A Multistage Stochastic Programming Approach for the Planning of Offshore Oil or Gas Field Infrastructure Under Decision Dependent Uncertainty

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Abstract

The planning of offshore oil or gas field infrastructure under uncertainty is addressed in this paper. The main uncertainties considered are in the initial maximum oil or gas flowrate, recoverable oil or gas volume and water breakthrough time of the reservoir, which are represented by discrete distributions. Furthermore, it is assumed that these uncertainties are not immediately realized, but are gradually revealed as a function of design and operation decisions. In order to account for these decision-dependent uncertainties, we propose a multistage stochastic programming model that captures the complex economic objectives and nonlinear reservoir behavior, and simultaneously optimizes the investment and operation decisions over the entire planning horizon. The proposed solution algorithm relies on a duality based branch and bound method involving subproblems as nonconvex mixed-integer nonlinear programs. Several examples involving nonlinear reservoir models are presented to illustrate the application of the proposed method.

Keywords: oil or gas field exploration, decision making under uncertainty, multistage stochastic programming, global optimization

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1. Introduction

Oil and gas field exploration and production operations consist of four major steps: exploration, appraisal, development and production. In each step, many decisions have to be made that affect the overall performance of the operation. In the beginning of exploration phase many uncertainties exist, and depending on the decisions made at each step, the uncertainty reduces gradually. Unfortunately, many crucial decisions related to planning of offshore oil or gas field infrastructure have to be made in the early steps where the uncertainty level is high. The quality of these decisions affects the overall profitability of the operation. The objective of this work is to develop and solve a model to optimize the decisions about planning of offshore oil or gas field infrastructure under gradual uncertainty resolution. For the sake of simplicity, we will refer throughout the paper to "oil and/or gas" shortly as oil, but the presented model and solution algorithm applies to any of them equally.

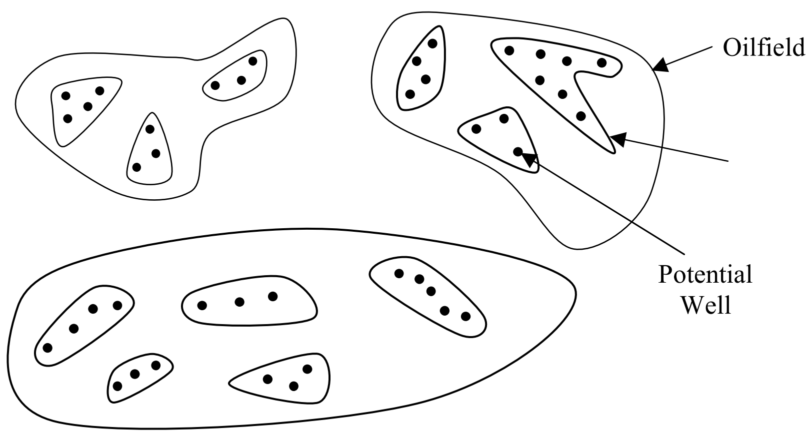

Fig. 1: Configuration of fields, reservoirs and wells

An oil field consists of several reservoirs where each of them contains a number of potential wells (Fig. 1). These potential wells can be drilled and exploited for oil using different facilities. Previous work on planning of oil field infrastructure can be classified into strategic, tactical and operational according to their timeframe, whether uncertainty is considered or not, and the way uncertainty is handled, i.e. simulation or optimization-based solution methods.

Haugland *et al.* (1988) proposed simultaneous MILP models for oil field design and production planning. These authors demonstrated that their model can be solved by commercial solvers for only small instances due to the computational burden, thus necessitating the use of customized algorithms that take advantage of the special problem structure.

Iyer, Grossmann, Vasantharajan & Cullick (1998) addressed the optimal planning and scheduling of offshore oil field infrastructure investment and operations. They proposed a multi-period mixed-integer linear programming model that optimizes the planning and scheduling of investment and operation decisions. These decisions are the selection of reservoirs to develop, selection of well sites and the production rates from wells of each time period. The model incorporates the nonlinear reservoir performance through piecewise linear approximations.

Van den Heever & Grossmann (2000a) proposed a multi-period mixed-integer nonlinear programming model for oil field infrastructure planning for which they developed a bi-level decomposition method. As opposed to Iyer & Grossmann (1998), a nonlinear reservoir model was explicitly incorporated into the formulation. In both papers one of the major assumptions was that there was no uncertainty in the parameters. Van den Heever *et al*. (2000b), (2001), extended the work to handle complex economic objectives such as royalties, tariffs and taxes. These complexities were incorporated into their model through disjunctions and converted the resulting disjunctive problem into a convex MINLP by obtaining the convex hull of each disjunction and using convex underestimators for nonconvexities in the constraints.

Lin & Floudas (2000) investigated the long term planning problem for gas field development. They optimized the investment and operation decisions using an MINLP model which incorporates nonlinear reservoir models and complex economic calculations in a continuous time formulation. They also proposed a two-stage algorithm for solving the MINLP model where they solve a simplified model at the first stage, and the detailed model in the second stage.

Carvalho et. al (2006a) presented reformulations of the MILP model by Tsarbopoulou (2000) and a bi-level decomposition algorithm for solving the larger instances of the problem. These authors use design cuts to avoid subsets and supersets as proposed by Iyer & Grossmann (1998), and report a significant improvement in solution time by the use of decomposition and design cuts in the algorithm. Furthermore, Carvalho et. al (2006b) extended their work to handle multiple reservoirs.

Behrenbruch (1993) presented a qualitative discussion on the development planning process, feasibility studies, cost estimating, economic evaluation and risk analysis for offshore oil field. Guidelines are given to evaluate the feasibility of offshore petroleum projects in terms of appraisal program, field development and facility options. It emphasizes the multidisciplinary nature of the problem, the need to consider the correct geological model and to incorporate flexibility into the decision process. Meister, Clark & Shah (1996) presented a model for selecting the optimal information-gathering process during the exploration phase, and simultaneously optimizing the operating policies.

Jonsbraten (1998) presented an MILP model for optimal development of an oil field under oil price uncertainty. The model predicts decisions related to both design and operation while maximizing the expected net present value of the project. The author used progressive hedging algorithm that is very similar to Lagrangean decomposition to solve the problem. Lund (2000) considered the value of flexibility in offshore petroleum projects by a stochastic dynamic programming model. The author assumed uncertainty in the oil price and recoverable oil volume, and illustrated the change in the objective value with respect to the changes in the flexibility of possible decisions. Similarly, Begg et. al (2002) extended the value of information concept to assess the value of flexibility in managing uncertainty in oil and gas investments. These authors used decision trees or asset models to find the optimum set of decisions, and presented three examples to show the relationship between uncertainty, flexibility, value of information and value of flexibility.

Cullick, Heath & Narayanan (2003) proposed an integration of global search optimization algorithm, finite-difference reservoir simulation, and economics. In the optimization algorithm, new decision variables were generated using meta-heuristics and uncertainties were handled through simulations for fixed design variables. They presented examples having multiple oil fields where the uncertainties are in reservoir volume, fluid quality, deliverability, and costs.

Aseeri, Gorman & Bagajewicz (2004) addressed the financial risk management in the planning and scheduling of offshore oil infrastructures. The authors introduced uncertainty, risk management and budgeting constraints to the model proposed by Iyer & Grossmann (1998). A sampling average algorithm was used to overcome the numerical difficulties and compare the results with optimum results found using upper bound risk curves. Ligero, Xavier & Schiozer (2005) proposed a new methodology to evaluate the value of information based on simulation of several reservoir models. They show the complexity to evaluate the value of information in appraisal and development phases.

Goel & Grossmann (2004) dealt with the gas field development problem under uncertainty in size and quality of reserves where decisions on the timing of field drilling are assumed to yield an immediate resolution of the uncertainty. Linear reservoir models were used, which provide a reasonable approximation for gas fields. In their solution strategy, the authors used a relaxed problem to predict upper bounds and solve multistage stochastic programs for fixed scenario tree for finding lower bounds. Goel *et al.* (2006) later proposed a branch and bound algorithm for solving the corresponding disjunctive/mixed-integer programming model where lower bounds are generated by Lagrangean duality.

Ulstein, Nygreen & Sagli (2007) presented a model for tactical planning of Norwegian petroleum production. The model maximizes the net income before taxes from the production and sale of petroleum products. Different cases with demand variations, varying quality constraints and system breakdowns are considered. The model is solved for different scenarios and solutions are compared with the base case scenario. The benefit of the model is to identify feasible ways to satisfy the demand for varying network configurations.

Ozdogan & Horne (2006) emphasized the value of time-dependent information to achieve better decisions in terms of reduced uncertainty and expected net present value. They use history-matched realizations and pseudo-history concepts to simulate and optimize the well placement decisions. The application of the proposed approach is rather limited because of the computational load of history-matching steps. This limits the application to fields with a small number of wells.

Besides extensive mathematical programming and deterministic global optimization algorithms, there are many papers using a combination of reservoir modeling, economics and decision making under uncertainty through simulation/optimization frameworks as a solution method. Such papers that we are aware include Begg *et al.* (2001), Cullick *et al.* (2003), Zabalza-Mezghani *et al.* (2004), Bailey *et al.* (2005).

Another important aspect of this paper is decision-dependent uncertainty. Most previous work dealing with uncertainty considers exogenous uncertainty where stochastic processes are independent of decisions (e.g. demands), whereas problems where stochastic processes are affected by decisions are said to possess endogenous uncertainty.

Decisions can affect the stochastic process in two different ways. One way is decisions can change the probability functions. Examples for such an effect has been considered by Ahmed (2000), Held *et al.* (2003) and Vishwanath *et al.* (2004). Ahmed (2000) presents examples on network design, server selection and facility location problem with decision dependent uncertainties. The author shows that these programs can be reformulated as MILP problems and solved by LP-based branch and bound algorithms. Held *et al.* (2003) have worked on another instance where the problem has endogenous uncertainty in the structure of network. In each stage of the problem, an operator tries to find the shortest path from a source to destination after the interdictor interdicts some of the nodes in the network. The aim is to maximize the probability of stopping the flow of goods or information in the network. Vishwanath *et al.* (2004) addressed a network problem having endogenous uncertainty in survival distributions. The problem is a two-stage stochastic program where first period investment decisions are made for changing the survival probability distribution of arcs after a disaster. The aim is to find the investments that will minimize the expected shortest path from source to destination after a disaster.

Another way the decisions can impact the stochastic process is that they can affect the resolution of uncertainty or the time uncertainty resolves. Such decisions have been considered in Pflug (1990), Jonsbraten *et al.* (1998) and Goel *et al.* (2004). Pflug (1990), addresses the problems in the context of discrete event dynamic systems where the underlying stochastic process depends on the optimization decisions. Jonsbraten *et al.* (1998) address endogenous uncertainty where project decisions reduce the uncertainty. These authors have proposed an implicit enumeration algorithm where decisions that affect the uncertain parameter values are made at the first stage.

Tarhan & Grossman (2008a) considered the synthesis of process networks. They assumed there were uncertainties in the yields of the processes and these uncertainties resolved gradually over time depending on the investment and operation decisions. Problem was modeled as a disjunctive program and then converted to an MILP using big-M type of transformation. For solving the model, authors use a duality based branch and bound procedure where each scenario subproblem is solved independently during the subgradient optimization for calculating upper bounds and heuristics are used for generating feasible solutions.

In this paper we address the planning of offshore oil field infrastructure involving endogenous uncertainty in the initial maximum oil flowrate, recoverable oil volume and water breakthrough time of the reservoir, where decisions affect the resolution of these uncertainties. We extend the work by Goel & Grossmann (2004, 2006) with three major differences. The first one is that the model will focus on only one field consisting of several reservoirs rather than multiple fields. The second difference is that nonlinear reservoir models will be considered instead of linear. Finally, the third difference is that the resolution of uncertainty is gradual over time instead of being resolved immediately.

This paper is organized as follows. In Section 2, we present the problem statement under consideration, and then in Section 3 we discuss the representation of planning horizon and uncertainty. In Section 4, the optimization model and in Sections 5, the proposed solution approach is explained in detail. Sections 6 discusses the implementation details and the results found by the proposed solution approach.

2. Problem statement

In this paper the problem that we consider is the design and planning of an offshore oil field infrastructure (Fig. 2) over a specified planning horizon. Specifically, we consider a field consisting of several reservoirs where a number of wells can be drilled and exploited for oil in every reservoir during the planning horizon.

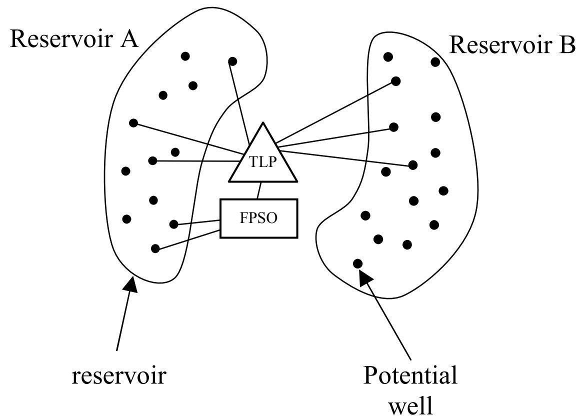

Fig. 2: A Typical oil field infrastructure.

