratic) and 48,000 variables (14,000 of which are integer). After 12 hours of computation, the global optimality cannot be proved.

Therefore, with the aim of overcoming significant limitations of the solution strategies presented in the literature so far, a systematic algorithm is proposed in this work to aid the optimal design of generalized gathering networks.

2. Problem statement

The optimization problem addressed in this work can be stated as follows. Given:

(a) A set of time periods comprising a multi-period, long-term planning horizon.

(b) A set of production sites (individual wells or wellpads), their production start times and expected productivity profiles (including gas-to-oil and water-to-oil ratios).

(c) Potential locations and existing sites for facilities, alternative capacities per component, capital expenditures and operating costs.

(d) Potential locations for junction nodes, capital expenditures and operating costs.

(e) Potential and existing locations for delivery points and their capacities.

(f) A superstructure of interconnections among nodes, where pipelines can be installed.

(g) A finite set of alternative pipeline diameters to build the network, their installation costs per unit of length and the operational expenditures.

(h) Estimated pressures at the well-heads, minimum inlet pressures at junctions, processing facilities and delivery points.

The goal is to design an integrated network of pipelines and surface facilities to gather flows, separate phases, process product streams and deliver both shale oil and shale gas production over the time horizon, while maximizing the net present value of the project. Wellpads need to be connected to separation and/or processing units through flowlines, and processing units are further connected among themselves through liquid and/or gas pipelines to finally deliver the product flows to the market. The problem is to optimally determine the number, location and size of processing facilities, together with the network of pipelines (diameters and lengths). The time for facility investments and expansions is also critical, and economies of scale also play a key role. We should note that typically the time periods are given in months, quarters or years. Figure 1 illustrates the components of the problem on a simple network.

In the most general case, the pressures at every node in the network are also variables of the problem, which are computed according to the flow properties, the selected diameters and the distances between the nodes, by applying fluid-dynamic equations. However, production curtailment is not allowed at any point in time to avoid undesirable effects in the response of the wells after bottom pressures manipulation. In other words, in line with Montagna et al. (2021), "choking" wells is not allowed and expected production should be fully processed.

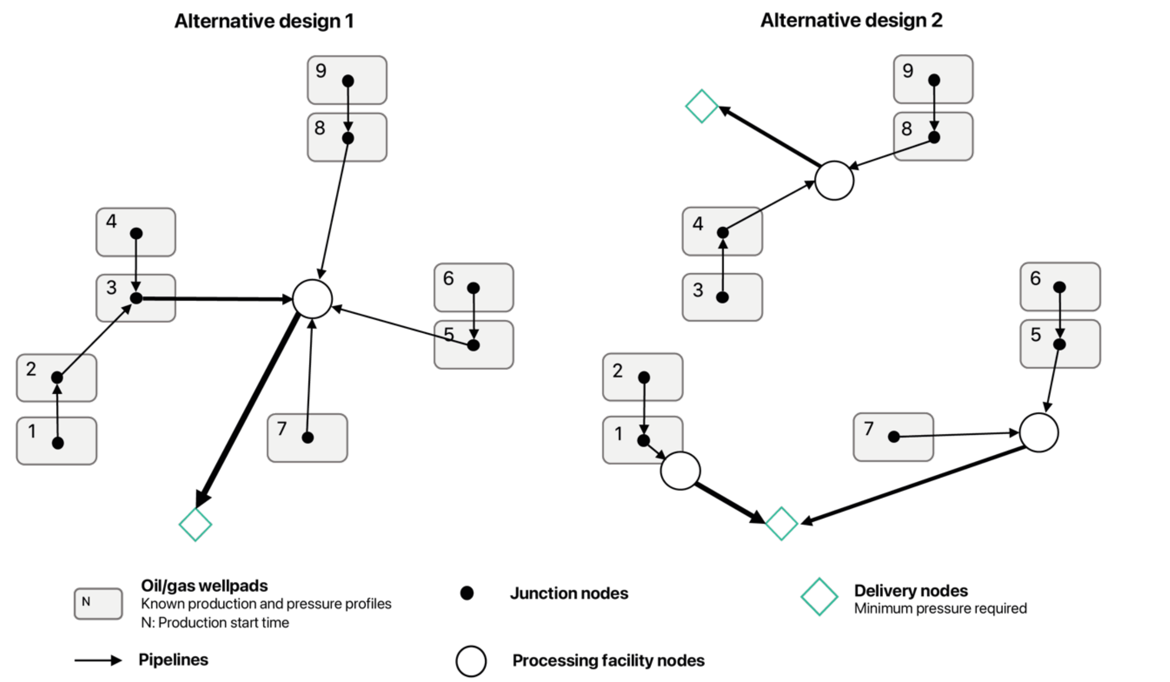

Figure 1 - Simple scheme of the components of the optimization problem for shale oil/gas gathering network design. Two alternative solutions from the superstructure are shown. Arrow widths represent different pipeline diameters.

3. Model assumptions

Optimization models for the design of pipeline networks can be distinguished among those for which the number of echelons (sequence of arcs or pipeline segments connecting a production node to a final delivery node) is predetermined, and others for which this number is undetermined (see Cafaro et al., 2022 for more details). Although formulations in both cases are different, the algorithms proposed in this work are general enough to tackle both. The following model assumptions are valid for any network structure:

1. Production nodes *i* may be individual wells, wellpads or rows of wellpads, depending on the level of aggregation of the problem. Geographical locations of production nodes are known data.

2. Development plan for shale oil and/or shale gas wells is given beforehand. It includes:

a. Number of wells to develop at each production node.

b. Drilling and completion period of the wells in the production node.

c. From the previous points, the productivity $pr_{c,i,t}$ of oil, gas and water at every node *i* for every period over the time horizon can be forecasted. Wellhead pressures $p_{i,t}^{up}$ are also given. Production curtailment through bottom pressure manipulation is not allowed at any time over the planning horizon.

3. A finite set of alternative sizes for processing facilities $fk_{q,c}$ is available. Some of them may be basic two-phase separators (horizontal or vertical vessels) to primarily split the liquids from the raw gas when the flow exits the wellheads while others may be integrated facilities featuring complex configurations meant to dewater, purify and desalt the oil, and/or conveniently dry the natural gas, before delivering them to midstream distributors. The latter are usually called tank-batteries.

4. Each production stream (comprising liquid and/or gas phases) leaving a wellpad needs to reach a processing facility through a series of pipeline segments called flowlines. After phase separation, further processing may be required and flows later converge into downstream facilities for storage, compression and/or delivery.

5. Possible locations for processing facilities, storage and delivery nodes (generically denoted by *j*) are given. Minimum flow pressures p_j^{lo} required at the inlet section of processing facilities are also known.

6. Pressures at each junction node are balanced down to the minimum pressure among the flows reaching that node on every time period.

7. The pressure of the flows cannot be boosted by building compressor stations at intermediate nodes. Furthermore, neither choking wells nor flaring gas is allowed.

8. Pipeline diameters are selected from a finite set of alternatives *($d \in D$)* that are commercially available. Only one pipeline segment can be installed between a pair of nodes along the time horizon.

4. Mathematical formulation

In general terms, optimization models for the design of gathering networks and surface facility planning are multi-period formulations that comprise three main parts: topological aspects (network design and surface facilities planning), fluid dynamics calculations and objective function. All the sets of parameters, variables and equations of the mixed-integer nonlinear programming (MINLP) models used in this work are presented in the Supplementary Material for completeness. In this section we just focus on variables and constraints that are relevant to the proposed solution algorithm.

4.1 Topological aspects

Depending on the flexibility of the network to build, we distinguish among problems with fixed and undetermined number of echelons. The first ones assume that there is a set of nodes (producers) that need to be connected to second level nodes (junctions), which in turn need to be connected to third level nodes, and so on and so forth until reaching the demand nodes in "n" unidirectional steps or echelons. Note that the number of segments can be less than the proposed number of echelons if nodes of subsequent levels are located in the same site. In the second case, there is a set of generic nodes (either sources, intermediate and/or processing nodes) that need to be connected among themselves to reach the final destinations. One of the major differences of the latter with regards to the models with a fixed number of echelons is that the flow direction may be reversed in any pipeline segment over the time horizon. In any case, facilities for merging, storing, separating and/or processing flows should be installed at some node/s to make the product flows be ready for delivery or use.

In both types of models there are basically three blocks of constraints: mass balances, facility planning and pipeline sizing constraints. As generically represented by Eq. (1), the aim of this optimization problem is to make source nodes $i \in I_{source}$ deliver their given production flows ($pr_{c,i,t}$) to one or more nodes j that collect the components $c \in C$ and subsequently send them to other nodes in the network until reaching the delivery nodes. The set of arcs A stands for potential interconnections between nodes where pipelines can be installed.

$$pr_{c,i,t} = \sum_{j:(i,j)\in A} F_{c,i,j,t} \qquad \forall i \in I_{source}, c \in C, t \in T \tag{1}$$

In turn, mass balances are imposed at every intermediate node j in the network and they state that the summation of all inlet flows equals the outlet flows (see Eq. 2). Note that volumetric units under standard conditions are often used, which is equivalent to handling mass flows (e.g., 10^3 scf = MSCF for gas flows, standing for thousand cubic feet of gas under standard conditions of temperature and pressure).

$$\sum_{i:(i,j)\in A} F_{c,i,j,t} = \sum_{j':(j,j')\in A} F_{c,j,j',t} \qquad \forall j \in J, c \in C, t \in T \tag{2}$$

If the node j in Eq. (2) splits the incoming flows towards two or more destinations, extra considerations should be made to ensure a homogenous mixture of components, requiring bilinear equations (Quesada and Grossmann, 1995). However, in oil and gas gathering networks, these equations are usually omitted given that the gathered flows need to converge to a single node, in a tree-like configuration (Cafaro and Grossmann, 2014).