Fig. 3: A typical FPSO facility[1]

Fig. 4: A typical TLP facility[2]

The field infrastructure can be composed of Floating Production Storage and Offloading (FPSO) (Fig. 3) and/or Tension Leg Platform (TLP) (Fig. 4) facilities. The FPSO facility can be either a small FPSO, converted from a retiring oil tanker, or a large FPSO, a newly constructed grassroots facility. An FPSO facility can process, store and offload the processed oil to other tankers. Processing means separating the oil and water that comes out of the well. Unlike FPSO, a TLP facility cannot process oil; it possesses only drilling and oil recovering capability. TLP and FPSO facilities can be connected to each other through pipes, called risers. Oil recovered from TLP facilities is pumped to FPSO facilities through these pipes. Each facility has a construction cost and a lead time between the construction decision and the actual start-up. There are two options for drilling wells. Each well can be drilled either as a sub-sea or a TLP well. Drilling ships are used to drill sub-sea wells, so there is no need to have a facility present to drill a sub-sea well. Unlike sub-sea wells, a TLP well has to be drilled by a TLP facility. Due to

[1] http://offalnews.blogspot.com/2007/03/producing-projects-terra-nova.html
[2] http://en.wikipedia.org/wiki/Image:Mars_Tension-leg_Platform.jpg

economic reasons, each type of well is drilled in groups consisting of fixed number of wells. In order to recover oil from a well, it must be connected to a facility. A sub-sea well has to be connected to an FPSO facility, whereas a TLP well has to be connected to a TLP facility.

The problem involves making investment and operation decisions over the planning horizon. Investment decisions are selection of the number, type and capacity of facilities and installation schedule of these facilities, as well as selection of types of wells and drilling schedule of wells. Operation decisions are amount of oil production for each time period given the limitations of the reservoirs. The goal is to capture the complex economic tradeoffs that arise from the investment and operation decisions in order to maximize the expected net present value of the project.

There are four major assumptions in the problem about the state of the reservoir. Firstly, there is no free gas in the reservoir. Secondly, there is a strong aqueous support which creates enough pressure in the reservoir. As an extension of the first two assumptions, we assume there is no need for enhanced recovery, i.e. no injection of gas or water into the reservoir. Finally, all wells in one reservoir are identical, which leads to the same maximum oil flowrate for all wells in the reservoir given the cumulative oil extracted from that reservoir. These assumptions are made for the sake of simplicity and both model and solution algorithm are flexible enough to incorporate more complex reservoir models.

Fig. 5 represents the nonlinear reservoir model. It shows oil and water flowrate from a single well versus the cumulative recovered oil. The maximum oil flowrate can be represent by a nonlinear function of cumulative recovered oil from the reservoir (eq. 1) as shown in Fig. 5. During oil recovery from a reservoir, the liquid coming from the well contains not only oil but also water, and the relative rates of these liquids can be characterized by the water-to-oil ratio. This ratio is approximated using a nonlinear function of the cumulative oil recovered and the recoverable oil volume of the reservoir (eq. 2). The water rate can then be calculated by multiplying the oil flowrate and the water-to-oil ratio as shown in eq. 3.

$$oil_t^{r,s} \leq \gamma_1^{r,s} \cdot \left(oil_{t-1}^{cum,r,s}\right)^2 + \gamma_2^{r,s} \cdot oil_{t-1}^{cum,r,s} + \gamma_3^{r,s} \cdot bw_t^{r,s} \qquad \forall t, \forall s, \forall r \qquad (1)$$

$$wor_t^{r,s} = \alpha^{r,s} \left(\frac{oil_{t-1}^{cum,r,s}}{REC^{r,s}} \right)^{\beta^r} \qquad \forall t, \forall s, \forall r \qquad (2)$$

$$water_t^{(\ ),r,s} = wor_t^{r,s} \ oil_t^{(\ ),r,s} \qquad \forall t, \forall s, \forall r \qquad (3)$$

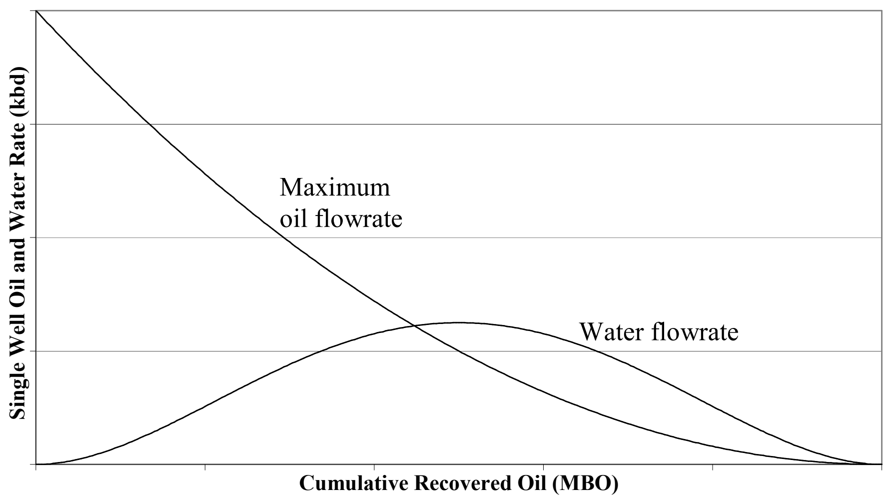

Fig. 5: Nonlinear reservoir model

Uncertain parameters in the system are initial maximum oil flowrate, recoverable oil volume and water breakthrough time of the reservoir. As we assumed that all wells in one reservoir are identical, we consider the uncertainty in initial maximum oil flowrate for only a single well in the reservoir. The three uncertain parameters affect the nonlinear reservoir behavior in Fig. 5 in different ways. Initial maximum oil flowrate affects the point where the oil flowrate starts on the vertical axis in Fig. 5. Recoverable oil of the reservoir affects where the oil flowrate becomes zero, i.e. the point where line that represents oil flowrate hits the horizontal axis. The third uncertain parameter, water breakthrough time is the time elapsed until the water rate exceeds a pre-specified value. We assume that water breakthrough time is correlated with the scalar α in the nonlinear function (eq. 2) representing water-to-oil ratio. Therefore, the uncertainty in water breakthrough time can be represented by the uncertainty in the parameter α. If α has a high value, water-to-oil ratio increases rapidly so does the water rate (eq. 3) making the water breakthrough time short. If α has a low value, water-to-oil ratio and water rate increases in a moderate way making the water breakthrough time longer.

In order to consider the gradual resolution of uncertainties, we make the following assumptions:

a) We assume that there is an appraisal program which resolves the uncertainty in initial maximum oil flowrate. Appraisal program consists of drilling N_1 number of wells. This appraisal program not only gives the actual value for the initial maximum oil flowrate, but also provides the posterior probabilities for recoverable oil from the reservoir depending on the outcome of initial maximum oil flowrate. Therefore, at the end of the appraisal program, we resolve the uncertainty only on the initial maximum oil flowrate but not on the recoverable oil of the reservoir or water breakthrough time.
b) We assume the uncertainty in recoverable oil volume can resolve in two ways. One way is to drill a total of more than N_2 number of wells, and the other is to produce from that reservoir for a duration of N_3 years. Either of these actions gives the same effect of resolving uncertainty in recoverable oil.
c) Uncertainty in water breakthrough time is resolved independently of the drilling decisions; it is affected by only the production. N_4 number of years of production from the reservoir resolves the uncertainty on the water breakthrough time. Therefore, it is possible to resolve the uncertainty in the initial maximum oil flowrate and recoverable oil by drilling wells, but there has to be production for resolving the uncertainty in water breakthrough time.

3. Representation of planning horizon and uncertainty

In this paper the planning horizon is discretized into time periods and the probability distributions of uncertain parameters are discrete because these specifications allow us to represent the stochastic process by scenario trees. Fig. 6 is a standard representation of a scenario tree having one uncertain parameter with two discrete values in two time periods, which leads to four scenarios. Uncertain parameters ξ_1 and ξ_2 reveal at the end of first and second time periods respectively generating four equally probable scenarios given the parameters have uniform distribution.

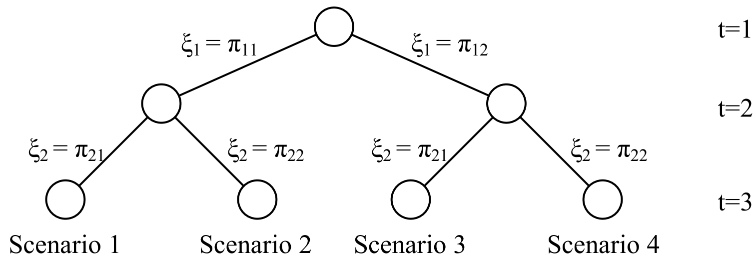

Fig. 6: Scenario tree with uncertain parameters ξ_1 and ξ_2.

In a standard scenario tree, each node for time period t represents a possible state for that time period. Each arc represents the possible transition from one state in time period t to another state in time period $t+1$. A path from the root node to a leaf node represents a scenario. Thus, a scenario is a combination of possible uncertain parameters in each of the time periods. The set of time periods, which has the same amount of information, define a stage. Problems that have more than two stages are called multi-stage stochastic programs.

Fig. 7 is an alternative representation of the scenario tree in Fig. 6, proposed by Ruszczynski (1997). In this representation each scenario is represented by a set of unique nodes. The horizontal lines connecting nodes in time period t mean that these nodes are indistinguishable and they have the same amount of information in that time period. The horizontal lines reduce the tree in Fig. 7 to the one in Fig. 6. For modeling the problem, the scenario trees will be considered according to this representation given by Ruszczynski (1997) (see Fig. 7) in order to explicitly handle the non-anticipativity constraints which give rise to different tree structures (see Goel and Grossmann, 2006).

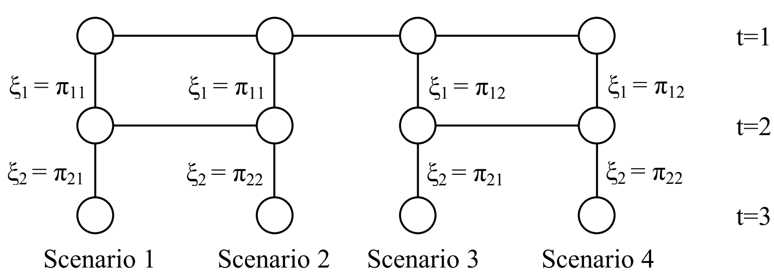

Fig. 7: Alternative scenario tree with uncertain parameters ξ_1 and ξ_2.

4. Mathematical model

In this section we present the optimization model for the planning of offshore oil field infrastructure under uncertainty. The model for the planning of field infrastructure under uncertainty optimizes investment decisions such as number of wells to drill, facilities to build and operational decisions such as oil production rate from the reservoirs to maximize the expected net present value. As explained in the previous sections, the interaction between decisions creates many trade-offs and makes the decision making process highly complex. The aggregate infrastructure is shown graphically in Fig. 8.

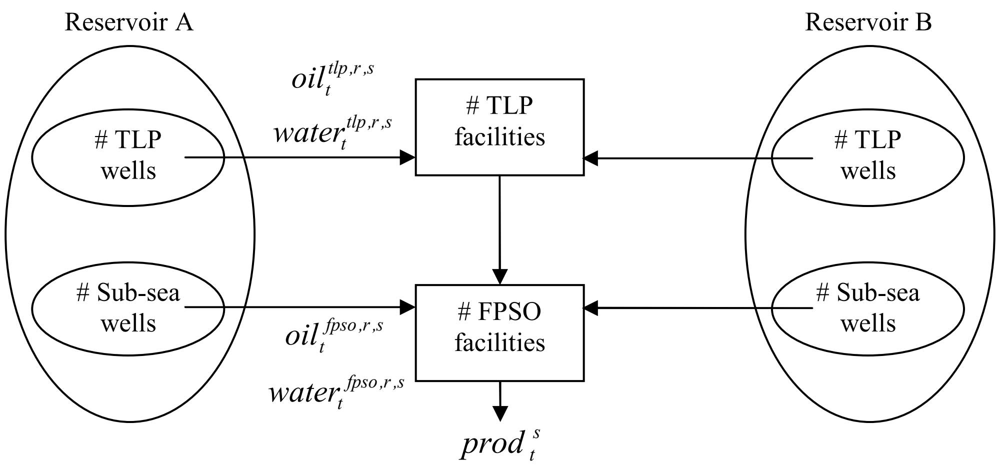

Fig. 8: Graphical representation of the aggregate infrastructure

The variables in the model can be classified as decision, state, and recourse variables. Decision variables are related to decisions that are made at the beginning of each time period t and scenario s (e.g. number of wells to drill and facilities to build). Recourse variables are related to decisions made after resolution of uncertainty (e.g. oil production from reservoirs). State variables are the variables that are calculated automatically when decision and recourse variables at previous time periods are selected (e.g. water-to-oil ratio, number of sub-sea/TLP wells available for production). The sequence of events is as follows. Decision variables are implemented at the beginning of time period t. This is followed by the resolution of uncertainty. Recourse decisions are made after resolution of uncertainty.

Given the sets, variables and parameters in the Nomenclature section, the model (P) is as follows. The objective is to maximize the expected net present value of the project given by,

$$(P) \qquad max \;\; enpv = \sum_{s \in S} P^s \cdot npv^s \tag{4}$$

Net present value of each scenario can be calculated using revenues and costs for that scenario,

$$npv^s = \sum_{t \in T} D_t \cdot \left(rev_t^s - cost_t^s \right) \qquad \forall s \tag{5}$$

The revenue at every time period t can be calculated using the total amount of oil produced and the sale price of oil at each time period,

$$rev_t^s = C_{1,t} \cdot prod_t^s \qquad \forall t, \forall s \tag{6}$$

Total cost at every time period t is the sum of capital and operating expenditures,

$$cost_t^s = capex_t^s + opex_t^s \qquad \forall t, \forall s \tag{7}$$

Capital expenditures at every time period t come from drilling group of TLP wells $\left(dtw_t^{r,s}\right)$, group of sub-sea wells $\left(dsw_t^{r,s}\right)$, building TLP $\left(bt_t^s\right)$, small FPSO $\left(bsf_t^s\right)$, large FPSO facilities $\left(blf_t^s\right)$ at that period,

$$capex_t^s = \sum_{r \in R} C_{2,t} \cdot dtw_t^{r,s} + \sum_{r \in R} C_{3,t} \cdot dsw_t^{r,s} + C_{4,t} \cdot bt_t^s + C_{5,t} \cdot bsf_t^s + C_{6,t} \cdot blf_t^s$$

$$\forall t, \forall s \tag{8}$$

The operating expenditure at every time period t is a linear function of the amount of oil produced,

$$opex_t^s = C_{7,t} \cdot prod_t^{total,s} \qquad \forall t, \forall s \tag{9}$$

Due to the limitations on the maximum number of drilling rigs on a TLP facility, the number of TLP wells to be drilled $\left(dtw_t^{r,s}\right)$ at time period t by each TLP facility is limited by some upper bounds $\left(U^{tw}\right)$,

$$\sum_{r \in R} dtw_t^{r,s} \le U^{tw} \cdot nt_t^s \qquad \forall t, \forall s \tag{10}$$

Similarly, the number of sub-sea wells to be drilled $\left(dsw_t^{r,s}\right)$ at time period t is limited by some upper bounds $\left(U^{sw}\right)$,

$$\sum_{r \in R} dsw_t^{r,s} \leq U^{sw} \qquad \forall t, \forall s \qquad (11)$$

A group of TLP wells (G^{tw}) in reservoir r can be drilled and connected to TLP facilities at time period t $(ntw_t^{r,s} > 0)$ only if at least one TLP facility has been built until that time period $(nt_t^s > 0)$. Also, there is a limit on the maximum number of wells to drill and connect per TLP facility (N_w^{max}),

$$\sum_{r \in R} ntw_t^{r,s} \leq N_w^{max} \cdot nt_t^s \qquad \forall t, \forall s \qquad (12)$$

Oil and water can be extracted from a well in reservoir r and pumped to TLP facilities at time period t if there is a TLP well available at time period t.

$$oil_t^{tlp,r,s} + water_t^{tlp,r,s} \leq M_1 \cdot ntw_t^{r,s} \qquad \forall t, \forall s, \forall r \qquad (13)$$

There is an upper bound on the total number of TLP and sub-sea wells that can be drilled in each reservoir,

$$ntw_t^{r,s} + nsw_t^{r,s} \leq U^{w,r} \qquad \forall t, \forall s, \forall r \qquad (14)$$

Oil can be recovered from reservoir r in time period t if there is a sub-sea well available for production at time period t,

$$oil_t^{fpso,r,s} + water_t^{fpso,r,s} \leq M_2 \cdot nsw_t^{r,s} \qquad \forall t, \forall s, \forall r \qquad (15)$$

The rate of oil recovered from a well in reservoir r at time period t is bounded above by a linear or nonlinear function of cumulative oil produced from the same reservoir until time period t. Also oil flowrate has to be zero if there is no well ready for production $(bw_t^{r,s} = 0)$.

$$oil_t^{r,s} \leq \gamma_1^{r,s} \cdot \left(oil_{t-1}^{cum,r,s}\right)^2 + \gamma_2^{r,s} \cdot oil_{t-1}^{cum,r,s} + \gamma_3^{r,s} \cdot bw_t^{r,s} \qquad \forall t, \forall s, \forall r \qquad (16)$$

Oil can be recovered from reservoir r at time period t if there is a well ready for production at time period t,

$$bw_t^{r,s} \leq ntw_t^{r,s} + nsw_t^{r,s} \qquad \forall t, \forall s, \forall r \qquad (17)$$

The amount of oil pumped to TLP and FPSO facilities at time period t can be calculated by the number of available TLP $(ntw_t^{r,s})$ and sub-sea $(nsw_t^{r,s})$ wells for production and the oil flowrate per well from the reservoir,

$$oil_t^{tlp,r,s} = oil_t^{r,s} \cdot ntw_t^{r,s} \qquad \forall t, \forall s, \forall r \qquad (18)$$

$$oil_t^{fpso,r,s} = oil_t^{r,s} \cdot nsw_t^{r,s} \qquad \forall t, \forall s, \forall r \qquad (19)$$

The water flowrate from reservoir r to TLP or FPSO facilities in time period t can be calculated using water-to-oil ratio and the oil flowrate for wells in that reservoir,

$$water_t^{tlp,r,s} = wor_t^{r,s} \cdot oil_t^{tlp,r,s} \qquad \forall t, \forall s, \forall r \qquad (20)$$

$$water_t^{fpso,r,s} = wor_t^{r,s} \cdot oil_t^{fpso,r,s} \qquad \forall t, \forall s, \forall r \qquad (21)$$

Water-to-oil ratio in reservoir r is a nonlinear function of the cumulative oil production from the reservoir and recoverable oil,

$$wor_t^{r,s} = \alpha^{r,s} \cdot \left(\frac{oil_{t-1}^{cum,r,s}}{REC^{r,s}} \right)^{\beta^r} \qquad \forall t, \forall s, \forall r \qquad (22)$$

The cumulative oil extracted from reservoir r until the end of time period t is calculated by,

$$oil_t^{cum,r,s} = oil_{t-1}^{cum,r,s} + \delta_t \cdot \left(oil_t^{tlp,r,s} + oil_t^{fpso,r,s} \right) \qquad \forall t, \forall s, \forall r \qquad (23)$$

The total amount of oil recovered by FPSO $\left(oil_t^{fpso,r,s} \right)$ and TLP $\left(oil_t^{tlp,r,s} \right)$ facilities has to be smaller than the total oil processing capacity of the small FPSO $\left(CAP_{sf}^{oil} \right)$ and large FPSO $\left(CAP_{lf}^{oil} \right)$ facilities,

$$CAP_{sf}^{oil} \cdot nsf_t^s + CAP_{lf}^{oil} \cdot nlf_t^s \geq \sum_{r \in R} \left(oil_t^{fpso,r,s} + oil_t^{tlp,r,s} \right) \qquad \forall t, \forall s \qquad (24)$$

The total liquid (oil and water) flowrate coming into the FPSO facilities including the liquid from TLP facilities has to be smaller than the total liquid capacity of small FPSO $\left(CAP_{sf}^{liq} \right)$ and large FPSO $\left(CAP_{lf}^{liq} \right)$ facilities,

$$CAP_{sf}^{liq} \cdot nsf_t^s + CAP_{lf}^{liq} \cdot nlf_t^s \geq \sum_{r \in R} \left(oil_t^{fpso,r,s} + water_t^{fpso,r,s} + oil_t^{tlp,r,s} + water_t^{tlp,r,s} \right) \qquad \forall t, \forall s \qquad (25)$$

The amount of production from reservoir r at time period t is calculated by summing the oil extracted by FPSO $\left(oil_t^{fpso,r,s} \right)$ and TLP facilities $\left(oil_t^{tlp,r,s} \right)$,

$$prod_t^{r,s} = \delta_t \cdot \left(oil_t^{fpso,r,s} + oil_t^{tlp,r,s} \right) \qquad \forall t, \forall s \qquad (26)$$

The amount of total production at time period t is calculated by summing productions over all reservoirs,

$$prod_t^{total,s} = \sum_r prod_t^{r,s} \qquad \forall t, \forall s \qquad (27)$$

The number of wells and facilities available to operate at time period t (e.g. nt_t^s) can be calculated by summing the available wells and facilities at the previous time period $t-1$ (e.g. nt_{t-1}^s) and the ones that started building earlier ($bt_{t-\tau_1}^s$) and expected to be ready at time period t,

$$nt_t^s = nt_{t-1}^s + bt_{t-\tau_1}^s \qquad \forall t, \forall s \qquad (28)$$

$$nsf_t^s = nsf_{t-1}^s + bsf_{t-\tau_2}^s \qquad \forall t, \forall s \qquad (29)$$

$$nlf_t^s = nlf_{t-1}^s + blf_{t-\tau_3}^s \qquad \forall t, \forall s \qquad (30)$$

$$ntw_t^{r,s} = ntw_{t-1}^{r,s} + dtw_{t-\tau_4}^{r,s} \qquad \forall t, \forall s, \forall r \qquad (31)$$

$$nsw_t^{r,s} = nsw_{t-1}^{r,s} + dsw_{t-\tau_5}^{r,s} \qquad \forall t, \forall s, \forall r \qquad (32)$$

There is an upper bound on the number of each type of facilities (TLP, small FPSO, large FPSO) that can be built during the entire planning horizon,

$$nt_t^s \le U^t \qquad \forall t \le T, \forall s \qquad (33)$$

$$nsf_t^s \le U^{sf} \qquad \forall t \le T, \forall s \qquad (34)$$

$$nlf_t^s \le U^{lf} \qquad \forall t \le T, \forall s \qquad (35)$$

Eqs. (36) –(40) relate the state variables and the binary/logic variables $\left(w_{(.),t}^{r,s}\right)$ which will be used for finding out the indistinguishable scenario pairs at every time period. The total number of wells are less than N_1 if and only if binary/logic variable $w_{1,t}^{r,s}$ is one/true. Similarly, the total is less than N_2 if and only if $w_{2,t}^{r,s}$ is one/true,

$$w_{1,t}^{r,s} \Leftrightarrow \left[ntw_t^{r,s} + nsw_t^{r,s} \le N_1 - 1 \right] \qquad \forall t, \forall s, \forall r \qquad (36)$$

$$w_{2,t}^{r,s} \Leftrightarrow \left[ntw_t^{r,s} + nsw_t^{r,s} \le N_2 - 1 \right] \qquad \forall t, \forall s, \forall r \qquad (37)$$

The production from reservoir r has been made less than N_3 years if and only if $w_{3,t}^{r,s}$ is one/true. And, it is less than N_4 years if and only if $w_{4,t}^{r,s}$ is one/true,

$$w_{3,t}^{r,s} \Leftrightarrow \left[\sum_{\tau=1}^{t-1} bp_{\tau}^{r,s} \leq N_3 - 1 \right] \qquad \forall t, \forall s, \forall r \qquad (38)$$

$$w_{4,t}^{r,s} \Leftrightarrow \left[\sum_{\tau=1}^{t-1} bp_{\tau}^{r,s} \leq N_4 - 1 \right] \qquad \forall t, \forall s, \forall r \qquad (39)$$

where, $bp_{\tau}^{r,s}$ is represented as,

$$bp_{t}^{r,s} \Leftrightarrow \left[prod_{t}^{r,s} \geq \varepsilon \right] \qquad \forall t, \forall s, \forall r \qquad (40)$$

Eqs. (41) - (47) are used to find the scenarios (s,s') which are indistinguishable at time period t depending on the decisions given until that time period. Actually, eqs. (41) - (47) are a shortened version of the equations in Appendix B using the proposition 3 presented in the same section. Two scenarios (s,s') which differ only in initial productivities (i.e. element of the set M_1) will be indistinguishable at time period t if and only if for each reservoir that distinguishes those scenarios (i.e. element of $D(s,s')$) $w_{1,t}^{r,s}$ is true (eq. (41)). Eqs. (42) - (47) can be interpreted similarly. Note that each possible scenario pairs (s,s') will be an element of exactly one of the sets M_k, $k = 1,...,7$, that refer to sets of scenarios that differ in the initial maximum oil flowrate, recoverable oil and water breakthrough time of the reservoir. (see Table 1)

$$z_{t}^{s,s'} \Leftrightarrow \bigwedge_{r \in D(s,s')} w_{1,t}^{r,s} \qquad \forall t, t \geq 2, \forall (s,s') \in M_1, s < s' \quad (41)$$

$$z_{t}^{s,s'} \Leftrightarrow \bigwedge_{r \in D(s,s')} w_{2,t}^{r,s} \wedge w_{3,t}^{r,s} \qquad \forall t, t \geq 2, \forall (s,s') \in M_2, s < s' \quad (42)$$

$$z_{t}^{s,s'} \Leftrightarrow \bigwedge_{r \in D(s,s')} w_{4,t}^{r,s} \qquad \forall t, t \geq 2, \forall (s,s') \in M_3, s < s' \quad (43)$$

$$z_{t}^{s,s'} \Leftrightarrow \bigwedge_{r \in D(s,s')} w_{1,t}^{r,s} \wedge w_{2,t}^{r,s} \wedge w_{3,t}^{r,s} \qquad \forall t, t \geq 2, \forall (s,s') \in M_4, s < s' \quad (44)$$

$$z_{t}^{s,s'} \Leftrightarrow \bigwedge_{r \in D(s,s')} w_{1,t}^{r,s} \wedge w_{4,t}^{r,s} \qquad \forall t, t \geq 2, \forall (s,s') \in M_5, s < s' \quad (45)$$

$$z_{t}^{s,s'} \Leftrightarrow \bigwedge_{r \in D(s,s')} w_{2,t}^{r,s} \wedge w_{3,t}^{r,s} \wedge w_{4,t}^{r,s} \qquad \forall t, t \geq 2, \forall (s,s') \in M_6, s < s' \quad (46)$$

$$z_t^{s,s'} \Leftrightarrow \bigwedge_{r \in D(s,s')} w_{1,t}^{r,s} \wedge w_{2,t}^{r,s} \wedge w_{3,t}^{r,s} \wedge w_{4,t}^{r,s} \qquad \forall t, t \geq 2, \forall (s,s') \in M_7, s < s' \quad (47)$$

Table 1: Sets that differ only in the specified parameters

	M_1	M_2	M_3	M_4	M_5	M_6	M_7
Initial maximum oil flowrate	√			√	√		√
Recoverable oil volume		√		√		√	√
Water breakthrough time			√		√	√	√

Note that it is enough to consider only the eqs. (41) – (43) that run over M_1, M_2 and M_3 instead of constraints over all subsets of M. Assume scenario pairs (s_1, s_2) in M_1 and (s_1, s_3) in M_2 are indistinguishable. Using the relation in Table 1, it can be inferred that scenario pairs (s_2, s_3), which is in M_4, will be indistinguishable as well. Similarly, all scenario pairs in subsets M_4 to M_7 can be inferred by the pairs in sets M_1 to M_3, making it possible to restrict constraints (41) – (47) to only subsets M_1, M_2 and M_3.