Finally, gathered flows converge to delivery/sink nodes $j' \in J_{sink}$, as shown in Eq. (3). Note that material balances usually yield linear equalities like (1), (2) and (3), of low complexity from the computational perspective. For completeness, a more detailed formulation for pipeline networks with a fixed number of echelons is presented in the Supplementary Material.

$$\sum_{j:(j,j')\in A} F_{c,j,j',t} = Q_{c,j',t} \qquad \forall j' \in J_{sink}, c \in C, t \in T \tag{3}$$

In the second block of constraints, the overall capacity of the processing facilities installed in a node is imposed as an upper bound for the flows reaching that node. Since pipeline connections and facility installation and expansions are handled through 0-1 variables, the referred constraints are complicating inequalities similar to those of the 0-1 knapsack problem, but under a multiperiod framework and with the possibility of expanding the upper bound on the right hand side. In Eq. (4), the binary variable $y_{q,j,\tau}$ stands for the decision of installing a new facility of size q (with a processing capacity of $fk_{q,c}$ units of component c per day) in node j at time period τ. The installation of more facilities in the same location increases the processing capacity of that node. Parameter *lt* is the lead-time to build the facility, given as an integer number of periods. We also introduce the subset $TI \subseteq T$ to account for particular periods τ along the time horizon in which new investments can be made.

$$\sum_{i:(i,j)\in A} F_{c,i,j,t} \leq \sum_{\substack{\tau \in TI \\ \tau \leq t-lt}} \sum_{q} fk_{q,c}\ y_{q,j,\tau} \qquad \forall j \in J, c \in C, t \in T \tag{4}$$

The third block of constraints deals with pipeline sizing and interconnection. We consider a finite set of alternative diameters that are available to build the network. Under the simplifying assumption that the pipeline transportation capacity $tc_{d,i,j}$ is a given parameter, only dependent on the pipeline section and length (regardless of the pressures at the inlet and outlet extremes of every segment), pipeline sizing constraints can be modelled in linear terms. Eq. (5) avoids flows to be sent through an arc *i-j* unless a pipeline with a proper diameter has been installed between those nodes.

$$\sum_{c} F_{c,i,j,t} \leq \sum_{\substack{\tau \in TI \\ \tau \leq t-lt}} \sum_{d} tc_{d,i,j}\ x_{d,i,j,\tau} \qquad \forall (i,j) \in A, t \in T \tag{5}$$

In more detailed MINLP representations like the one used in this work, fluid dynamic conditions across the pipeline network are also included in the model and the transportation capacity $tc_{d,i,j}$ turns into a decision variable. Note that the set of arcs *A* is given by the superstructure of alternative paths that can be used to build the network, which in the most general case grows quadratically with the number of nodes. Therefore, cutting off elements from the set *A* has a significant impact on the computational performance of the MINLPs as will be shown in later sections.

4.2 Fluid-dynamic considerations

According to the fluid state, the pipeline length and the selected diameter, managing pressures at the inlet and outlet extremes of a segment ($P_{i,t}, P_{j,t}$) allows to optimally determine the flowrates at every period *t*. Flowrates and pressure drops are usually correlated by algebraic fluid dynamic equations that have been extensively used and validated in practice. For simplicity, we present the nonlinear equations that need to be added into the optimization model for the particular case of natural gas pipeline networks, based on the Weymouth correlation (Weymouth, 1912). However, the algorithm presented later in this work is general enough to handle more complex equations, like the calculation of pressure loss along multiphase pipelines (Lockhart and Martinelli, 1949).

For natural gas pipelines (*c = gas*), the transportation capacity of a segment *i-j* with length $l_{i,j}$ (in km) and diameter $D_{i,j,t}$ (available by period *t*, in m) is usually estimated by the Weymouth correlation (Weymouth, 1912), as stated by Eq. (6). Pressures are given in 10^6 Pa and flowrates in 10^6 m^3/day.

$$F_{i,j,t} \leq \gamma^{-0.5}\, {l_{i,j}}^{-0.5}\left({P_{i,t}}^2 - {P_{j,t}}^2\right)^{0.5} \sum_{\substack{\tau \in TI \\ \tau \leq t-lt}} {D_{i,j,\tau}}^{2.667} \qquad \forall (i,j) \in A\colon l_{i,j} > 0, t \in T \tag{6}$$

The parameter γ can be obtained from the expression $s_g\, T\, [P_o/(0.375\, T_o)]^2$, where s_g is the specific gravity of the gas in standard conditions (P_o = 0.1013 MPa; T_o = 298.15 K) and T is its average temperature in K. Figure 2 illustrates that correlation for 5 km long pipelines with two different diameters, assuming $T = T_o$.

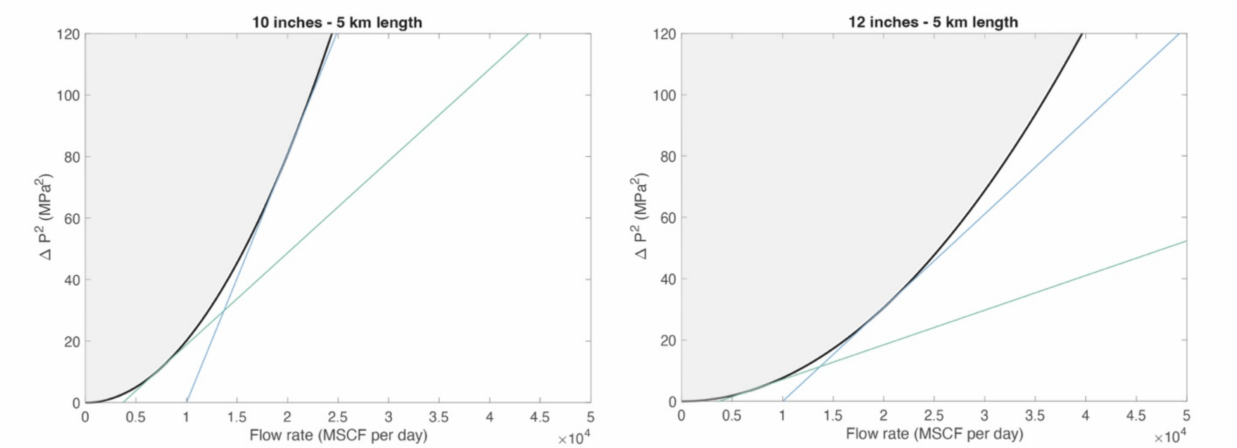

Figure 2 – Weymouth fluid dynamics constraints and linearizations for gas pipelines of 5 km length and different pipeline diameters

Since diameters are chosen from a finite set of alternatives (namely $d \in D$, as stated by assumption 8), Eq. (6) can be rewritten as Eq. (7). The 0-1 variable $x_{d,i,j,t}$ stands for the installation of a pipeline segment of internal diameter δ_d in the arc *(i,j)* at period *t*.

$$F_{i,j,t} \leq \gamma^{-0.5}\, {l_{i,j}}^{-0.5}\left({P_{i,t}}^2 - {P_{j,t}}^2\right)^{0.5} \sum_{\substack{\tau \in TI \\ \tau \leq t-lt}} \sum_{d \in D} {\delta_d}^{2.667} x_{d,i,j,\tau} \qquad \forall (i,j) \in A\colon l_{i,j} > 0, t \in T \tag{7}$$

Given that, by assumption, only one pipeline segment can be installed in an arc over the time horizon ($\sum_d \sum_t x_{d,i,j,t} \leq 1$), at most one term of the summation in Eq. (7) may take a non-zero value. From the latter, if we replace the expressions ${P_{i,t}}^2$ and ${P_{j,t}}^2$ with the variables $P_{i,t}^{sq}$ and $P_{j,t}^{sq}$, respectively, and we raise both sides of Eq. (7) to the power of 2, we obtain the constraint presented in Eq. (8). Note that by assumptions 2 and 5, $\left(p_i^{lo}\right)^2 \leq P_{i,t}^{sq} \leq \left(p_{i,t}^{up}\right)^2$, where p_i^{lo} is the minimum pressure admitted for a flow reaching a processing facility (if installed) at node *i*, and $p_{i,t}^{up}$ represents the wellhead pressure at the source node *i* at period *t*.

$$ {F_{i,j,t}}^2 \leq \gamma^{-1}\, {l_{i,j}}^{-1}\left(P_{i,t}^{sq} - P_{j,t}^{sq}\right) \sum_{\substack{\tau \in TI \\ \tau \leq t-lt}} \sum_{d \in D} {\delta_d}^{5.334} x_{d,i,j,\tau} \qquad \forall (i,j) \in A\colon l_{i,j} > 0, t \in T \tag{8}$$

Through a reformulation of the right-hand side of Eq. (8) based on a big-M relaxation, a linear expression is used to account for the selection of diameters (Glover, 1975). In Eq. (9), the big-M parameter ($dM^{5.334}\ \Delta sp_{i,j}^{Max}$) can be calculated from the maximum diameter in the set D and the maximum admissible difference between the square pressures at the extremes of the segment *i-j*. Both values are usually known before solving the optimization problem.

$$F_{i,j,t}{}^{2} \leq \gamma^{-1}\, l_{i,j}{}^{-1}\, [\delta_d^{5.334}\, (P_{i,t}^{sq} - P_{j,t}^{sq}) + dM^{5.334}\, \Delta sp_{i,j}^{Max} (1 - \sum_{\substack{\tau \in TI \\ \tau \leq t - lt}} x_{d,i,j,\tau})] \qquad \forall (i,j) \in A: l_{i,j} > 0, d \in D, t \in T \tag{9}$$

4.3 Objective function

In one of its simplest forms, the network design problem for gathering shale production seeks to minimize the net present cost of investments comprising pipeline and facility costs, as stated by Eq. (10). Such capital expenditures are brought to the present time by means of the discount rate *r*. Solving the tradeoff between these two main terms implies to optimally decide among distributed or centralized network designs. The first alternative minimizes pipeline costs by reducing pipeline lengths and diameters, while the second prioritizes facility costs, taking advantage of economies of scale associated with larger sizes.

$$\text{Min NPC} = \sum_t (1+r)^{-t} \left[\sum_{j,q} cf_{q,j}\ \ y_{q,j,t} + \sum_{(i,j)\in A} \sum_d cp_{d,i,j}\ \ x_{d,i,j,t} \right] \tag{10}$$

In summary, the MINLP formulation seeks to minimize Eq. (10) subject to topological and fluid dynamic constraints. For more details on specific equations for liquid, gas and multi-phase flows, under different network topologies, we refer the reader to Cafaro et al. (2022). Regardless of the complexity of the nonlinear correlations used to manage pressure drops along the pipeline segments, it is quite clear at this point that avoiding calculations of that kind in certain arcs of the network could be helpful for the optimization model. We will discuss this strategy with more detail in the following sections.

5. Selective Tightening Algorithm for global optimization

In the MINLP formulation described in previous sections the number of nonlinear constraints is significantly large because they are imposed for every potential arc in the superstructure of alternatives. From the latter, the global optimization of gathering networks of real size is a major challenge. By relaxing all or a set of nonlinear constraints, we obtain MILP or MINLP models of reduced size that, although combinatorically complex, can be solved in reasonable computational times, even for large instances of the problem. Taking advantage of this fact, we aim to develop an algorithm that successively adds on the fly linear and nonlinear constraints on the current relaxed model in order to converge to the global optimal solution.

5.1 Building blocks

Valid relaxations of the MINLP

A valid lower bound to the global optimal solution can be obtained by solving an MILP approximation under the following conditions:

1. Fluid dynamic constraints (for pipeline sizing, based on pressure/flow correlations) are relaxed. This implies that for every arc *i-j* used to build the network, the cheapest (smallest) diameter will be naturally selected by the solution procedure.
2. The maximum admissible flow for any arc *i-j* is limited to the capacity of a pipeline with the maximum diameter, under the maximum admissible difference of pressures between the nodes *i* and *j*, assuming that the arc is selected to build the network.