The disjunctive constraint shows the decisions that are the same for time period t if s and s' are indistinguishable at the beginning of time period t. Due to the same argument, the set of scenario pairs (s_1, s_2) are restricted to only subsets M_1, M_2 and M_3.

$$\begin{bmatrix} z_t^{s,s'} \\ bt_t^s = bt_t^{s'} \\ bsf_t^s = bsf_t^{s'} \\ blf_t^s = blf_t^{s'} \\ dtw_t^{r,s} = dtw_t^{r,s'} \qquad \forall r \\ dsw_t^{r,s} = dsw_t^{r,s'} \qquad \forall r \end{bmatrix} \vee \left[\neg z_t^{s,s'} \right] \qquad \begin{array}{l} \forall t \geq 2, \forall (s,s') \in M_1 \cup M_2 \cup M_3, \\ \forall (s,s',t) \in N_C \end{array} \qquad (48)$$

Eqs. (49) - (53) represent the initial non-anticipativity constraints. Initial non-anticipativity constraints include the first period non-anticipativity constraints and some further time non-anticipativity constraints which appear as a result of the gradual resolution of uncertainty. For instance, the scenarios which differ only in the water breakthrough time will be indistinguishable until there is production for N_4 number of periods. Given restrictions on production such as availability of wells and facilities and delays for building infrastructure, it can be shown that these scenarios must be indistinguishable for certain number of years. These non-anticipativity constraints are also included in the initial non-anticipativity constraints.

$$bt_t^s = bt_t^{s'} \qquad \forall (s, s', t) \in N_I \tag{49}$$

$$bsf_t^s = bsf_t^{s'} \qquad \forall (s, s', t) \in N_I \tag{50}$$

$$blf_t^s = blf_t^{s'} \qquad \forall (s, s', t) \in N_I \tag{51}$$

$$dtw_t^{r,s} = dtw_t^{r,s'} \qquad \forall (s, s', t) \in N_I, \forall r \tag{52}$$

$$dsw_t^{r,s} = dsw_t^{r,s'} \qquad \forall (s, s', t) \in N_I, \forall r \tag{53}$$

The model given by eqs. (4) – (53), corresponds to a mixed-integer nonlinear/disjunctive programming model. The most direct way of solving this problem is to convert it into an mixed-integer nonlinear model (see Appendix C) by converting the logic propositions into linear inequalities and disjunctions into mixed-integer constraints (e.g. see Raman & Grossmann, 1994). Also, note that the model has been generated using the scenario tree approach proposed by Ruszczynski (1997) so that each scenario is represented by a set of unique variables and conditional and first period non-anticipativity constraints relate these variables for different scenarios. Also, most of the constraints are linear, except a few relating the water-to-oil ratio and cumulative oil production; water flowrate, oil flowrate and water-to-oil ratio. These constraints are nonconvex which necessitate the use of a global optimization algorithm as will be explained in the solution strategy section.

5. Solution strategy

The size of model (P) increases exponentially with the number of scenarios and time periods, making this nonconvex MINLP model extremely difficult to solve in fullspace for real size problems using commercial solvers. The model P is composed of subproblems connected through initial (eqs. (49) - (53)) and conditional non-anticipativity constraints (eqs. (41) - (48)). Each subproblem is a nonconvex mixed-integer nonlinear program and can be solved independently when these non-anticipativity constraints are relaxed. Therefore, we propose a duality-based branch and bound algorithm that takes advantage of the special problem structure.

In the following sections, the upper and lower bounding procedures used at each node of the branch and bound tree, branching scheme and, the proposed solution algorithm are presented.

5.1. Upper bounding procedure

In the proposed algorithm, the upper bound at the root node of the branch and bound tree is found by optimizing model P_0^{LR} in which the logic constraints (41) – (47) and disjunction (48) have been removed and the initial non-anticipativity constraints (49) - (53) are dualized as follows:

$$\left(P_0^{LR}\right) \quad \phi_0^{LR}\left(\lambda_{bt,t}^{s,s'},\lambda_{bsf,t}^{s,s'},\lambda_{blf,t}^{s,s'},\lambda_{dtw,r,t}^{s,s'},\lambda_{dsw,r,t}^{s,s'}\right)$$

$$= max \ \sum_{s\in S} P^s \ npv^s + \sum_{(s,s',t)\in N_I}\Big[\lambda_{bt,t}^{s,s'}\left(bt_t^s - bt_t^{s'}\right) + \lambda_{bsf,t}^{s,s'}\left(bsf_t^s - bsf_t^{s'}\right) + \lambda_{blf,t}^{s,s'}\left(blf_t^s - blf_t^{s'}\right)$$

$$+ \sum_{r\in R}\lambda_{dtw,r,t}^{s,s'}\left(dtw_t^{r,s} - dtw_t^{r,s'}\right) + \sum_{r\in R}\lambda_{dsw,r,t}^{s,s'}\left(dsw_t^{r,s} - dsw_t^{r,s'}\right)\Big] \tag{54}$$

s.t. (5) – (40).

The parameters $\lambda_{bt,t}^{s,s'}$, $\lambda_{bsf,t}^{s,s'}$, $\lambda_{blf,t}^{s,s'}$, $\lambda_{dtw,r,t}^{s,s'}$, $\lambda_{dsw,r,t}^{s,s'}$ represent the Lagrange multipliers corresponding to constraints (49) – (53), respectively. In order to find the tightest upper bound generated by model P_0^{LR} at the root node, we consider the Lagrangean dual problem P_0^{LD} which minimizes the model P_0^{LR} in the space of multipliers,

$$\left(P_0^{LD}\right) \quad \phi_0^{LD} = \min_{\lambda} \ \phi_0^{LR}\left(\lambda_{bt,t}^{s,s'},\lambda_{bsf,t}^{s,s'},\lambda_{blf,t}^{s,s'},\lambda_{dtw,r,t}^{s,s'},\lambda_{dsw,r,t}^{s,s'}\right) \tag{55}$$

For finding upper bounds at any node n other than root node in the branch and bound tree, model P_n is considered. In model P_n besides the initial non-anticipativity constraints that are added to the model regardless of any decision, some conditional non-anticipativity constraints coming from the relaxed disjunction are also included. The conditional non-anticipativity constraints which apply in node n are included in the dynamic set N_C^n. The selection of which conditional non-anticipativity constraint to include into set N_C^n as well as some necessary cuts to be added to P_n will be discussed in Section 5.2.2. The model P_n, not including any of such cuts, is given as follows,

$$(P_n) \quad \phi_n = max \sum_{s \in S} P^s \cdot npv^s \tag{56}$$

s.t. (4) – (40) and

$$bt_t^s = bt_t^{s'} \qquad \forall (s,s',t) \in N_I \cup N_C^n \tag{57}$$

$$bsf_t^s = bsf_t^{s'} \qquad \forall (s,s',t) \in N_I \cup N_C^n \tag{58}$$

$$blf_t^s = blf_t^{s'} \qquad \forall (s,s',t) \in N_I \cup N_C^n \tag{59}$$

$$dgtw_t^{r,s} = dgtw_t^{r,s'} \qquad \forall (s,s',t) \in N_I \cup N_C^n, \forall r \tag{60}$$

$$dgsw_t^{r,s} = dgsw_t^{r,s'} \qquad \forall (s,s',t) \in N_I \cup N_C^n, \forall r \tag{61}$$

The upper bound at node n is generated by optimizing model P_n^{LR} in which the non-anticipativity constraints (57) – (61) are dualized as follows:

$$\begin{aligned}(P_n^{LR}) \quad \phi_n^{LR} = max \ & \sum_{s \in S} P^s \ npv^s \\ & + \sum_{(s,s',t) \in N_I \cup N_C^n} \Big[\lambda_{bt,t}^{s,s'} \left(bt_t^s - bt_t^{s'} \right) + \lambda_{bsf,t}^{s,s'} \left(bsf_t^s - bsf_t^{s'} \right) + \lambda_{blf,t}^{s,s'} \left(blf_t^s - blf_t^{s'} \right) \\ & + \sum_{r \in R} \lambda_{dtw,r,t}^{s,s'} \left(dtw_t^{r,s} - dtw_t^{r,s'} \right) + \sum_{r \in R} \lambda_{dsw,r,t}^{s,s'} \left(dsw_t^{r,s} - dsw_t^{r,s'} \right) \Big] \end{aligned} \tag{62}$$

s.t. (4) – (40)

Similar to the root node, in order to find the tightest upper bound, we again consider the Lagrangean dual problem P_n^{LD} which is minimization of the model P_n^{LR} in the space of multipliers,

$$(P_n^{LD}) \quad \phi_n^{LD} = \min_{\lambda} \ \phi_n^{LR} \left(\lambda_{bt,t}^{s,s'}, \lambda_{bsf,t}^{s,s'}, \lambda_{blf,t}^{s,s'}, \lambda_{dtw,r,t}^{s,s'}, \lambda_{dsw,r,t}^{s,s'} \right) \tag{63}$$

where $\lambda_{bt,t}^{s,s'}$, $\lambda_{bsf,t}^{s,s'}$, $\lambda_{blf,t}^{s,s'}$, $\lambda_{dtw,r,t}^{s,s'}$, $\lambda_{dsw,r,t}^{s,s'}$ represent the Lagrange multipliers corresponding to constraints (57) – (61).

For fixed values of the multipliers both models P_0^{LR} and P_n^{LR} can be decomposed into independent scenarios each of which is represented by a nonconvex MINLP subproblem. Since both models are relaxations of the model P for any fixed values of the Lagrange multipliers, they yield a valid upper bound if all the nonconvex subproblems are globally optimized. The validity of these upper bounds is proved in Appendix A.

Minimization of the Lagrangean dual models P_0^{LD} and P_n^{LD} in the space of multipliers is performed by the subgradient method proposed by Fisher (1985).

5.2. Branching

In general at any node in the branch and bound tree, the solution of the Lagrangean dual may not satisfy the dualized non-anticipativity constraints (57) – (61), or the conditional non-anticipativity constraints in relaxed disjunction (48) inferred by decisions. In this case, new branches are generated from the current node by considering the violations in the dualized non-anticipativity constraints or the relaxed disjunction.

5.2.1. Branching on the dualized non-anticipativity constraints:

At any node of the branch-and-bound tree, the solution of model P_n^{LD} may not satisfy the dualized initial or conditional non-anticipativity constraints ($N_I \cup N_C^n$). In this case, branching is performed over the constraints in violation. The branching strategy divides the feasible region into two, where one of the regions is restricted to $\left(d_t^s, d_t^{s'}\right) \leq \left\lfloor \tilde{d}_t \right\rfloor$, and the other $\left(d_t^s, d_t^{s'}\right) \geq \left\lceil \tilde{d}_t \right\rceil$. $\tilde{d}_t$ is calculated by the probability weighted average of the two variables, $\tilde{d}_t = \dfrac{p^s d_t^s + p^{s'} d_t^{s'}}{p^s + p^{s'}}$.

A special case occurs during the branching of the first period non-anticipativity constraints. As these constraints should hold for each scenario pair, we can rewrite these equations as one expression, $d_1^{s_1} = d_1^{s_2} = \ldots = d_1^{s_k}$. The branching can then be performed so that the entire expression is restricted to $\left\lfloor \tilde{d}_t \right\rfloor$ (i.e. $\left(d_1^{s_1}, d_1^{s_2}, \ldots, d_1^{s_k}\right) \leq \left\lfloor \tilde{d}_t \right\rfloor$) or to $\left\lceil \tilde{d}_t \right\rceil$ (i.e. $\left(d_1^{s_1}, d_1^{s_2}, \ldots, d_1^{s_k}\right) \geq \left\lceil \tilde{d}_t \right\rceil$).

5.2.2. Branching on the disjunctions:

Once the dualized non-anticipativity constraints are satisfied, it is possible to bifurcate the feasible space into two using the disjunction (48) to continue searching. The branching is performed using the conditional non-anticipativity constraints in relaxed disjunction which are supposed to hold but do not do so given the values of the variables at the previous time periods. For instance, assume the decisions satisfy the initial non-anticipativity constraints which include the first period non-anticipativity constraints then indistinguishable scenario pairs are inferred by using (36) – (47). Given the indistinguishable scenario pairs, if the variable values do not satisfy the conditional non-anticipativity constraints in disjunction (48), the branching is performed on $z_t^{s,s'}$ as shown in Fig. 9.

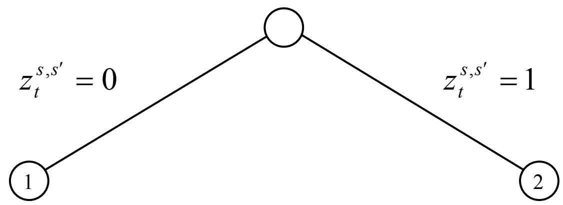

Fig. 9: Branching on the disjunction (48).

The two generated nodes in Fig. 9 (nodes 1 and 2) have different properties. At node 1 $z_t^{s,s'}$ is fixed to false (or equal to zero) and at node 2 $z_t^{s,s'}$ is fixed to true (or equal to one). At node 1 due to fixing $z_t^{s,s'}$ to false, it is required to add some cuts which will guarantee the distinguishability of the scenario pair (s,s'). These cuts will be generated using constraints (36) – (47). For example if we fix $z_t^{s,s'}$ to false, where the scenario pair (s,s') belongs to the set M_3, then logic constraint (64) and (65) can be inferred using constraint (43).

$$\bigwedge_{r\in D(s,s')} w_{4,t}^{r,s} \Leftrightarrow FALSE \tag{64}$$

$$\bigwedge_{r\in D(s,s')} w_{4,t}^{r,s'} \Leftrightarrow FALSE \tag{65}$$

Along with constraints (64) and (65), constraints (39) - (40) need to be added to the model P_n. Different from node 1, some non-anticipativity constraints coming from the relaxed disjunction are added to the set N_C^n in model P_n at node 2 due to fixing $z_t^{s,s'}$ to true by the branching cuts. Similar to node 1, several cuts are generated and added to model P_n for forcing $z_t^{s,s'}$ to be true. These cuts are necessary to make the solution algorithm convergent as will be explained in sections 5.4.

5.3. Lower bounding procedure

The lower bound at each node is found by a heuristic, based on the solution found by the upper bound. Usually, the solution found by the upper bound generation does not satisfy the non-anticipativity constraints. In this case, a feasible solution is generated using a rolling horizon approach (Dimitriadis *et al.*, 1997). Starting from the first period, the probability weighted average of variables in indistinguishable scenarios are found (similar to one performed in branching, see step 7 in the algorithm). Assuming that these variables are fixed and considering the resolution of uncertainty depending on the fixed decisions, the next period decisions are found iteratively by calculating the probability weighted average of the variables in indistinguishable scenarios. This approach continues until all scenarios are distinguishable or the end of the planning horizon is reached, whichever comes first. Then the model is solved in fullspace with these fixed decisions using outer approximation procedure. This solution yields a feasible set of decisions on the number of wells to drill, facilities to build, time to drill and build them, and the oil production for every time period.

5.4. Solution Algorithm (SP-GO)

For convenience in the presentation, generic variable names are used in the explanation of the algorithm. There are two criteria taken into consideration for selecting the generic variable names. The first one is a variable can be discrete or continuous, and the second is a variable can be a decision, state or recourse variable. The discrete decision variables are denoted by d, whereas discrete state variables denoted by y. Continuous variables are denoted by w, z or x: w if it is a decision variable; z if it is a state variable, x if it is a recourse variable. Note that there is no discrete recourse variable. Based on the above definitions, the proposed algorithm is presented below.

$\mathscr{P}$ denotes the list of open nodes each having an upper bound ϕ_n^{UB} found by the Lagrangean dual problem, while ϕ^{LB} represents the objective value of the best feasible solution obtained so far. Given these variables, the major steps of the algorithm is given below.

Step 1: *Initialization:* $\phi^{LB} = -\infty$.

Step 2: *Bound generation:* For the root node, generate upper $\left(\phi_0^{UB}\right)$ and lower bounds $\left(\phi^{LB}\right)$ with solutions $\left(\hat{d},\hat{y},\hat{w},\hat{z},\hat{x}\right)$ and $\left(\overline{d},\overline{y},\overline{w},\overline{z},\overline{x}\right)$ respectively. (Details of this step (steps a-g) will explained below.)

Step 3: *Branching:* Branch on the dualized non-anticipativity constraints or disjunctions that are violated by the solution $\left(\hat{d}, \hat{y}, \hat{w}, \hat{z}, \hat{x}\right)$ of the relaxed problem P_0 and generate two children nodes.

Step 4: *Bound generation:* For each child node n, generate upper $\left(\phi_n^{UB}\right)$ and lower bounds $\left(\phi^{LB}\right)$ with solutions $\left(\hat{d}, \hat{y}, \hat{w}, \hat{z}, \hat{x}\right)$ and $\left(\overline{d}, \overline{y}, \overline{w}, \overline{z}, \overline{x}\right)$ respectively. (Details of this step for each child node are similar to Step 2)

Step 5: *Termination:* If $\mathscr{P} = \emptyset$, stop. The current best solution is optimal. Otherwise, repeat steps 6 to 8.

Step 6: *Node selection:* Select and delete node n from $\mathscr{P}$ based on the best bound.

Step 7: *Branching:* Branch on the dualized non-anticipativity constraints or disjunctions that are violated by the solution $\left(\hat{d}, \hat{y}, \hat{w}, \hat{z}, \hat{x}\right)$ of the relaxed problem P_n. Generate two children nodes, add them to $\mathscr{P}$.

Step 8: *Bound generation:* For each child node n, generate upper $\left(\phi_n^{UB}\right)$ and lower bounds $\left(\phi^{LB}\right)$ with solutions $\left(\hat{d}, \hat{y}, \hat{w}, \hat{z}, \hat{x}\right)$ and $\left(\overline{d}, \overline{y}, \overline{w}, \overline{z}, \overline{x}\right)$ respectively. (Details of this step for each child node are similar to Step 2)

Note that steps 1 to 4 are a part of initialization where the bounds for the root node and two children nodes are found. The algorithm iterates between steps 5 to 8 during the rest of the algorithm. Steps 2, 4 and 8 share the same procedure for finding bounds. The steps of this common procedure (steps a to g) is explained below.

Step a: *Set iteration* $i = 0$.

Step b: While i is less than or equal to $max\_iteration$, $i = i + 1$, repeat steps b through g.

Step c: *Generate upper bound:* For fixed multipliers, use global optimizer for each subproblem to solve P_n^{LR} to obtain solution $\left(\hat{d}, \hat{y}, \hat{w}, \hat{z}, \hat{x}\right)$ with objective function value $\hat{\phi}$.

Step d: *Update upper bound:* Update the upper bound by $\phi^{UB} = \min\left\{\phi^{UB}, \hat{\phi}\right\}$.

Step e: *Generate lower bound:* Find a heuristic solution based on $\left(\hat{d},\hat{y},\hat{w},\hat{z},\hat{x}\right)$ to generate a feasible solution $\left(\overline{d},\overline{y},\overline{w},\overline{z},\overline{x}\right)$ with objective value $\overline{\phi}$.

Step f: *Update lower bound:* Update the lower bound by $\phi^{LB} = \max\left\{\phi^{LB},\overline{\phi}\right\}$. Delete from $\mathscr{P}$ all problems P' with $\hat{\phi}(P') \leq \phi^{LB}$.

Step g: *Update multipliers:* Update multipliers using subgradient method.

6. Results

In this section, we present results for specific example problems to show the effectiveness of the proposed solution algorithm, SP-GO (For generic problem statement, see section 2). In the first example (Section 6.1), planning decisions are optimized for a single reservoir for 10 years where uncertainty is represented using 8 different scenarios. The oil flowrate from a single well in the reservoir is assumed to be a given by a linear function of cumulative oil production from the same reservoir. In the second example problem (Section 6.2), different from the example 1, the oil flowrate from a single well in the reservoir is approximated using a nonlinear function of the cumulative production from the reservoir. Note that in both examples the reservoir model is nonlinear due to the nonlinearity caused by water-to-oil-ratio and water rate. All the results have been obtained on a Pentium-IV, 3.20 GHz Windows machine. Also, we employed AIMMS 3.8 for implementing the solution algorithm using solvers Cplex 10.1 for MILP, Conopt 3.14 and Snopt 6.1 for NLP, Baron 7.5.3 for global optimization of MINLP, AOA (Aimms Outer Approximation Module) for local optimization of MINLP.

6.1. Example 1

In this example, our aim is to optimize the infrastructure planning decisions for an offshore oil field having a single reservoir for 10 years. The planning decisions are the number, capacity and installation schedule of FPSO/TLP facilities; the number and drilling schedule of sub-sea/TLP wells; and the oil production profile over time. There are certain specifications about the actions that can be taken during the planning process. Sub-sea and TLP wells are drilled in groups of three. Because of the limitations on the number of drilling rigs, the maximum number of sub-sea wells that can be drilled each year is 12. Also, each TLP facility can drill at most 6 TLP wells each year. A maximum of 30 TLP wells can be connected to each TLP facility. The construction time delays for different types of wells and facilities are given in Table 2.

Table 2: Construction lead time (years)

Wells		Facilities		
TLP	**Sub-sea**	**TLP**	**Small FPSO**	**Large FPSO**
1	1	1	2	4

The uncertainties in initial maximum oil flowrate, the size of the reservoirs and water breakthrough time (see section 2 for the correlation between water breakthrough time and parameter α) are represented by discrete distributions consisting of two values (high and low). These uncertainties are incorporated into the model using eight different scenarios. Each scenario is a unique combination of the possible values of uncertain parameters. Possible values for each uncertain parameter high and low values of parameters, (see Table 3). In example 1, each scenario is assumed to be equally likely.

Table 3: Representation of scenarios using uncertain parameters

	Scenarios							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Initial Productivity per well (kbd)	10	10	20	20	10	10	20	20
Reservoir Size (Mbbl)	300	300	300	300	1500	1500	1500	1500
Water Breakthrough Time	5	2	5	2	5	2	5	2

We assume a linear relation between the maximum oil flowrate and the cumulative oil production from the reservoir as shown in Fig. 10. Therefore, combination of two different values (low and high) of initial productivities and two different values (low and high) of reservoir sizes gives four different lines representing maximum oil production flowrate. For each maximum oil flowrate line there can be two possible curves for water flowrate depending on the value of water breakthrough parameter. This leads to eight different water flowrate curves each of which corresponds to a scenario as shown in Fig. 11.

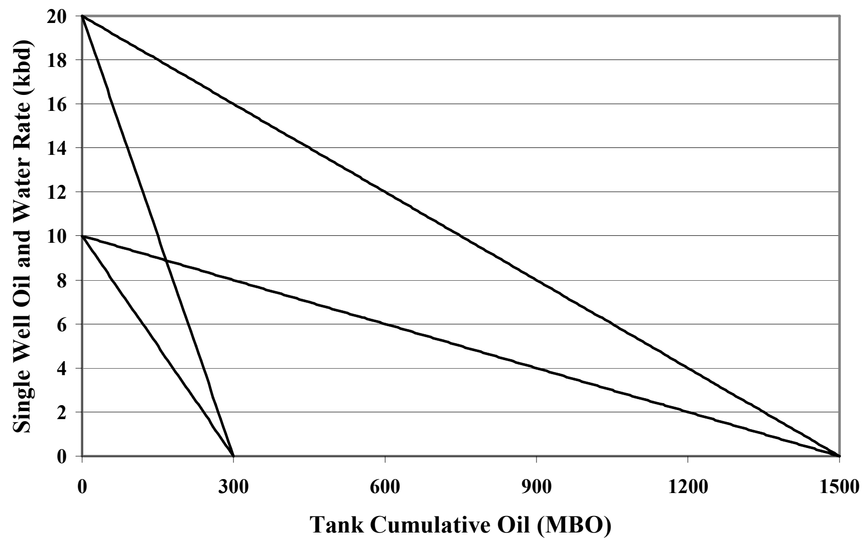

Fig. 10: Maximum oil flowrate behavior under uncertainties in initial productivities and reservoir sizes

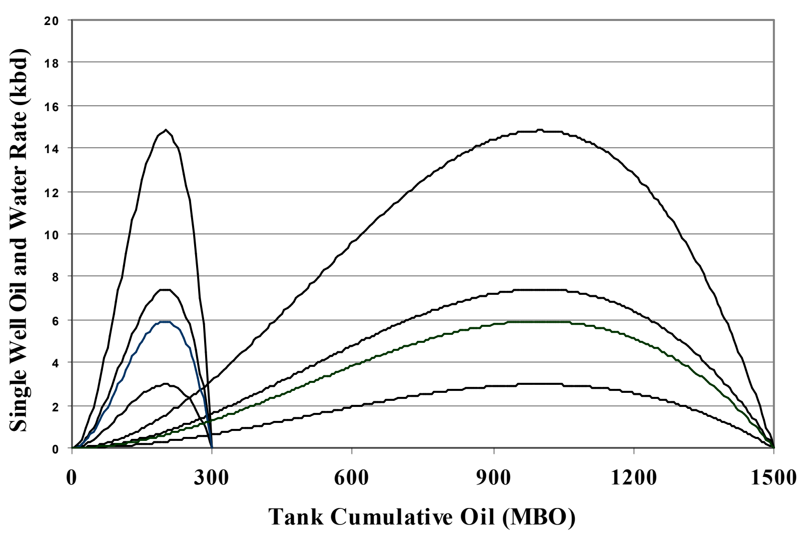

Fig. 11: Water flowrates for eight different scenarios

The specific values used for describing the uncertainty resolution are given as follows. The appraisal program is completed when a total of three wells are drilled $(N_1 = 3)$ in one reservoir. As explained before, this appraisal program not only gives the actual value for the initial maximum oil flowrate, but also provides the posterior probabilities of reservoir sizes depending on the outcome. The uncertainty in reservoir size can be resolved if either a total of nine $(N_2 = 9)$ or more wells are drilled, or production is made from that reservoir for a duration of one year $(N_3 = 1)$. Uncertainty in water breakthrough time is resolved after one year of production from the reservoir $(N_4 = 1)$.

In Table 4, the comparison of model statistics is given for full space model and individual scenarios for 10 time periods. We should note that the size of the full space model increases exponentially as a result of the increase in the number of binary variables for representing uncertainty resolution and the non-anticipativity constraints that relate the decisions in indistinguishable scenarios.

Table 4: Model size for the example problem.

	Individual Scenario	**Full Space Model**
Integer Variables	100	800
Binary Variables	10	800
Continuous Variables	121	869
Constraints	271	8088

A simplified approach for finding a feasible solution to this planning problem is to use expected values, in which all discrete decisions for the entire planning horizon are first optimized using one point estimates (mean, most-likely, etc.) of the uncertain parameters. Then starting from the first time period, discrete decisions are fixed successively, resolution of uncertainties are observed and the rest of the planning horizon is re-optimized throughout the planning horizon. The expected value solution (see Table 5) proposes building 5 small FPSO, 2 TLP facilities and drilling 9 sub-sea wells in the first year. These decisions resolve the uncertainty in initial productivity and reservoir size. Depending on the values of reservoir size and initial productivity, different decisions are implemented. This expected value approach gives an objective function value of \$5.81 x 10^9.