Note that the optimal solution from the MILP approximation yields a network configuration (set of interconnections and facility investment planning) without details on the pipeline diameters that are required. Also note that if conditions 1, 2 and 3 apply on a subset of arcs *i-j* in the superstructure, the resulting model will also be a valid relaxation of the full-scale MINLP (imposing fluid-dynamic constraints on all arcs), although the reduced model will still be an MINLP. Finally, linear underestimations of the pressure drop in the fluid-dynamics equations also yield valid relaxations.

Finding good feasible solutions

One of the simplest strategies to find a feasible solution to the full-scale problem is by solving a simplified MILP model that assumes direct link between sources and sinks. This is equivalent to assuming a network with a single echelon, with no chance to merge flows coming from different wells at intermediate nodes. Inlet and outlet pressures are fixed at their maximum and minimum admissible values, respectively, thus making it possible to preselect the minimum diameter that can handle production flows from every individual node over the time horizon. The resulting network usually has a higher-than-needed cost in pipelines because of the following reasons:

a) Pipelines cannot be shared by nearby wells. This is particularly inefficient when production profiles have almost no interference since development times are set apart from each other.
b) The solution takes no advantage of economies of scale of pipelines.

As a result, the MILP model with one echelon favors decentralized configurations. From that, such initial allocation of wellpads to facilities can be taken as a reference to limit the number of nodes for facility location. Note that by progressively increasing the number of echelons, and thus the size of the network superstructure, pipeline costs are reduced and lower-cost solutions can be found but the model complexity increases significantly. Good feasible solutions can be also found in reasonable times by solving smaller MINLPs over a reduced set of arcs in the network superstructure. In fact, if good configurations (network topologies) are derived by MILP approximations, they may be used to select specific arcs, size the corresponding pipelines and finally build the network.

A third strategy to find good feasible solutions is to prefix pressures at intermediate nodes, thus leading to simpler (linear) pipeline sizing equations (Montagna et al., 2021). Finally, linear approximations (not necessarily underestimations) of the pressure drops can also be useful to find efficient configurations. However, solving a reduced MINLP model will be also required to finally adopt proper pipeline diameters for every segment. Further details on all these strategies are given in the following sections.

5.2 Selective Tightening Algorithm (STA)

A global optimization algorithm can be drawn by successively solving valid relaxations of the full-scale MINLP (see Section 5.1). The so-called Selective Tightening Algorithm (STA) seeks to progressively add constraints that are potentially binding in the optimal solution. In this way, the number of nonlinear constraints is considerably smaller than the ones in the monolithic model, and the solution of the full-scale problem can be avoided. In the most basic form of the algorithm, successive lower bounds are obtained for the problem until both feasibility and optimality are simultaneously reached. The lower bound approach of the STA is summarized in Figure 3 and can be stated as follows:

A. **Initialization**. Iteration counter *k* is set to 0 and the subset of arcs $SC_k \subseteq A$ on which fluid dynamic constraints are imposed is empty. For a problem with a given number of echelons, specify *E* as this number (see Supplementary Material).

B. **Main step**. **Solving successive relaxations.**

B1. A relaxation $\mathcal{R}_k$ of the full-scale problem is solved by only imposing fluid dynamic constraints over the arcs included in SC_k. The optimal pipeline network (triplets *i-j-d*) yielded by this relaxation is stored in the solution $\mathcal{R}_k^*$.

B2. Fix the network topology and pipeline sizes (triplets *i-j-d* in $\mathcal{R}_k^*$) and impose fluid dynamic constraints over all the selected arcs. If $\mathcal{R}_k^*$ is feasible, then the network proves to be optimal because unconstrained links (not in SC_k) can manage flows with the minimum diameter. If that is the case, the procedure ends; otherwise, it continues as in **B3**.

B3. Add arcs in $\mathcal{R}_k^*$ into SC_{k+1} and increase iteration counter *k* by 1 (see Figure 2). The main step is repeated until convergence.

Notice that the first iteration yields an MILP formulation since this set SC_k of constrained arcs is initially empty. It is also important to note that step **B1** yields a valid lower bound of the original problem at every iteration, and for all the selected segments that are not in SC_k the solution procedure will automatically assign the smallest possible diameter, to reduce cost.

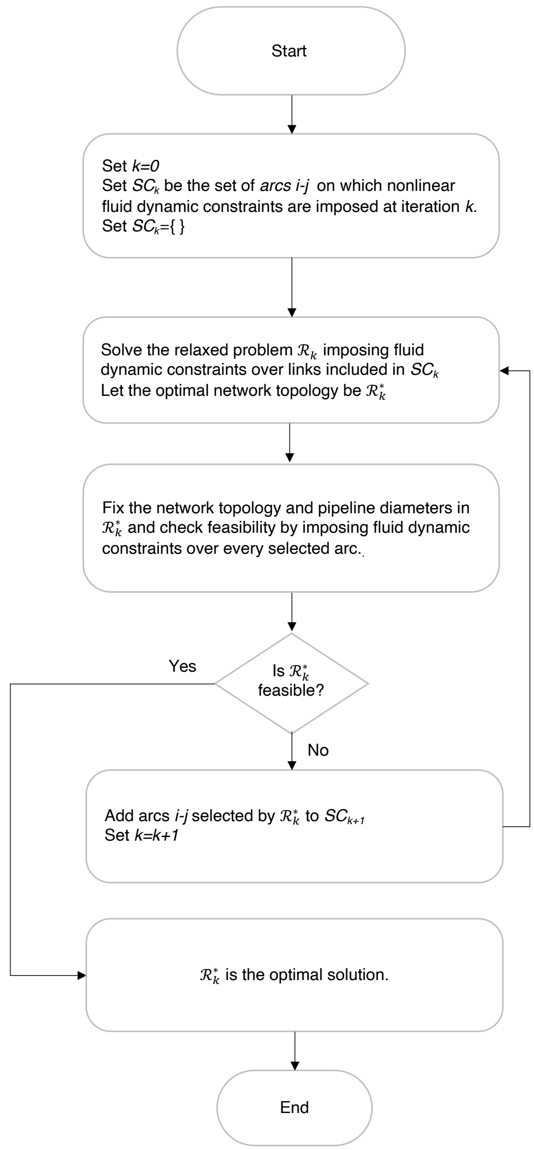

Figure 3 – Basic form of the Selective Tightening Algorithm (STA)

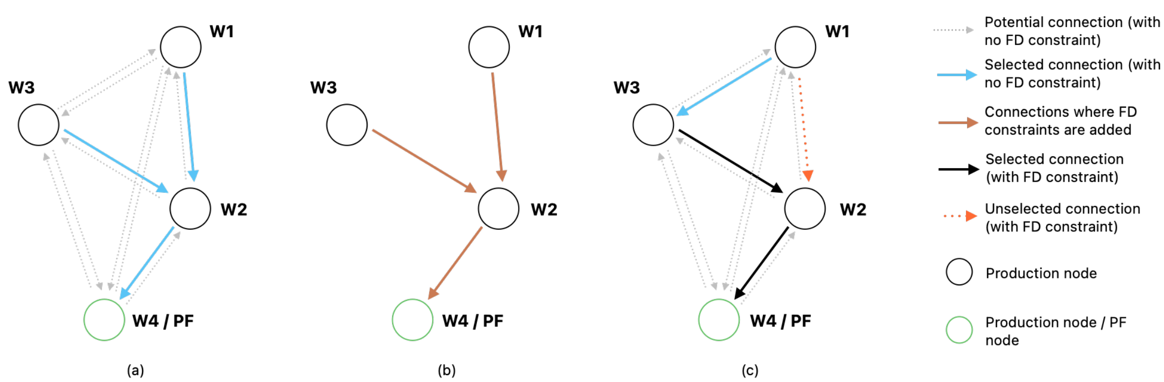

Figure 4 – (a) Initial solution with no arc in the set SC_k. (b) Links added to SC_{k+1}. (c) Solution from the second relaxation still using an unconstrained link (in blue). FD stands for fluid dynamic.

5.3 Generalized Solution Approach based on Selective Tightening

When a specified optimality gap is sufficient from the optimization perspective, relaxation strategies (lower bounds) and heuristics to find good feasible solutions (upper bounds) can be combined for faster convergence. Strategies presented in Section 5.1 can be added to the procedure with this aim. By comparing upper and lower bounds for the problem, the algorithm can be interrupted prematurely after reaching a gap termination criterion. This strategy leads to a generalized approach that can be summarized as follows:

A. Initialization. The algorithm is initialized in the same way as in Section 5.2, with no arcs in the subset SC_k. The initial upper bound z^{BF} is set to infinity.

B. Main step: Solving successive relaxations and finding feasible solutions

B1. A relaxation $\mathcal{R}_k$ of the full-scale problem is solved by only imposing fluid dynamic constraints over the arcs included in SC_k. The optimal solution of the relaxation and the value of the objective function are stored in $\mathcal{R}_k^*$ and z_k^{LB}, respectively.

B2. If solution $\mathcal{R}_k^*$ is feasible then it is also optimal and the procedure ends. Else, the current solution provides a new lower bound for the problem ($z^{LB} = z_k^{LB}$). In addition, a feasible solution can be obtained from $\mathcal{R}_k^*$ by fixing the network topology and optimizing pipeline diameters in a subproblem $\mathcal{F}_k$. Let the objective value of this solution be $\mathcal{F}_k^* = z_k^{UB}$.

B3. The feasible solution derived from $\mathcal{R}_k^*$ (step **B2**) can be stored as a new "best-found" solution if $z_k^{UB} < z^{BF}$.

B4. By comparing the current bounds, the optimality gap can be calculated for the best available feasible solution. If the gap satisfies the tolerance ε, the current feasible solution is reported as the optimal solution. Otherwise, fluid-dynamic constraints are enforced for unconstrained links still present in $\mathcal{R}_k^*$ and added to SC_{k+1}, before updating k. The main step is repeated until convergence.

Note that a single-echelon feasible solution strategy as described in Section 5.1 can be used to set the initial upper bound z^{BF} to a finite value, usually in short CPU times. Furthermore, other heuristic strategies can be run in parallel with the successive relaxations (see parallel task block in Figure 5), which can be used to feed the step **B3** of the algorithm with improved feasible solutions (upper bounds). Only some of them have been presented in Section 5.1. For completeness, some others are described in further sections.

Parallel tasks

In parallel with the main procedure described in previous sections, reduced optimization problems can be solved to feed new feasible solutions into the main program and thus speeding up convergence. This section describes two heuristic strategies that may yield good quality feasible solutions.