Table 5: Expected value solution found for example 1.

<table>
<tr><th></th><th colspan="8">Scenario</th></tr>
<tr><th>Year</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th></tr>
<tr><td>1</td><td colspan="8"><u>Build</u>
5 small FPSO
2 TLP
<u>Drill</u>
9 sub-sea well</td></tr>
<tr><td>2</td><td colspan="2"><u>Drill</u>
12 sub-sea well
12 TLP well</td><td colspan="2"><u>Drill</u>
3 sub-sea well
12 TLP well</td><td colspan="2"><u>Build</u>
2 small FPSO
3 TLP
<u>Drill</u>
12 sub-sea well
12 TLP well</td><td colspan="2"><u>Build</u>
2 small FPSO
<u>Drill</u>
6 sub-sea well
12 TLP well</td></tr>
<tr><td>3</td><td colspan="2"><u>Drill</u>
12 TLP well</td><td colspan="2"></td><td colspan="2"><u>Build</u>
1 small FPSO
<u>Drill</u>
12 sub-sea well
30 TLP well</td><td colspan="2"><u>Drill</u>
12 TLP well</td></tr>
<tr><td>4</td><td></td><td></td><td></td><td></td><td><u>Drill</u>
27 TLP well</td><td><u>Drill</u>
27 TLP well</td><td><u>Build</u>
2 small FPSO
<u>Drill</u>
12 TLP well</td><td><u>Drill</u>
9 TLP well</td></tr>
<tr><td>5</td><td></td><td></td><td></td><td></td><td><u>Drill</u>
12 TLP well</td><td><u>Drill</u>
30 TLP well</td><td><u>Drill</u>
12 TLP well</td><td><u>Drill</u>
12 TLP well</td></tr>
<tr><td>6</td><td></td><td></td><td></td><td></td><td></td><td><u>Drill</u>
30 TLP well</td><td></td><td><u>Build</u>
1 TLP
<u>Drill</u>
12 TLP well</td></tr>
<tr><td>7</td><td></td><td></td><td></td><td></td><td></td><td><u>Drill</u>
21 TLP well</td><td></td><td><u>Drill</u>
15 TLP well</td></tr>
<tr><td>8</td><td></td><td></td><td></td><td></td><td><u>Drill</u>
3 TLP well</td><td></td><td><u>Drill</u>
3 TLP well</td><td><u>Drill</u>
18 TLP well</td></tr>
<tr><td>9</td><td></td><td></td><td></td><td></td><td></td><td></td><td><u>Drill</u>
9 TLP well</td><td></td></tr>
<tr><td>10</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>NPV ($ x 10^9)</td><td>0.13</td><td>0.13</td><td>0.83</td><td>0.83</td><td>9.36</td><td>10.46</td><td>11.60</td><td>13.11</td></tr>
<tr><td>ENPV ($ x 10^9)</td><td colspan="8">5.81</td></tr>
</table>

As seen in Table 6, the optimal stochastic programming solution yields an expected net present value of \$6.37 x 10^9, which is higher than the expected value solution (\$5.81 x 10^9). The solution proposes building 2 small FPSO, 1 TLP facilities and drilling 9 sub-sea wells in the first year. Uncertainty in initial oil flowrate and reservoir size resolves after drilling 9 wells. For scenarios (5-6) the solution proposes building 4 more small FPSO facilities, 1 large FPSO, 5 TLP facilities, drilling 12 sub-sea and 3 TLP wells. For scenarios (7-8), the solution proposes building 6 more small FPSO facilities, 1 TLP facilities, drilling 6 sub-sea wells.

For solving each subproblem globally in a reasonable timeframe, we customized the options for solving different scenarios depending on the time spent at different parts of the solver BARON (i.e. preprocessing, local search, bounds tightening, marginals testing, probing). This leads to savings up to 85% in solution time compared to the default settings.

Table 6: Stochastic programming solution found for example 1.

<table>
<tr><th></th><th colspan="8">Scenario</th></tr>
<tr><th>Year</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th></tr>
<tr><td>1</td><td colspan="8">Build
2 small FPSO
1 TLP
Drill
9 sub-sea well</td></tr>
<tr><td>2</td><td colspan="2">Drill
6 TLP well
6 sub-sea well</td><td colspan="2">Drill
3 TLP well</td><td colspan="2">Build
4 small FPSO
1 large FPSO
5 TLP
Drill
3 TLP well
12 sub-sea well</td><td colspan="2">Build
6 small FPSO
1 TLP
Drill
6 sub-sea well</td></tr>
<tr><td>3</td><td colspan="2">Drill
6 TLP well</td><td colspan="2">Drill
3 TLP well</td><td colspan="2">Drill
12 sub-sea well
30 TLP well</td><td colspan="2">Drill
12 sub-sea well
12 TLP well</td></tr>
<tr><td>4</td><td>Drill
6 TLP well</td><td>Drill
6 TLP well</td><td></td><td>Drill
3 TLP well</td><td>Drill
18 TLP well</td><td>Build
1 small FPSO
Drill
18 TLP well</td><td>Drill
12 TLP well</td><td>Drill
12 TLP well</td></tr>
<tr><td>5</td><td></td><td>Drill
6 TLP well</td><td></td><td>Drill
6 TLP well</td><td>Drill
36 TLP well</td><td>Drill
36 TLP well</td><td>Drill
12 TLP well</td><td>Build
1 TLP
Drill
12 TLP well</td></tr>
<tr><td>6</td><td></td><td></td><td></td><td></td><td></td><td>Drill
36 TLP well</td><td></td><td>Drill
3 sub-sea well
18 TLP well</td></tr>
<tr><td>7</td><td>Drill
3 TLP well</td><td></td><td>Drill
3 TLP well</td><td></td><td></td><td>Drill
36 TLP well</td><td></td><td>Drill
18 TLP well</td></tr>
<tr><td>8</td><td></td><td></td><td>Drill
3 TLP well</td><td></td><td>Drill
3 TLP well</td><td></td><td>Drill
3 TLP well</td><td>Drill
18 TLP well</td></tr>
<tr><td>9</td><td></td><td></td><td></td><td></td><td></td><td></td><td>Drill
12 TLP well</td><td></td></tr>
<tr><td>10</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>NPV ($ x 10^9)</td><td>2.05</td><td>2.15</td><td>2.41</td><td>2.60</td><td>8.69</td><td>9.67</td><td>10.98</td><td>12.40</td></tr>
<tr><td>ENPV ($ x 10^9)</td><td colspan="8">6.37</td></tr>
</table>

Fig. 12 shows the branch and bound tree generated by the proposed algorithm which found the optimal solution with an expected net present value of $\$6.37\times10^9$. The root node is denoted by 0 and the rest of the nodes are numbered according to the order of exploration. At the root node, the Lagrangean problem yields an upper bound of $\$6.49\times10^9$ after 5 iterations of subgradient optimization and the heuristic for finding a feasible solution yields a lower bound of $\$6.33\times10^9$. In the optimal solution of the Lagrangean problem at the root node, the highest violation in the initial non-anticipativity constraints are drilling group of sub-sea wells. Therefore, the feasible region is separated into two which are represented by Nodes 1 and 2.

The best feasible solution proposed by the algorithm is guaranteed to be within 9% optimality gap, and the proposed branch and bound algorithm required 23 hours since at each node 40 MINLP problems were solved to global optimality. A total of 7 nodes have been traversed and the best feasible solution was found at Node 5. As mentioned before, in this instance the modified options of the solver BARON (Sahinidis, 2000) have been used for solving each nonconvex MINLP.

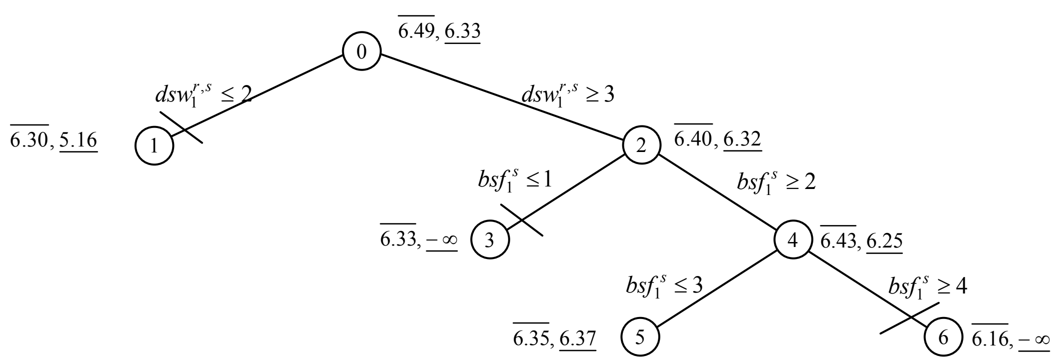

Fig. 12: Resulting Branch and bound tree for example 1.

Fig. 13 compares the net present values of the expected value solution (columns in pattern) and the stochastic programming solution (columns in black) over the 8 scenarios in the 10 year period. The added value of stochastic programming is due to the conservative initial investment strategy compared to the expected value solution strategy. The stochastic programming approach considers all 8 scenarios before making the initial investment, therefore, it proposes building 2 small FPSO facilities instead of 5 and 1 TLP facility instead of 2. Also, it builds more facilities and drills wells only after it finds out that the reservoir size is 1500 Mbbl. The comparison of net present values of the expected value solution and stochastic programming shows the added value of stochastic programming comes from handling the downside risk much better than the expected value solution.

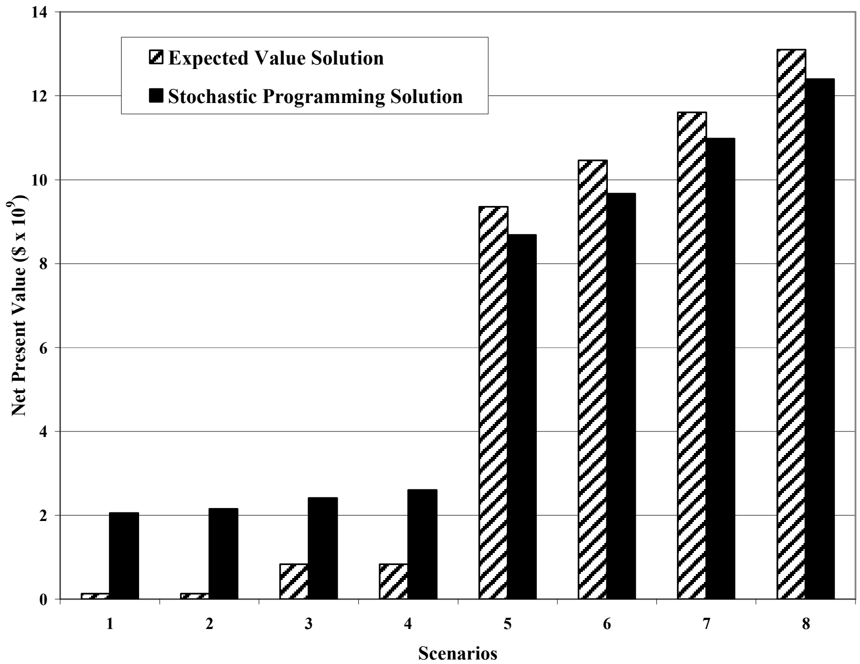

Fig. 13: Comparison of Net Present Values

Table 7 compares minimum, maximum and average capital investment decisions made by expected value and stochastic programming approaches over all scenarios. The variation in the amount of capital investment is higher in the stochastic programming approach because a smaller investment is made in unfavorable scenarios, whereas larger investments are made in favorable scenarios. Furthermore, the average investment proposed by stochastic programming is less than the one from the expected value solution. Therefore, stochastic programming solution proposes smaller investments and creates higher expected net present value.

Table 7: Comparison of Capital Investments

		Number of units in solutions					
		Expected value approach			**Stochastic programming Approach (GO)**		
	Cost per unit ($ x 10^6)	**Min**	**Average**	**Max**	**Min**	**Average**	**Max**
TLP facilities	250	2	2.8	5	1	2.6	6
Small FPSO Facilities	700	5	6.5	8	2	4.6	6
Large FPSO facilities	1200	0	0	0	0	0.25	1
TLP wells	20	12	57	150	12	57.7	159
Sub-sea wells	30	12	20.3	33	9	20.6	33
Total Capital Expenditure ($ x 10^9)		4.6	7	10.8	2.16	5.9	11.07

6.2. Example 2

In this example problem, the maximum oil flowrate from a single well in the reservoir is approximated using a nonlinear function of the cumulative production from the reservoir. The basic idea is to see the sensitivity of the solution over this oil flowrate curve.

$$\gamma_1^{r,s} = \frac{\gamma_3^{r,s}}{\left(REC^{r,s}\right)^2} \frac{1-\lambda-\mu}{\lambda(1-\lambda)} \qquad \forall s, \forall r \qquad (66)$$

$$\gamma_2^{r,s} = \frac{\gamma_3^{r,s}}{REC^{r,s}} \frac{\lambda^2+\mu-1}{\lambda(1-\lambda)} \qquad \forall s, \forall r \qquad (67)$$

$\gamma_1^{r,s}$, $\gamma_2^{r,s}$, $\gamma_3^{r,s}$ are as given in eq. (16), λ and μ are parameters specifying the third point ($\lambda \cdot REC^{r,s}$, $\mu \cdot \gamma_3^{r,s}$) that the oil flow rate passes besides two prespecified points (0, $\gamma_3^{r,s}$) and ($REC^{r,s}$,0). It is also more realistic to assume a convex curve such that the oil flowrate decreases faster initially. This necessitates the third point satisfying $\lambda + \mu < 1$. For $\lambda = 0.5$ and $\mu = 0.25$, the maximum oil flowrate from a single well in a reservoir for different scenarios are shown in Fig. 14.

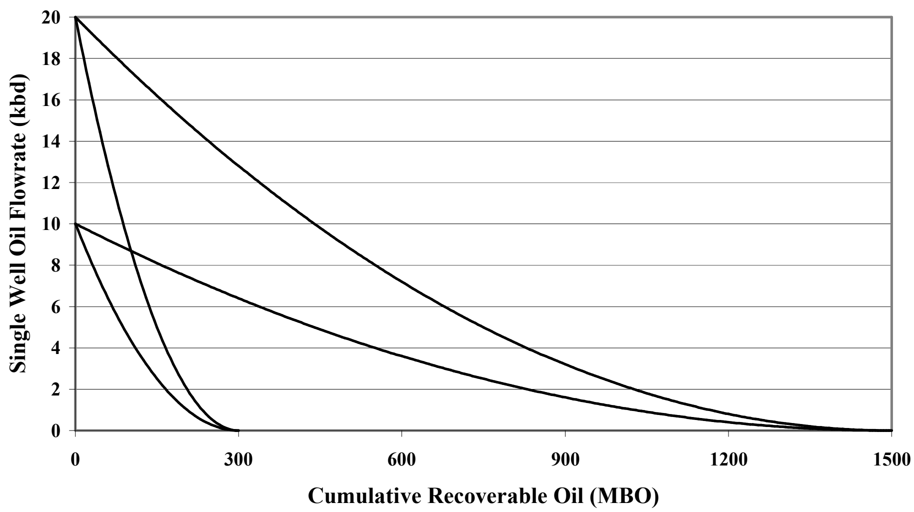

Fig. 14: Maximum oil flowrate behavior under uncertainties in initial productivities and reservoir sizes

Given the eight scenarios over the 10 year horizon, the problem data used in example 1 and specifications for nonlinear oil profile, the expected value solution (see Table 8) proposes building 4 small FPSOs, 3 TLP in the first year and start drilling 12 TLP wells in year 2. These decisions resolve the uncertainty in initial productivity and size in year 3. The production starts at year 3 and after all the scenarios become distinguishable at year 4, different decisions are implemented for the rest of the time horizon to maximize the net present value. This expected value approach gives an objective function value of \$3.76 x 10^9.

Table 8: Expected value solution found for example 2.

	Scenario							
Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	Build 4 small FPSO 3 TLP							
2	Drill 12 TLP well							
3	Drill 18 TLP well		Drill 18 TLP well		Drill 12 sub-sea well 18 TLP well		Build 3 small FPSO Drill 15 TLP well	
4	Drill 18 TLP well	Drill 18 TLP well	Drill 9 TLP well	Drill 9 TLP well	Build 1 small FPSO Drill 18 TLP well	Drill 18 TLP well	Drill 18 TLP well 3 sub-sea well	Build 1 TLP Drill 18 TLP well 3 sub-sea well
5					Drill 18 TLP well	Build 1 TLP Drill 18 TLP well	Drill 18 TLP well	Drill 24 TLP well
6					Drill 18 TLP well	Drill 21 TLP well	Drill 18 TLP well	Drill 9 sub-sea well 21 TLP well
7					Build 1 TLP Drill 18 TLP well	Drill 6 sub-sea well 24 TLP well	Build 1 TLP Drill 15 TLP well	Drill 24 TLP well
8					Drill 12 TLP well		Drill 9 TLP well	
9-10								
NPV ($\$ \times 10^9$)	-0.40	-0.40	0.41	0.41	6.25	6.30	8.64	8.91
ENPV ($\$ \times 10^9$)	3.76							

As seen in Table 9, the optimal stochastic programming solution yields an expected net present value of $\$4.59 \times 10^9$ versus the $\$3.76 \times 10^9$ from the expected value solution. The solution proposes building 2 small FPSO, 1 TLP facilities and drilling 9 sub-sea wells initially. Drilling these 9 wells will resolve the uncertainty in initial oil flowrate and reservoir size. For scenarios (5-6) the solution proposes building 4 small FPSO facilities, 5 TLP facilities and drilling 12 sub-sea wells. For scenarios (7-8), the solution proposes building 4 small FPSO facilities, 3 TLP facilities and drilling 6 TLP and 6 sub-sea wells.

Table 9: Stochastic programming solution found for example 2.

Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	Build 2 small FPSO 1 TLP Drill 9 sub-sea well							
2			Drill 3 TLP well		Build 4 small FPSO 5 TLP Drill 12 sub-sea well		Build 4 small FPSO 3 TLP Drill 6 TLP well 6 sub-sea well	
3	Drill		Drill 6 TLP well		Drill 12 sub-sea well 27 TLP well		Drill 6 sub-sea well 18 TLP well	
4	Drill 12 sub-sea well 6 TLP well	Drill 12 sub-sea well 6 TLP well	Drill 6 TLP well	Drill 3 sub-sea well 6 TLP well	Drill 24 TLP well	Drill 30 TLP well	Build 1 small FPSO Drill 3 TLP well	Drill 9 TLP well
5	Drill 6 TLP well	Drill 6 TLP well	Drill 6 TLP well	Drill 6 TLP well	Drill 30 TLP well	Drill 36 TLP well	Drill 24 TLP well	Drill 24 TLP well
6	Drill 6 TLP well	Drill 6 TLP well			Drill 24 TLP well	Drill 36 TLP well	Drill 15 TLP well	Drill 3 sub-sea well 24 TLP well
7					Drill 21 TLP well	Drill 36 TLP well	Drill 18 TLP well	Drill 24 TLP well
8							Drill 3 TLP well	Drill 15 TLP well
9-10								
NPV ($ x 10^9)	0.83	0.83	1.86	1.87	6.69	6.89	8.56	9.17
ENPV ($ x 10^9)	4.59							

The best feasible solution proposed by the algorithm after 120 hours is guaranteed to be within 12% optimality gap. The reason for such a gap is each subproblem is solved with 10% gap, therefore in the worst case the upper bound generated will be 10% better. A total of 14 nodes have been traversed and the best feasible solution was found after 90 hours at Node 6. Notice that in this instance some of the default options of the solver BARON (i.e. preprocessing, local search, bounds tightening, marginals testing, probing) have been modified for solving each scenario subproblem. This leads to one order of magnitude reduction in solution time compared to the default options.

Fig. 15 compares the net present values of the expected value solution (columns in pattern) and the stochastic programming solution (columns in black) over the 8 scenarios in the 10 year horizon. Similar to the example 1, the added value of stochastic programming is due to the conservative initial investment strategy compared to the expected value solution strategy. Comparing the result of example 2 with example 1 shows that the improvement by stochastic programming on the expected value solution is higher (22% vs. 10%) when the conditions become harder to extract oil from the reservoir. Note that in the nonlinear case it becomes harder to extract oil as the decrease in oil flowrate is much faster compared to the linear case.

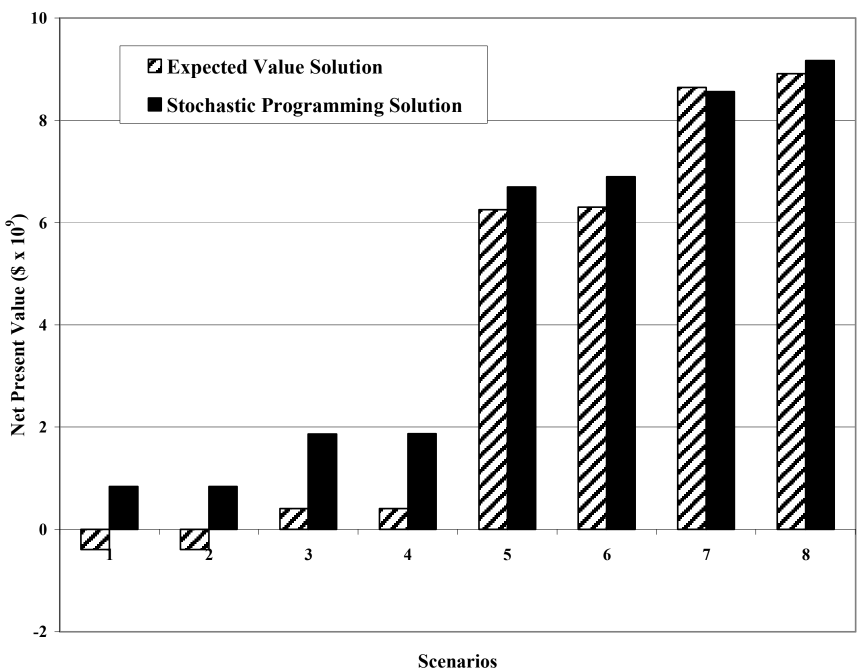

Fig. 15: Comparison of Net Present Values

Table 10 compares capital investment decisions made by the expected value and the stochastic programming approaches for example 2. Although the average capital investment is close in the two approaches, the stochastic programming approach gives lower minimum and higher maximum investments. This smaller investment is due to the conservative initial investment strategy proposed by the stochastic programming approach. Also, stochastic programming proposes higher investments only when the uncertainty parameters is found to be favorable.

Comparing the solution quality of the two examples, the stochastic programming solution is more clearly robust with respect to the uncertainty in the shape of the curve representing maximum oil flowrate versus cumulative production from the reservoir. The first year decisions proposed by stochastic programming is the same in the two examples, whereas the expected value solution decisions change. Stochastic programming is not only generating solutions with higher expected net present values, but also more robust solutions with respect to the uncertainties that were not included explicitly in the uncertainty space (e.g. uncertainty in shape of maximum oil flowrate). Although this result was not included in the objective explicitly, it supports the advantage of using stochastic programming.

Table 10: Comparison of Capital Investments

		Number of units in solutions					
		Expected value approach			**Stochastic programming Approach (GO)**		
	Cost per unit ($ x 10^6)	**Min**	**Average**	**Max**	**Min**	**Average**	**Max**
TLP facilities	250	3	3.50	4	1	3	6
Small FPSO Facilities	700	4	5.00	5	2	4.1	6
Large FPSO facilities	1200	0	0.00	0	0	0	0
TLP wells	20	39	77.25	111	21	72	165
Sub-sea wells	30	0	5.63	18	9	21.4	33
Total Capital Expenditure ($ x 10^9)		4.33	6.08	7.26	2.34	5.72	9.99

7. Conclusion

We have presented in this paper a multistage stochastic programming model for planning of offshore oil field infrastructure under uncertainty where the uncertainties, the initial maximum oil flowrate, size and water breakthrough time of the reservoir, reduce gradually as a function of design and operating decisions. The proposed model considers possible investment strategies for reducing uncertainties before making the major facility investments. The proposed model is a disjunctive/mixed-integer nonlinear programming model that is converted to an MINLP using big-M type of transformations. We have proposed a duality-based branch and bound algorithm that enables solving the scenario subproblems independently. Feasible solutions are generated using the infeasible solutions coming from individual scenarios.

Implementation issues and results for two examples were presented to illustrate the application of the proposed method (SP-GO). In examples 1 and 2, the proposed solution algorithm (SP-GO) found solutions that are nearly 10 and 22% better than the solution found by the expected value approach, respectively. The reason is that stochastic programming incorporates the uncertainty and value of information analysis directly into the mathematical model, and proposes decisions which hedge against possible outcomes of uncertain parameters. The solution times in both examples are rather long. In the forthcoming paper we address how to improve the computational efficiency (Tarhan and Grossmann, 2008b).

Acknowledgments

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Nomenclature

In this section the sets, variables and parameters for infrastructure planning model is presented.

Sets and indices:

S	: Set of scenarios
s	: Scenario $s \in S$
R	: Set of reservoirs
r	: Reservoir $r \in R$
T	: Planning horizon
t, τ	: Time periods $t \in T$
$D(s, s')$	: Set of reservoirs that differentiate scenarios s and s'
M_1	: Set of scenario pairs (s, s') which differs only in initial productivities.
M_2	: Set of scenario pairs (s, s') which differs only in reservoir sizes.
M_3	: Set of scenario pairs (s, s') which differs only in water breakthrough times.
M_4	: Set of scenario pairs (s, s') which differs only in initial productivities and reservoir sizes.
M_5	: Set of scenario pairs (s, s') which differs only in initial productivities and water breakthrough times.
M_6	: Set of scenario pairs (s, s') which differs only in reservoir sizes and water breakthrough times.
M_7	: Set of scenario pairs (s, s') which differs in initial productivities, reservoir sizes and water breakthrough times.
N_I	: Set of initial non-anticipativity constraints.
N_C^n	: Set of conditional non-anticipativity constraints at node n.