Two-step solution approach: Allocating wells' production to processing facilities configures a class of multiperiod bin packing problem that is complemented with the pipeline network design. Pre-allocating wellpads to facilities in a master allocation problem may drastically reduce the computational time of multi-echelon instances. This strategy comprises two subsequent optimization problems. First, a single echelon network configuration with precalculated pipeline diameters is solved. Based on the suggested allocation, a second step model adds strong topological cuts into the multi-echelon model. Equations (11) to (14) illustrate topological constraints for a 2-echelon formulation. We introduce routing variables $w_{i,i',j} \in \{0;1\}$ to indicate that nodes *i* and *j* are connected through an intermediate node *i'*. Eqs. (11), (12) and (13) model the equivalence $w_{i,i',j} = 1 \Leftrightarrow \sum_{d\in D} x_{d,i,i',t=st} = 1 \wedge \sum_{d\in D} \sum_{t'\leq st(i)} x_{d,i',j,t'} = 1$, determining the route *i-i´-j* according to the upstream and downstream segments *i-i'* and *i'-j*, respectively. Note that the upstream segment *i-i'* is assumed to be built at the production start time of node *i* (*t = st(i)*).

$$w_{i,i',j} \geq \sum_{d\in D} x_{d,i,i',t=st(i)} + \sum_{d\in D} \sum_{t'\leq st(i)} x_{d,i',j,t'} - 1 \qquad \forall (i,i') \in A, (i',j) \in A \tag{11}$$

$$w_{i,i',j} \leq \sum_{d\in D} \sum_{t'\leq st(i)} x_{d,i',j,t'} \qquad \forall (i,i') \in A, (i',j) \in A \tag{12}$$

$$w_{i,i',j} \leq \sum_{d\in D} x_{d,i,i',t=st(i)} \qquad \forall (i,i') \in A, (i',j) \in A \tag{13}$$

In Eq. (14), $\theta_{i,j}$ is a parameter taking value 1 when production node *i* should be allocated to a processing facility *j* (a result from the first step). If $\theta_{i,j}$ = 1, one and only one routing variable $y_{i,i',j}$ (i.e. using a single intermediate node *i'*) must be activated.

$$\sum_{i':(i,i')\in A \wedge (i',j)\in A} w_{i,i',j} = \theta_{i,j} \qquad \forall (i,j) \in I : i \neq j \tag{14}$$

Note that the second step model is still an MINLP formulation including fluid dynamic constraints on every arc. It is solved to obtain a feasible solution, and later used as an upper bound for the solution algorithm. This strategy can be iteratively applied for an increasing number of echelons by generalizing constraints (11) to (14), adding intermediate indices *i'', i'''*, etc., into variables *w*. Once again, it is important to highlight that in the first step, when the number of echelons is reduced, pipeline network costs are overweighted and decentralized solutions may be erroneously favored. To mitigate this effect, pipeline cost terms in the objective function of the first model can be multiplied by a factor *f* < 1 such that the total costs in both steps are comparable.

Allocation-Within-Allocation (AWA) procedure: When the number of echelons in the second step of the two-step approach is large, the number of integer cuts required for fixing the well-facility allocation grows quickly. The AWA is a heuristic algorithm that deals with a larger number of echelons by generating clusters of wellpads whose production flows are gathered into a single node. The collecting node of a cluster is arbitrary located, typically closer to the most productive wells in the group. In turn, these junction nodes can be linked to junction nodes of other clusters, yielding a multi-echelon configuration with reduced degrees of freedom. At each iteration, different wellpad clusters are proposed based on vicinity (following geographical or temporal criteria), and a reduced set of junction nodes is made available at every cluster. Then, using the two-step approach described in the previous section, the AWA algorithm: (a) allocates clusters to processing facilities, (b) selects a junction node for production flows coming from the wellpads in the same cluster, and (c) builds a pipeline network based on decisions (a) and (b). The

AWA is solved in parallel to produce feasible solutions that are fed into the generalized solution procedure depicted on Figure 5.

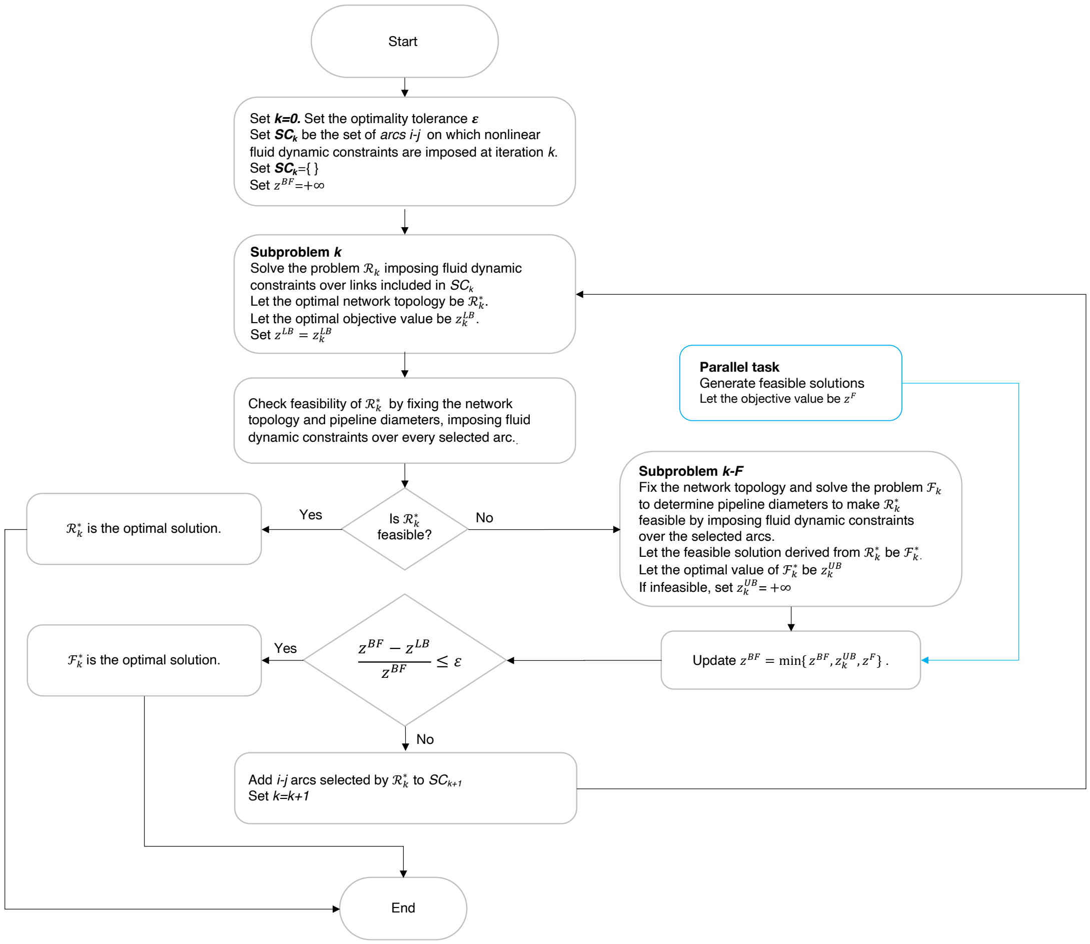

Figure 5 – Generalized Solution Approach based on Selective Tightening Algorithm (STA)

Fixed pressures at intermediate nodes: A simplified model where flow pressures are fixed at intermediate nodes, in addition to the production and delivery points, is another strategy to obtain good feasible solutions. This feature reduces the complexity in pipeline sizing equations at the expense of trimming the feasible region. Since the pressure drops are imposed beforehand, some suitable gathering configurations may be omitted. Although the models maintain a very high combinatorial complexity, nonlinear constraints are avoided and good quality solutions can be obtained in relatively short CPU times. For further details see Montagna et al. (2021) and Cafaro et al. (2022).

6. Results

This section presents two case studies that demonstrate the capabilities of the Selective Tightening Algorithm against monolithic approaches to the pipeline network design problem. We focus on natural gas gathering networks that rely on fluid-dynamic correlations of quadratic, convex form. For the sake of brevity, more complex fluid-dynamic equations like those modeling multiphase flows will be addressed in future work. The optimal design of the gas pipeline network is sought to gather the production from shale wells and to deliver raw gas to the drying and purification plants at a minimum pressure. In addition to the modeling assumptions presented in Section 3, it is assumed that the primary separation of gas from liquids (hydrocarbons and water) is performed at the source nodes, and therefore pipelines transport a single gas phase. Fluid dynamic correlations suggested by Weymouth (1912) are used to restrict and manage pressure drops along the pipeline segments (see Section 4.3). Like in previous works (Montagna et al., 2021), splitting flows is not allowed at any node in the network and facilities/pipeline of different sizes cannot be installed in the same node/arc at a single time period. The complete MINLP model used in both cases is given in the Supplementary Material. Several STA-based solution strategies are implemented and compared against monolithic instances to draw conclusions on their effectiveness.

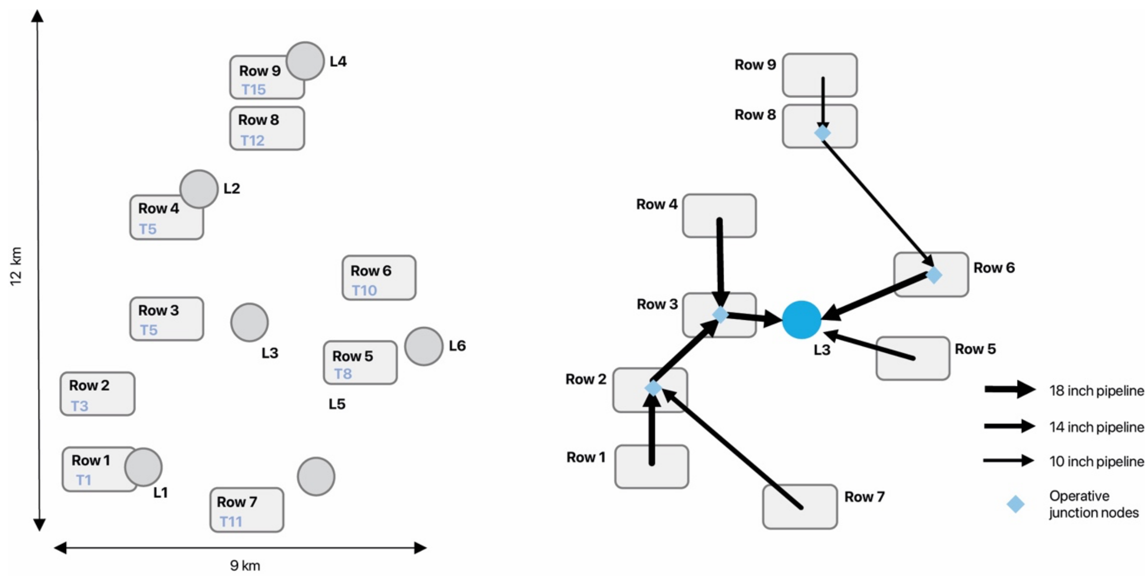

Figure 6 – Left: Spatial arrangement of rows and potential locations for facilities in Case 1. Right: Optimal solution for Case 1

6.1 Case 1: Illustrative network design problem

The first case study comprises 9 rows of shale gas wells arranged over a geographical region with an area of 108 km^2 as shown in Figure 6. We consider 15 planning periods of one year for each of them, gas production rates and wellhead pressures at every row are given. Junction nodes can be placed at any row in the network. Regarding the installation of processing plants, 6 locations are proposed where successive capacity expansions can be carried out over time. Expansions can be made at any point in the time horizon, reducing present cash flows both by deferring investments (11% annual rate) and/or by taking advantage of economies of scale with larger expansions. As for the pipeline network design, 3 alternative diameters (10, 14 and 18 inches) can be selected to meet pressure drop constraints. The network configuration is set to a maximum of 3 echelons, i.e., the maximum number of pipeline segments between a row and a processing facility is 3. Distances between nodes, pipeline installation costs, facility expansion costs and reference pressures are shown in the Supplementary Material.

The transportation capacity and utilization of reference pipeline segments that converge and emerge to/from the junction node in Row 3 (R3) are shown in Figure 7. The times of maximum pipeline utilization are between periods 8 and 12, when Rows 1 and 4 reach their maximum production rates, respectively. Figure 8 shows the pressure profiles in these segments, revealing that when the Rows 1 and 2 together deliver their maximum flowrate on the path R1-R2-R3, the outlet pressure of Row 4 must be decreased to enter the network. The effect is reversed when production coming from Row 4 produce a large pressure drop on the segment R4-R3 during time period 12 and pressure at rows R1 and R2 (producing low flow rates) need to be reduced. In this way, pressures are managed to reach the processing plant under specification. Besides, Figure 9 illustrates the optimal expansion planning for the selected processing plant.

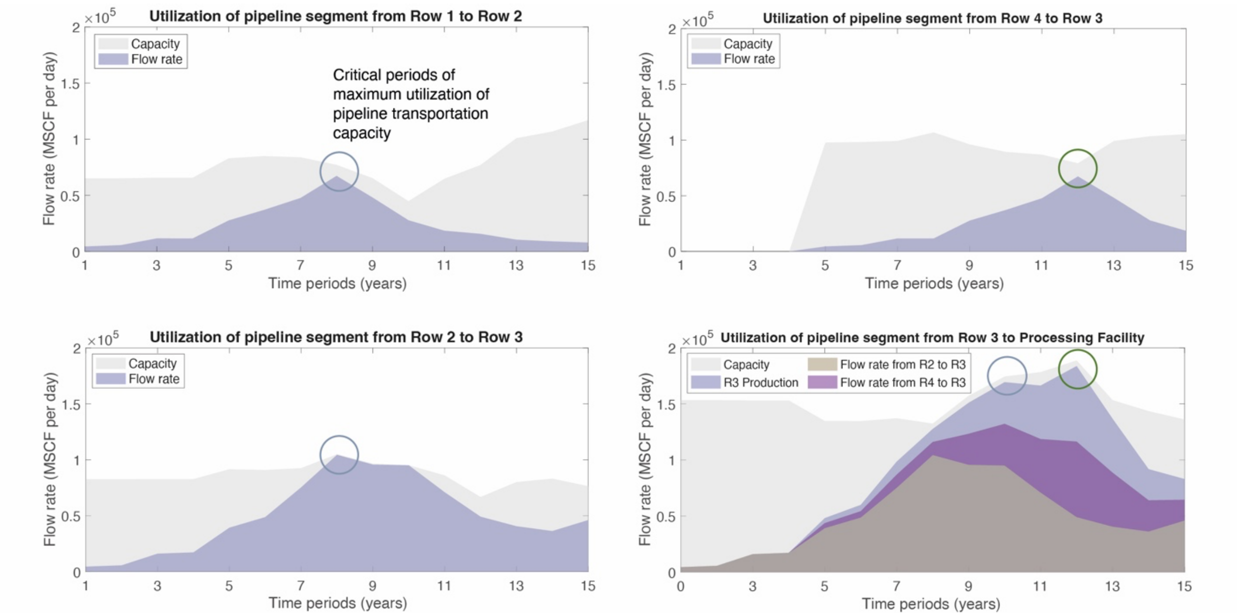

Figure 7 – Utilization of pipeline segments that converge/emerge to/from the junction node in Row 3.

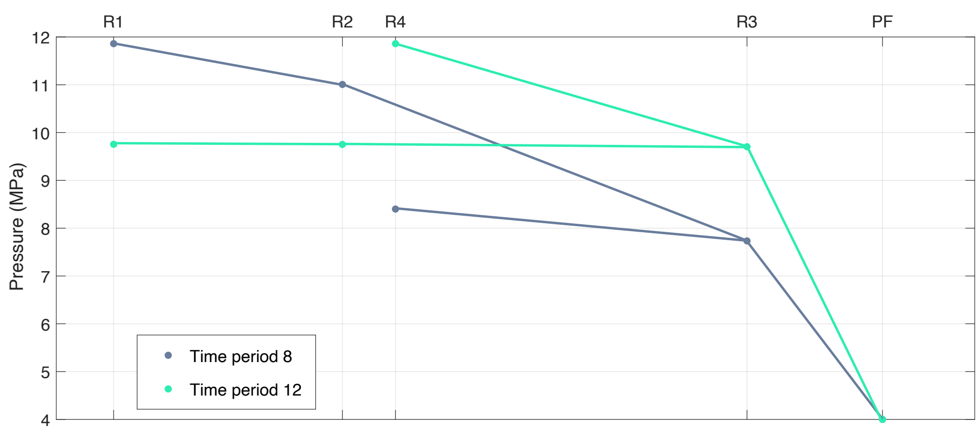

Figure 8 – Pressure profile for pipeline segments that converge/emerge to/from the junction node in Row 3.

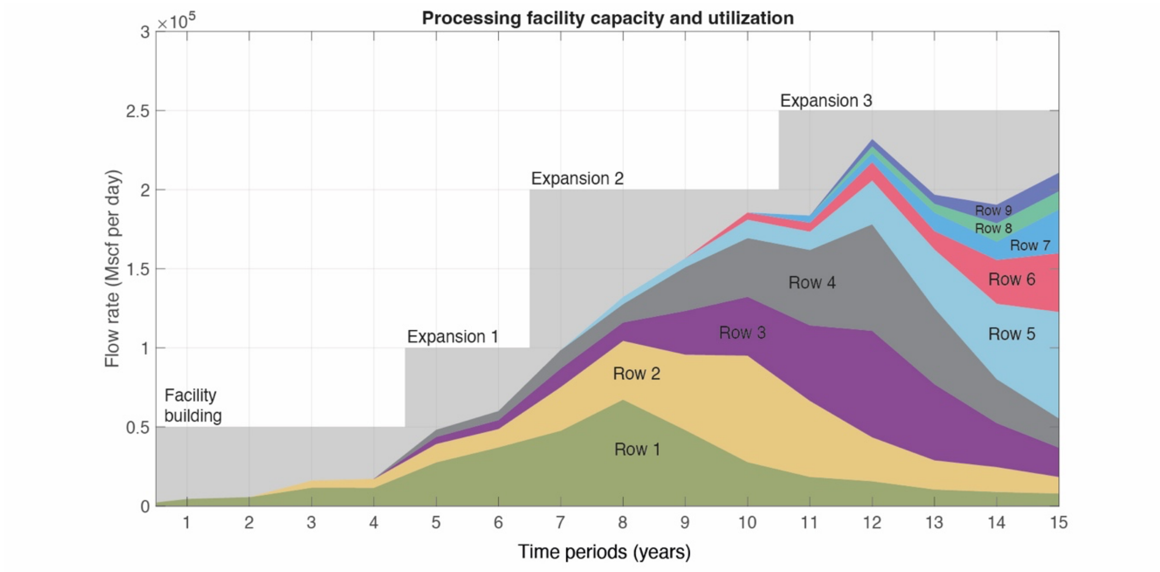

Figure 9 – Processing facility expansion planning in the optimal solution of Case 1

The monolithic MIQCP optimization model for Case Study 1 involves 14,888 linear constraints, 12,015 quadratic constraints, 13,861 continuous variables and 3,651 binary variables. When approaching the problem in one step, 4,972 seconds of CPU time are required to reach an optimality gap of 10^{-16} using GUROBI 10.0.1 using 16 threads on an Intel Core i9 CPU at 2.9 GHz with 16 GB RAM. The optimal solution is shown at the right of Figure 6 and has an objective value of 52.59 MUSD. The problem is then tackled by means of two variants of the STA algorithm. First, the most basic form of STA is used for the design of surface facilities by successively improving the lower bounds. The optimal solution is obtained after 5 iterations and 261 seconds of computation as detailed in Table 1. The last iteration of the STA comprises 3,195 quadratic constraints, with a reduction of 73.4% in the number of constraints compared to the monolithic formulation. Note that the optimal solution of every step is automatically used by the MIQCP solver as a warm start for the next iteration. Furthermore, the generalized STA solution framework can be used to derive feasible solutions from each relaxation, and to compute the optimality gap. Results from this procedure are also shown in Table 1. Figure 10 illustrates convergence steps of the methods.

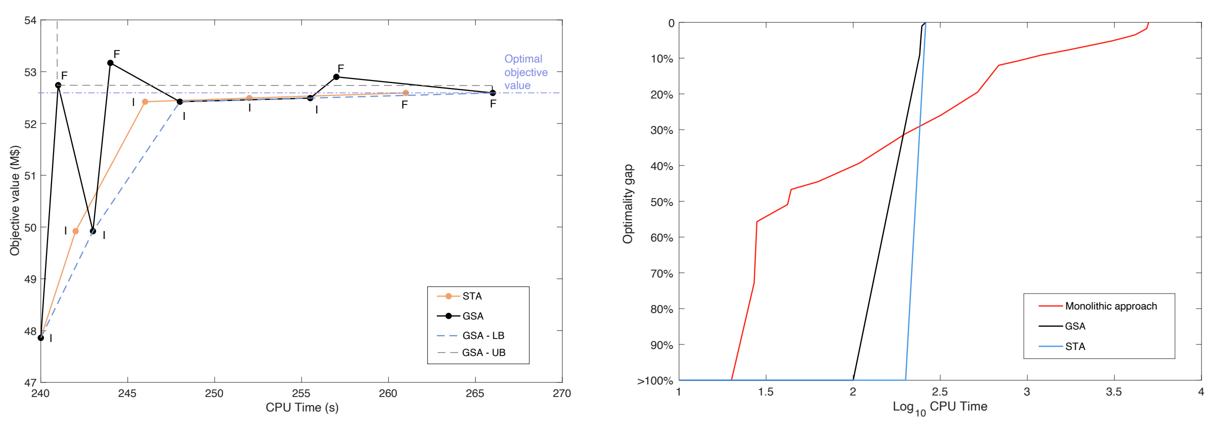

Figure 10 – Convergence to optimal solution (left) and optimality gap evolution (right) in the solution approaches used to tackle the problem

Table 1 – Summary of solution strategies for Case 1.

Approach	Iteration	Model Type	CPU Time (s)	Number of quadratic constraints	*Objective value (M$)*	*Optimality gap (%)*
Monolithic solution	-	*MIQCP*	*4,972.0*	*12,015*	*52.59 (F)†*	*0.00*
Selective Tightening Algorithm	*1*	*MILP*	*240.0*	*0*	*47.86 (I)*	-
	2	*MIQCP*	*2.0*	*1,755*	*49.92 (I)*	-
	3		*4.0*	*2,655*	*52.42 (I)*	-
	4		*6.0*	*2,925*	*52.49 (I)*	-
	5		*9.0*	*3,195*	*52.59 (F)†*	*0.00*
	Total CPU time		*261.0*			
Generalized Solution Approach	*1*	*MILP*	*240.0*	*0*	*47.86 (I)*	-
	1-F	*MIQCP*	*1.0*	*1,755*	*52.74 (F)*	*9.00*
	2		*2.0*	*1,755*	*49.92 (I)*	*5.30*
	2-F		*1.0*	*2,655*	*53.17 (F)*	*5.30*
	3		*4.0*	*2,655*	*52.42 (I)*	*0.60*
	3-F		*1.5*	*2,925*	*- (I)*	*0.60*
	4		*6.0*	*2,925*	*52.49 (I)*	*0.47*
	4-F		*1.4*	*3,195*	*52.90 (F)*	*0.47*
	5		*9.0*	*3,195*	*52.59 (F)†*	*0.00*
	Total CPU time		*265.9*			

(F): Feasible solution in the original space. (I): Infeasible solution in the original space.
† Optimal solution

6.2 Case 2: Real world network design problem

This section explores the capabilities of the STA to solve real world instances of the surface facilities network design problem. Figure 11 shows the spatial arrangement of wellpads and potential locations for processing facilities involved in this case study, which is a modified version of a real-world case study presented by Montagna et al. (2021). Figure 11 also shows the time periods T when start of production of each well takes place. The region comprises 40 individual wellpads, 20 potential junction nodes and 9 locations for processing facilities over an area of 3,600 km^2. A maximum of 3 echelons is also considered to build the network, together with 3 alternative pipeline diameters of 10, 16 and 22 inches. After construction, the processing plants can be expanded in their processing capacities by 64×10^3, 128×10^3 and 256×10^3 Mscfd (10^6 standard cubic feet per day) at any time period. Economies of scale in pipelines and processing facility costs are important. Starting times of wellpads production and their corresponding decline curves are given for a planning horizon comprising 120 monthly periods. As in the previous case, the network is intended for shale gas gathering, transportation and processing, thus the Weymouth (1912) correlation is used to calculate pressure drops at every segment.

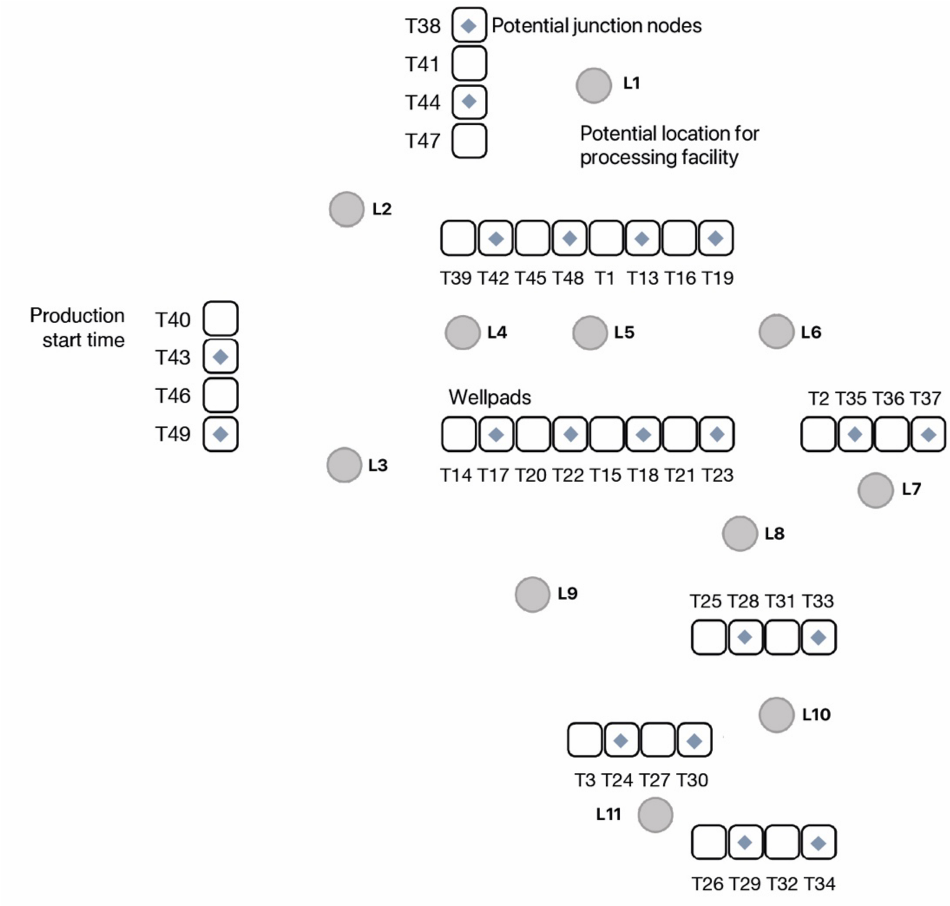

Figure 11 - Spatial arrangement of wellpads and potential locations for facility placement in Case Study 2

In its monolithic form, the convex MIQCP formulation involves more than 16,300 binary variables and 95,000 quadratic constraints. Attempting its solution in a single step, GUROBI 10.0.1 solver yields an optimality gap of 2.6% after 20,000 seconds of computation, using 16 threads on an Intel Core i9 CPU at 2.9 GHz with 16 GB RAM. In this context, four STA-based algorithms are proposed for tackling the problem:

(1) Basic STA, generating increasingly tighter lower bounds by successively adding hydraulic constraints on arcs until convergence to the global optimum.

(2) Linear STA, adding an ad-hoc linear outer-approximation of the quadratic constraint at every arc suggested by relaxations. After convergence, a reduced MIQCP is solved to properly size the selected pipeline segments based on quadratic correlations.

(3) Linear-then-Quadratic STA. When a link is selected for the first time in the optimal solution of a relaxation, an ad-hoc linear outer-approximation of the quadratic constraints for fluid dynamics is imposed. Later, if another relaxation suggests the same connection, the quadratic constraint is finally imposed for that arc.

(4) Generalized STA, using all capabilities described on Figure 5.

Table 2 – Summary of solution strategies for Case 2.

Approach	Iteration	Model Type	CPU Time (s)	Cumulative CPU Time (s)	Number of quadratic constraints	*Objective value (M$)*	*Optimality gap (%)*
Monolithic solution	-	*MIQCP*	*20,000*	*20,000*	*95,640*	*356.23 (F)*	*2.13**
Selective Tightening Algorithm	*1*	*MILP*	*126*	*126*	*0*	*305.16 (I)*	-
	2	*MIQCP*	*145*	*271*	*12,960*	*328.26 (I)*	-
	3		*372*	*643*	*18,960*	*346.51 (I)*	-
	4		*1,322*	*1,965*	*24,360*	*353.04 (I)*	-
	5		*1,413*	*3,378*	*25,440*	*355.76 (I)*	-
	6		*1,439*	*4,817*	*27,000*	*356.12 (F)†*	*0.00*
	Total CPU Time (s)		*4,817*				
Linear STA	*1*	*MILP*	*126*	*126*	-	*305.16 (I)*	-
	2		*74*	*200*	-	*320.25 (I)*	-
	3		*75*	*275*	-	*333.91 (I)*	-
	4		*117*	*392*	-	*334.77 (I)*	-
	5		*124*	*516*	-	*334.77 (I)*	-
	6	*MIQCP*	*5*	*521*	-	*364.10 (F)*	*2.21***
	Total CPU Time (s)		*521*				
Linear then quadratic STA	*1*	*MILP*	*126*	*126*	-	*305.16 (I)*	-
	2		*74*	*200*	-	*320.25 (I)*	-
	3		*75*	*275*	-	*333.91 (I)*	-
	4	*MIQCP*	*272*	*547*	*4,800*	*342.53 (I)*	-
	5		*493*	*1,040*	*11,040*	*344.98 (I)*	-
	6		*576*	*1,616*	*12,000*	*351.01 (I)*	-
	7		*1,109*	*2,725*	*16,080*	*354.67 (I)*	-
	8		*1,195*	*3,920*	*17,960*	*356.12 (F)†*	*0.00*
	Total CPU Time (s)		*3,920*				
Generalized solution strategy	*1*	*MILP*	*126*	*126*	-	*305.16 (I)*	-
	1-F	*MIQCP*	*8*	*134*	*1,655*	*360.82 (F)*	*15.27*
	2		*145*	*279*	*12,960*	*328.26 (I)*	*9.02*
	2-F		*12*	*291*	*1,695*	*366.40 (F)*	*9.02*
	3		*372*	*663*	*18,960*	*346.51 (I)*	*3.97*
	3-F		*10*	*673*	*1,508*	*379.31 (F)*	*3.97*
	4		*1,322*	*1,995*	*24,360*	*353.04 (I)*	*2.16*
	4-F		*12*	*2,007*	*1,734*	*373.86 (F)*	*2.16*
	5		*1,413*	*3,420*	*25,440*	*355.76 (I)*	*1.40*
	5-F		*5*	*3,425*	*1,443*	*359.51 (F)*	*1.04*
	6		*1,439*	*4,864*	*27,000*	*356.12 (F)†*	*0.00*
	Total CPU Time (s)		*4,864*				

(F): Feasible solution in the original space. (I): Infeasible solution in the original space.
** Relative to the best bound provided by the solver*
*** Relative to global optimal solution*
† Optimal solution

The basic version of the STA (focusing on lower bounds) yields the optimal solution for the problem with a total cost of 356.12 M$, which is found after 6 iterations and 4,817 CPU s (see Table 2). Figure 12 shows the three optimal locations for the processing facilities, as well as the sizes of the planned expansions and their timing. The utilization of the processing plants based on the production streams coming from the allocated wellpads is presented in Figure 13. The solution is obtained by including only 28% (27,000) of the quadratic constraints comprised in the original problem, in the last iteration.

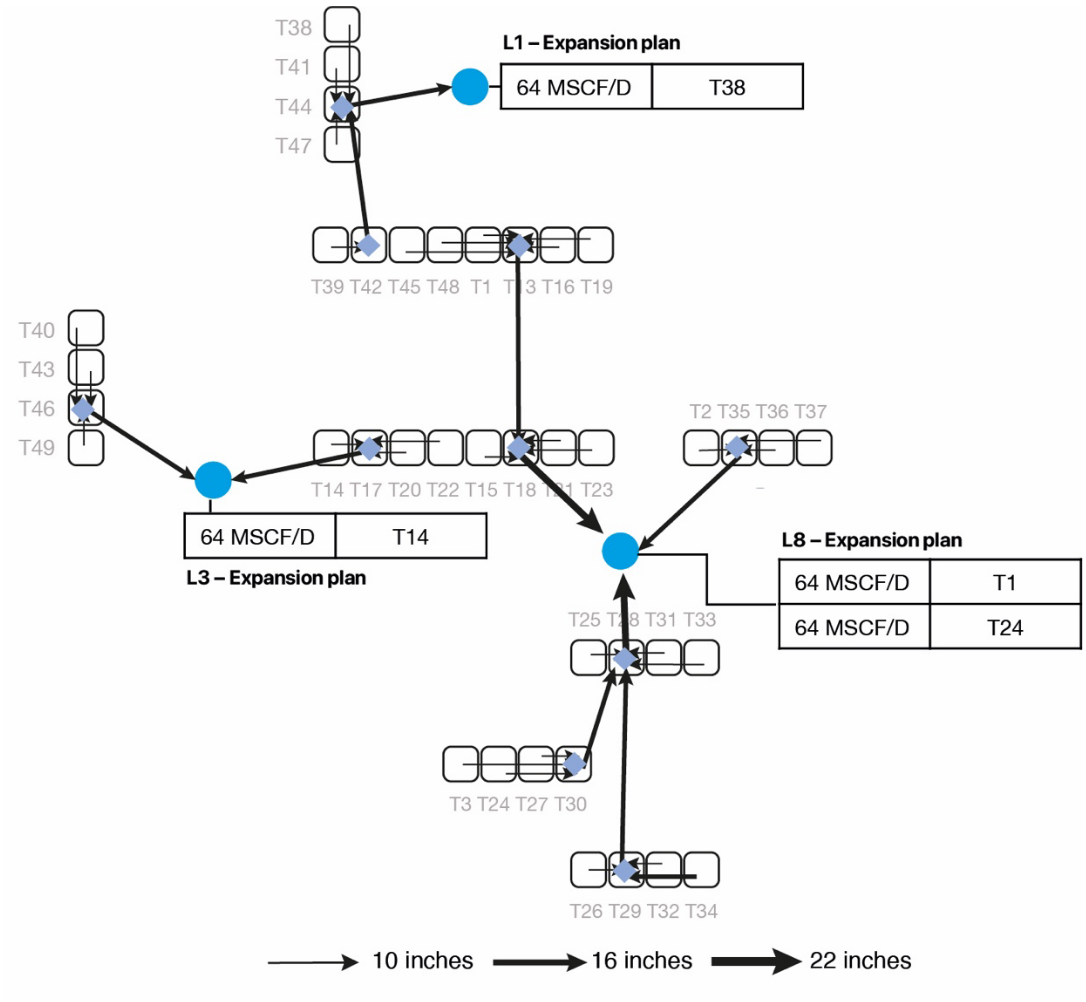

Figure 12 – Optimal network design for Case 2

Following the Linear STA strategy, the solution of successive MILP formulations with increasing numbers of linear constraints converges to an approximate total cost of 334.77 M$ (lower bound). Upon convergence, a reduced MIQCP model is solved to properly size the pipelines by fixing the network topology and imposing accurate pressure drop constraints over every segment. This approach leads to a feasible solution of 364.10 M$ (2.2% gap relative to the actual global optimal solution) after a total time of 521 CPU s and 6 iterations before completion. Note that the Linear STA approach yields a better solution than the best feasible solution found by the monolithic approach, in less than 3% of the total wall time. Third, the Linear-then-Quadratic STA is applied, aiming at imposing linear underestimations of the actual quadratic constraints to build tighter linear relaxations before accounting for non-linear, accurate fluid dynamic correlations. This approach leads to the global optimal solution, since iterations are executed over successive relaxations until convergence to feasibility in the original space. However, linear constraints may be enough to exclude network configurations that are clearly unfavorable under fluid dynamic conditions. This strategy yields the global optimum depicted in Figure 12 after 3,920 seconds of computation, including less than 18,000 quadratic constraints and 26,000 linear fluid dynamic underestimations, after 8 iterations. Figure 14 presents a comparison between the proposed approaches.

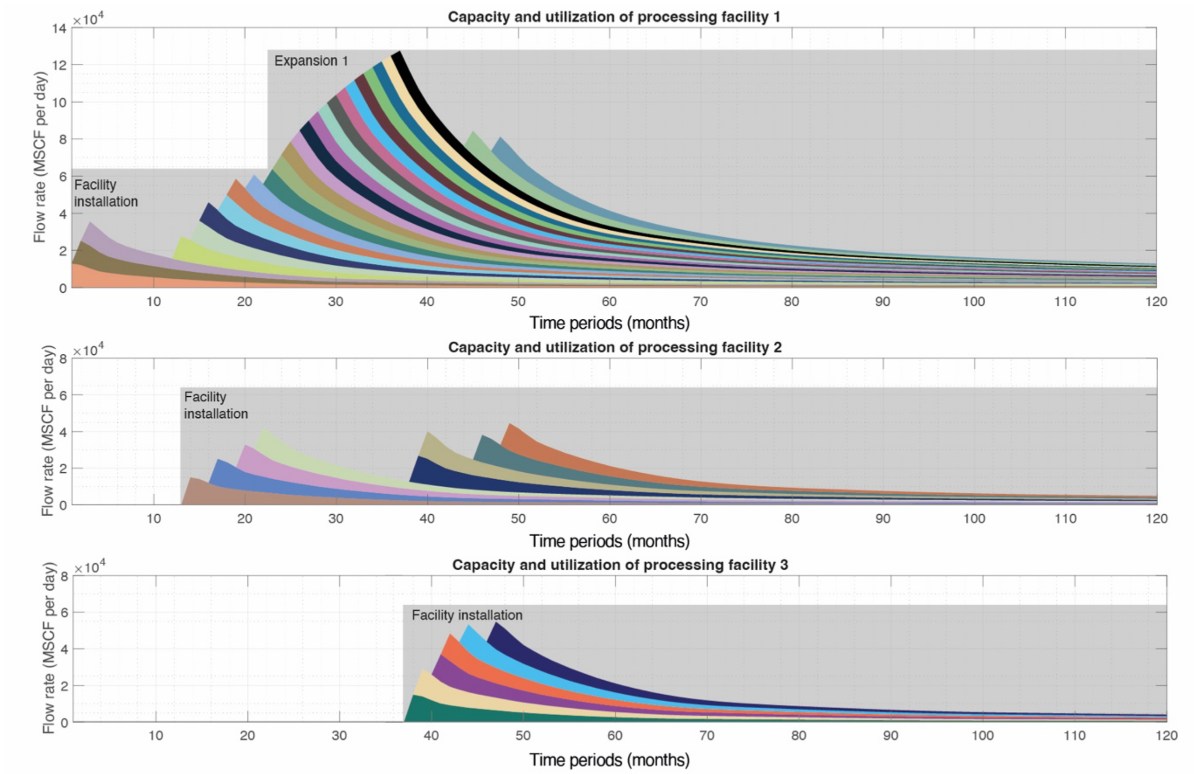

Figure 13 – Processing facility expansion strategy in the optimal solution of Case 2

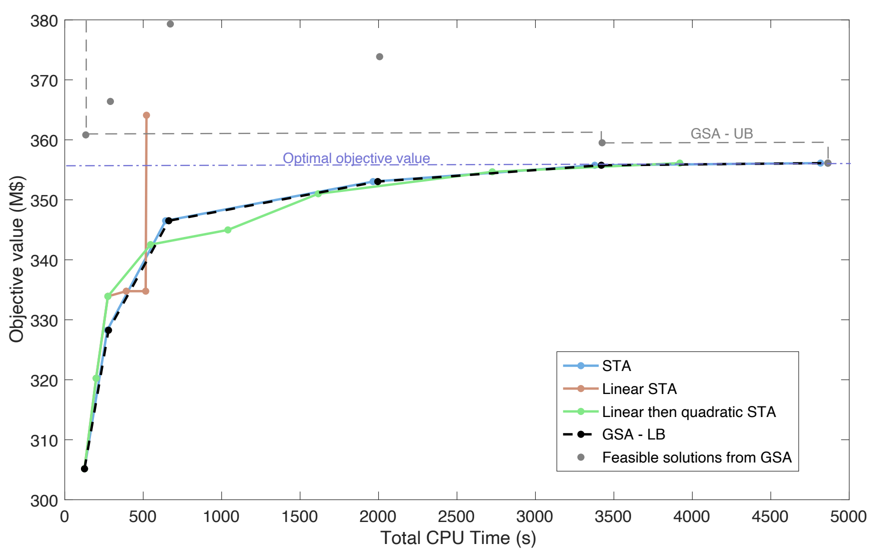

Figure 14 – Comparison between different approaches for Case Study 2

Finally, the Generalized STA depicted on Figure 5 yields a feasible solution of 360.15 M$ in the first iteration (134 s), which, after 4 iterations adding quadratic constraints (3410 s), guarantees an optimality gap of merely 1.4% (see Figure 14).

Regarding the network design, the optimal solution determines the installation of processing facilities in three different locations. As shown in Figure 12, the location L8 is selected for the initial placement of a facility with a processing capacity of 64 MSCF/D (time period T1). Such capacity is then expanded by 64 MSCF/D in the time period T24. Besides, the wellpads are optimally divided by the model into three clear regions. Most of the production of the field is sent to the central facility located at L8, reaching its full capacity utilization by T38 (see Figure 13). Two additional gathering lines are installed at the west and north-west of the map to accommodate the production of the remaining wells in their closest locations (L3 and L1, respectively). The solution makes use of three echelons to convey production from the most distant points, but only two are required to gather production from the wells closest to the processing plants. Finally, the optimal topology takes advantages of the three alternative diameters, keeping the largest and more expensive ones only for final segments, with high flowrates and close to the central processing plant in L8.

7. Conclusions

This work has addressed one of the challenging problems since the early times of mathematical programming for process systems engineering (Mah, 1978), which is how to integrally solve network design, pipeline sizing and multiperiod flow allocation problems as a whole, to global optimality. In fact, even under the assumption that the pipelines carry single components like natural gas, solving the mixed-integer nonlinear formulation in a single step takes hours to reduce optimality gap to 2% in moderate-size instances. Interestingly enough, monolithic formulations for natural gas pipeline networks have a convex relaxation, but the model size can be extremely large (more than 700,000 quadratic constraints). Moreover, due to conditional terms involving big-M parameters, relaxations are loose.

In contrast to nonlinear counterparts, linear estimations converge in minutes but are usually infeasible for the original problem and require adjustments to the diameters of the pipes. Furthermore, feasible solutions guided by approximate MILPs are still far from optima. Decomposing the problem in successive, hierarchical stages is a typical industrial practice aiming at finding feasible solutions in reasonable times. However, we have shown that such approaches may have a significant impact on the total costs when compared with the global optimal solutions. For real-sized unconventional projects, cost savings can reach up to 10%, i.e., tens of millions of dollars in net present costs, which has motivated the algorithmic tools developed along this contribution.

We have devised the so-called Selective Tightening Algorithm (STA) that systematically adds fluid dynamic constraints to model relaxations until convergence to the global optimum is achieved. The aim of the algorithm is to impose pressure drop (nonlinear) correlations just on particular segments of the network superstructure. Every iteration yields a tighter lower bound for the main problem. This process is repeated until the pressure drop constraints are satisfied by all the segments comprising the network. The proposed algorithm converges in few MIQCP iterations, in reasonable CPU times. STA is able to reduce both the total CPU time and the number of quadratic constraints in the last (largest) model by roughly 75%. We have also extended the capabilities of STA by adding parallel computations that help finding very good feasible solutions (upper bounds), reducing the optimality gap to less than 1% in 10 minutes for real world instances of the problem. Such parallel tasks are based on heuristic strategies and linear estimators that are inferred from network designs that have been already explored. Additional strategies such as adding linear underestimations and/or tighter constraints in blocks, or the pre-preprocessing of the set of candidate pipeline diameters can also accelerate convergence.

In general terms, we have proved that the STA is an excellent approach for the global optimization of large-scale pipeline networks design and operation problems with convex fluid-dynamic correlations. However, finding valid relaxations, even for a reduced set of arcs, is far more complex if pressure drop estimations follow a non-convex relation with the flow rates. Extensions of the STA aimed at solving non-convex MINLPs is currently under development. For future work, addressing the risk of failing in the sizing of facilities through stochastic programming formulations seems to be an interesting new dimension in the tradeoff between economies-of-scale and financial cost. The addition of new wells that were not part of the original field development plan can also be accounted for with stochastic programming strategies. Finally, pipeline flow reversals can be relevant when dealing with uncertainty, yielding more robust solutions. In the effort to address more complex and realistic instances of the problem, results from STA are certainly promising.

Nomenclature

Sets/Indices

$A = \{(i,j)\}$	Subset of arcs in the superstructure of alternatives
$C = \{c\}$	Components in the fluid stream
$D = \{d\}$	Alternative pipeline diameters
$I, J = \{i, j\}$	Nodes in the network
$I_{source} \subseteq I$	Source/Production nodes
$J_{sinks} \subseteq J$	Sink/Delivery nodes
$K = \{k\}$	Iteration counter for algorithms
$SC_k \subseteq A$	Subset of fluid dynamic constrained arcs at iteration *k*
$T = \{t\}$	Time periods
$TI \subseteq T$	Time periods when investments are allowed

Parameters

$cf_{q,j}$	Cost of a facility expansion of size *q* in node *j*
$cp_{d,i,j}$	Cost of a pipeline of diameter *d* from node *i* to node *j*
dM	Maximum available diameter
$fk_{q,c}$	Capacity of a facility of size *q* for processing the component *c*
$l_{i,j}$	Length of the pipeline connecting *i* and *j*
lt	Lead-time for pipeline and facility installation
P_o, T_o	Pressure and temperature at standard conditions

p_j^{lo} Minimum flow pressure at processing facility in node *j*

$p_{i,t}^{up}$ Wellhead pressure at node *i* during time period *t*

$pr_{c,i,t}$ Production of component *c* from node *i* during time period *t*

r Discount rate for cashflows

sg Specific gravity of the gas

st_i Production start time of source node *i*

$tc_{d,i,j}$ Transportation capacity of a pipeline with diameter *d* connecting *i* and *j*

γ Constant of the Weymouth correlation for gas flows

δ_d Numerical value (in inches) of the internal diameter of a pipeline of diameter *d*

$\Delta sp_{i,j}^{Max}$ Maximum difference of square pressures allowed between nodes *i* and *j*

$\theta_{i,j}$ Equal to 1 if node *i* must be connected to facility located in node *j* and 0 otherwise

Non-negative Variables

$D_{i,j,t}$ Diameter of a pipeline connecting *i* and *j* built in period *t*

$F_{c,i,j,t}$ Amount of component *c* flowing from *i* to *j* during period *t*

NPC Total net present cost

$P_{i,t}$ Pressure at node *i* during period *t*

$P_{i,t}^{sq}$ = ${P_{i,t}}^2$

Binary variables

$w_{i,i',j}$ = 1 if source node *i* is connected to facility *j* through an intermediate node *i'*

$x_{d,i,j,t}$ = 1 if a pipeline with diameter *d* is built between *i* and *j* in period *t*

$y_{q,j,t}$ = 1 if facility of size *q* is installed at node *j* at time period *t*

Optimization problems

$\mathcal{F}_k$ Subproblem *k-F* of the generalized STA algorithm at iteration *k*

$\mathcal{F}_k^*$ Optimal solution of the subproblem $\mathcal{F}_k$

$\mathcal{R}_k$ Problem relaxation solved at iteration *k*

$\mathcal{R}_k^*$ Optimal solution of the relaxed problem $\mathcal{R}_k$

z^{BF} Objective value of the best solution found for the original problem

z^{LB} Best-possible objective value for the original problem

z^F Objective value of a feasible solution from parallel process

z_k^{LB} Lower bound at iteration k

z_k^{UB} Upper bound at iteration

References

1. Allen, R.C., Allaire, D., El-Halwagi, M.M., 2019. Capacity Planning for Modular and Transportable Infrastructure for Shale Gas Production and Processing. Ind. Eng. Chem. Res. 58, 5887-5897.
2. Cafaro, D.C., Grossmann, I.E., 2014. Strategic planning, design, and development of the shale gas supply chain network. AIChE J. 60, 2122-2142.
3. Cafaro, D.C., Presser, D.J., Grossmann, I.E., 2022. Recent Contributions to the Optimal Design of Pipeline Networks in the Energy Industry using Mathematical Programming. TOP 30, 618-648.
4. Drouven, M.G. and Grossmann, I.E., 2017. Mixed-integer programming models for line pressure optimization in shale gas gathering systems. J. Pet. Sci. Eng. 157, 1021-1032.
5. Drouven, M.G., Cafaro, D.C., Grossmann, I.E., 2023. Mathematical programming models for shale oil & gas development planning: A review and perspective. Comput. Chem. Eng. https://doi.org/10.1016/j.compchemeng.2023.108317.
6. Gao, J., You, F., 2017. Design and optimization of shale gas energy systems: overview, research challenges, and future directions. Comput. Chem. Eng. 106, 699-718.
7. Glover, F., 1975. Improved Linear Integer Programming Formulations of Nonlinear Integer Problems. Management Science *22*, 455–460. https://doi.org/10.1287/mnsc.22.4.455
8. He, G., Chen, D., Liao, K., Sun, J., Nie, S., 2019. A methodology for the optimal design of gathering pipeline system in old oilfield during its phased development process. Computers & Industrial Engineering 130, 14-34.
9. Hong, B., Li, X., Di, G., Song, S., Yu, W., Chen, S., Li, Y., Gong, J., 2020a. An int. MILP model for opt. planning of multi-period onshore gas field gathering pipeline system. Comp. Ind. Eng. 146, 106479.
10. Hong, B., Li, X., Song, S., Chen, S., Zhao, C., Gong, J., 2020b. Optimal planning and modular infrastructure dynamic allocation for shale gas production. Applied Energy 261, 114439.
11. Lockhart, R. and Martinelli, R.C., 1949. Proposed correlation of data for isothermal two-Phase, two-component flow in pipes. Chem. Eng. Prog. 45, 38–48.
12. Montagna, A.F., Cafaro, D.C., Grossmann, I.E., Burch, D., Shao, Y., Wu, X.H., Furman, K., 2021. Pipeline network design for gathering unconventional oil and gas production using mathematical optimization. Opt. Eng. 24, 539-589.
13. Montagna, A.F., Cafaro, D.C., Grossmann, I.E., Ozen, O., Shao, Y., Zhang, T., Guo, Y., Wu, X.H., Furman, K., 2022. Surface facility optimization for combined shale oil and gas development strategies. Opt. Eng. doi: 10.1007/s11081-022-09775-8
14. Quesada, I., Grossmann I.E. (1995) Global optimization of bilinear process networks with multicomponent flows. Comput. Chem. Eng. 19, 1219-1242.
15. Ryoo, H.S. and Sahinidis, N.V., 1995. Global optimization of nonconvex NLPs and MINLPs with applications in process design, Comput. Chem. Eng. 19, 551-566.
16. Tan, S.H. and Barton, P.I., 2016. Optimal dynamic allocation of mobile plants to monetize associated or stranded natural gas, part II: Dealing with uncertainty. Energy 96, 461-467.
17. Wang, B., Yuan, M., Zhang, H., Zhao, W., Liang, Y., 2018. An MILP model for optimal design of multi-period natural gas transmission network. Chemical Engineering Research and Design 129, 122-131.

18. Wen, K., Lu, Y., Lu, M., Zhang, W., Zhu, M., Qiao, D., Meng, F., Zhang, J., Gong, J., Hong, B., 2022. Multi-period optimal infrastructure planning of natural gas pipeline network system integrating flowrate allocation. Energy 257, 124745.
19. Weymouth, T.R., 1912. Problems in Natural Gas Engineering. Transactions of the ASME 34, 185-234.
20. Zhou, J., Zhang, H., Li, Z. and Liang, G., 2022. A MINLP model for combination pressurization optimization of shale gas gathering system. J Petrol. Explor. Prod. Technol. 12, 3059-3075.