Continuous variables:

$oil_t^{r,s}$	: Oil flowrate from one well in reservoir r, in time period t, scenario s
$oil_t^{cum,r,s}$	: Cumulative amount of oil extracted from reservoir r until the end of time period t, scenario s

$oil_t^{tlp,r,s}$: Amount of oil extracted from reservoir r and pumped to TLP facilities, in time period t, scenario s

$oil_t^{fpso,r,s}$: Amount of oil extracted from reservoir r and pumped to FPSO facilities in time period t, scenario s

$water_t^{tlp,r,s}$: Amount of water extracted from reservoir r and pumped to TLP facilities in time period t, scenario s

$water_t^{fpso,r,s}$: Amount of water extracted from reservoir r and pumped to FPSO facilities in time period t, scenario s

$prod_t^{r,s}$: Amount of oil produced from reservoir r in time period t, scenario s

$prod_t^{total,s}$: Total amount of oil produced in time period t, scenario s

rev_t^s : Revenue coming from the sale of produced oil in time period t, and in scenario s

$cost_t^s$: Total of capital and operating expenditures in time period t, and in scenario s

$capex_t^s$: Capital expenditures in time period t, and in scenario s

$opex_t^s$: Operating expenditures in time period t, and in scenario s

npv^s : Net present value of the project in scenario s

$enpv$: Expected net present value of the project

Integer variables:

bt_t^s : Number of TLP facilities built in time period t, scenario s

bsf_t^s : Number of small FPSO facilities built in time period t, scenario s

blf_t^s : Number of large FPSO facilities built in time period t, scenario s

$dtw_t^{r,s}$: Number of TLP wells drilled in reservoir r, in time period t, and in scenario s. It takes a value multiple of G^{tw}.

$dsw_t^{r,s}$: Number of sub-sea wells drilled in reservoir r, in time period t, and in scenario s. It takes a value multiple of G^{sw}.

nt_t^s : Number of TLP facilities available in time period t, scenario s

nsf_t^s : Number of small FPSO facilities available in time period t, scenario s

nlf_t^s : Number of large FPSO facilities available in time period t, scenario s

$ntw_t^{r,s}$: Number of TLP wells available (ready to operate) in reservoir r in time period t, scenario s. It takes a value multiple of G^{tw}.

$nsw_t^{r,s}$: Number of sub-sea wells available (ready to operate) in reservoir r in time period t, scenario s. It takes a value multiple of G^{sw}.

Binary/Logic variables:

$bw_t^{r,s}$: Whether or not there is a well ready for extracting oil from reservoir r, in time period t, and in scenario s

$bp_t^{r,s}$: Whether or not there is production from reservoir r, in time period t, and in scenario s

$w_{1,t}^{r,s}$: True if and only if the number of wells drilled in reservoir r, until time period t in scenario s is less than N_1

$w_{2,t}^{r,s}$: True if and only if the number of wells drilled in reservoir r, until time period t in scenario s is less than N_2

$w_{3,t}^{r,s}$: True if the number of years of production is less than the amount of time needed to resolve uncertainty in reservoir size (i.e. N_3) at beginning of time period t in scenario s

$w_{4,t}^{r,s}$: True if the number of years of production is less than amount of time needed to resolve uncertainty in water breakthrough time (i.e. N_4) at beginning of time period t in scenario s

$z_t^{s,s'}$: True if scenario s and s' are indistinguishable at the beginning of time period t

Parameters:

$C_{1,t}$: Price of oil at time period t

$C_{2,t}$: Capital cost of drilling a TLP well connecting it to a TLP facility at time period t

$C_{3,t}$: Capital cost of drilling a sub-sea well at time period t

$C_{4,t}$: Capital cost of building a TLP facility at time period t

$C_{5,t}$: Capital cost of building a small FPSO at time period t

$C_{6,t}$: Capital cost of building a large FPSO at time period t

$C_{7,t}$: Cost of production per unit of product at time period t

D_t : Discounting factor at time period t

$REC^{r,s}$: The amount of recoverable oil in reservoir r, in scenario s

N_w^{max} : Maximum number of wells to be connected to a TLP facility

G^{tw} : Number of TLP wells to be drilled together as one group

G^{sw} : Number of sub-sea wells to be drilled together as one group

U^{tw} : Upper bound on the number of TLP wells to drill in one time period per TLP facility

U^{sw} : Upper bound on the number of sub-sea wells to drill in one time period

$U^{w,r}$: Upper bound on the number of wells to drill in reservoir r

U^{t} : Upper bound on the number of TLP facilities to build

U^{sf} : Upper bound on the number of small FPSO facilities to build

U^{lf} : Upper bound on the number of large FPSO facilities to build

CAP_{sf}^{oil} : Maximum capacity of small FPSO for oil

CAP_{lf}^{oil} : Maximum capacity of large FPSO for oil

CAP_{sf}^{liq} : Maximum capacity of small FPSO for liquid (oil and water)

CAP_{lf}^{liq} : Maximum capacity of large FPSO for liquid (oil and water)

N_1 : Number of wells needed to complete appraisal program

N_2 : Total number of wells needed to be drilled to resolve uncertainty in reservoir size.

N_3 : Number of years of production needed to resolve uncertainty in reservoir size.

N_4 : Number of years of production needed to resolve uncertainty in water breakthrough time.

P^s : Probability of scenario s

$\gamma_1^{r,s}$: Quadratic term coefficient of the equation representing oil flowrate from a well in reservoir r, in scenario s

$\gamma_2^{r,s}$: Linear term coefficient of the equation representing oil flowrate from a well in reservoir r, in scenario s

$\gamma_3^{r,s}$: Constant coefficient of the equation representing oil flowrate from a well in reservoir r, in scenario s. Also, represents initial oil flowrate from a well in reservoir r, in scenario s

δ_t : Operating days per period (Usually assumed to be one year)

$\tau_{(\,)}$: Time delays

Appendix A: On the validity of the upper bound from Lagrangean duality

A stochastic program can be represented by eqs. (A1) - (A4), where the objective function is the sum of convex or nonconvex functions over all scenarios S. The constraint set (A2) represents a nonlinear, nonconvex feasible space for scenario s. The constraint set (A3) represents non-anticipativity constraints linking decisions in different scenarios.

$$\max_{x_s} \quad \sum_{s\in S} f_s(x_s) \tag{A1}$$

$$g_s(x_s) \le 0 \qquad \forall s \in S \tag{A2}$$

$$x_s - x_{s'} = 0 \qquad \forall (s,s') \in N \tag{A3}$$

$$x_s \in R^n \qquad \forall s \in S \tag{A4}$$

Variables for different scenarios are linked through the non-anticipativity constraints (A3) and if they are dualized, the model can be decomposed into independent subproblems for each scenario s.

Proposition 1

If the non-anticipativity constraints (A3) are dualized, the Lagrangean dual function is convex in the space of the multipliers.

Proof

Dualizing the non-anticipativity constraints (A3) yields.

$$\max_{x_s} \quad \sum_{s\in S} f_s(x_s) + \sum_{(s,s')\in N} \lambda_{s,s'}(x_s - x_{s'}) \tag{A5}$$

$$g_s(x_s) \le 0 \qquad \forall s \in S \tag{A6}$$

Then the model can be written as the sum of independent subproblems.

$$\max_{x_s} \quad \sum_{s\in S} \left(f_s(x_s) + x_s \left(\sum_{s<s'} \lambda_{s,s'} - \sum_{s'<s} \lambda_{s',s} \right)\right) \tag{A7}$$

$$g_s(x_s) \le 0 \qquad \forall s \in S \tag{A8}$$

For the sake of simplicity the term $\sum_{s<s'} \lambda_{s,s'} - \sum_{s'<s} \lambda_{s',s}$ in objective function can be replaced with $h_s(\lambda)$. Then, the model becomes,

$$\max_{x_s} \quad \sum_{s \in S} \left(f_s(x_s) + x_s h_s(\lambda) \right) \tag{A9}$$

$$g_s(x_s) \leq 0 \qquad \forall s \in S \tag{A10}$$

The Lagrangean dual can be written as,

$$\min_{\lambda} \quad \max_{x_s} \quad \sum_{s \in S} \left(f_s(x_s) + x_s h_s(\lambda) \right) \tag{A11}$$

$$g_s(x_s) \leq 0 \qquad \forall s \in S \tag{A12}$$

The inner problem can be written as a model that is parametric in λ,

$$\psi(\lambda) = \max_{x_s} \quad \sum_{s \in S} \left(f_s(x_s) + x_s h_s(\lambda) \right) \tag{A13}$$

$$g_s(x_s) \leq 0 \qquad \forall s \in S \tag{A14}$$

Replacing the inner problem of (A11) - (A12) with (A13) - (A14) gives the Lagrangean dual as,

$$\min_{\lambda} \quad \psi(\lambda) \tag{A15}$$

For any x_s satisfying the constraints (A2), constraint (A16) must hold true,

$$\psi(\lambda) \geq \sum_{s \in S} f_s(x_s) + \sum_{s \in S} x_s h_s(\lambda) \tag{A16}$$

$\sum_{s \in S} f_s(x_s)$ and $\sum_{s \in S} x_s h_s(\lambda)$ are constant and linear terms (in λ), respectively. Since their sum is a linear support for each x_s, it follows that these yield a convex function. ■

Lemma 1

If $\hat{x}_s$ is the optimal solution of the model represented by the eqs. (A1) - (A4), and x_s^g is the global optimal solution of each scenario of the model represented by eqs. (A1) - (A2) and (A4), then eq. (A17) must hold.

$$\sum_{s \in S} f_s(\hat{x}_s) \leq \sum_{s \in S} f_s(x_s^g) \tag{A17}$$

This follows from the fact that the optimal objective function value of the entire model found by the solution $\hat{x}_s$ has to be smaller or equal to the sum of individual objective function values of global optimal solutions (i.e. wait-and-see solution).

Lemma 2

If x_s^f is any feasible solution to the model represented by the eqs. (A1) - (A4), then eq. (A18) must hold.

$$\sum_{s \in S} x_s^f h_s(\lambda) \geq 0 \tag{A18}$$

The problem (eqs. (A9) - (A10)) is a relaxation of the original model (eqs. (A1) - (A4)). For each feasible solution x_s^f of the original model, eq. (A19) has to hold because the objective function value of the relaxed problem has to be at least as large as the original model.

$$\sum_{s \in S} \left(f_s\left(x_s^f\right) + x_s^f h_s(\lambda) \right) \geq \sum_{s \in S} f_s\left(x_s^f\right) \tag{A19}$$

Canceling the $\sum_{s \in S} f_s\left(x_s^f\right)$ terms on each side of the inequality proves that the term $\sum_{s \in S} x_s^f h_s(\lambda)$ has to be greater than or equal to zero.

Proposition 2

In order to generate a valid upper bound from the Lagrangean dual, each nonconvex subproblem has to be globally optimized (Westerberg and Shah, 1978).

Proof

Let us assume that the upper bound generated using a local solver will be valid. In order to have a valid upper bound, eq. (A20) has to hold. The left hand side of the inequality represents the upper bound generated by the Lagrangean dual problem (subject to constraints (A1) - (A2), (A4)), whereas the right hand side is the objective function of the original model (subject to constraints (A1) - (A4)). As the local solver can get trapped in a local feasible solution during the upper bound generation, in order to prove the validity of the bound, we need to show that eq. (A20) holds for any feasible solution as shown in eq. (A21).

$$\min_{\lambda} \max_{x_s} \sum_{s \in S} \left(f_s(x_s) + x_s h_s(\lambda) \right) \geq \max_{x_s} \sum_{s \in S} f_s(x_s) \tag{A20}$$

$$\min_{\lambda} \sum_{s \in S} \left(f_s\left(x_s^f\right) + x_s^f h_s(\lambda) \right) \geq \max_{x_s} \sum_{s \in S} f_s(x_s) \tag{A21}$$

Eq. (A21) can be rearranged,

$$\sum_{s\in S} f_s\left(x_s^f\right) + \min_{\lambda} \left(\sum_{s\in S} x_s^f h(\lambda)\right) \geq \max_{x_s} \sum_{s\in S} f_s\left(x_s\right) \tag{A22}$$

The right hand side takes its maximum value when x_s equals the optimal solution $\hat{x}_s$ of the original model (eqs. (A1) - (A4)). From Lemma 1, the most extreme case occurs when the optimum solution is actually the wait-and-see solution $\left(\hat{x}_s = x_s^g\right)$, then the right hand side equals to $\sum_{s\in S} f_s\left(x_s^g\right)$. Also from Lemma 2, the term $\sum_{s\in S} x_s^f h(\lambda)$ can take the minimum value zero. In this case (A21) can be rewritten as shown in constraints (A23) and (A24).

$$\sum_{s\in S} f_s\left(x_s^f\right) + \min_{\lambda} \left(\sum_{s\in S} x_s^f h(\lambda)\right) \geq \sum_{s\in S} f_s\left(x_s^g\right) \tag{A23}$$

$$\sum_{s\in S} f_s\left(x_s^f\right) \geq \sum_{s\in S} f_s\left(x_s^g\right) \tag{A24}$$

Eq. (A24) contradicts with the fact that the global optimal solution is at least as large as any feasible solution. Therefore, each subproblem has to be globally optimized to generate a valid upper bound. ■

Appendix B: On the indistinguishability of scenarios

Two scenarios (s,s') which differ only in initial productivities (i.e. element of the set M_1) will be indistinguishable if and only if for each reservoir that distinguishes those scenarios (i.e. element of $D(s,s')$), $w_{1,t}^{r,s}$ and $w_{1,t}^{r,s'}$ are true (eq. (B1)). Eqs. (B2) - (B7) can be interpreted similarly.

$$z_t^{s,s'} \Leftrightarrow \bigwedge_{r\in D(s,s')} \left(w_{1,t}^{r,s}\right)\wedge\left(w_{1,t}^{r,s'}\right) \qquad \forall t,\forall(s,s')\in M_1, s<s' \tag{B1}$$

$$z_t^{s,s'} \Leftrightarrow \bigwedge_{r\in D(s,s')} \left(w_{2,t}^{r,s}\wedge w_{3,t}^{r,s}\right)\wedge\left(w_{2,t}^{r,s'}\wedge w_{3,t}^{r,s'}\right) \qquad \forall t,\forall(s,s')\in M_2, s<s' \tag{B2}$$

$$z_t^{s,s'} \Leftrightarrow \bigwedge_{r\in D(s,s')} \left(w_{4,t}^{r,s}\right)\wedge\left(w_{4,t}^{r,s'}\right) \qquad \forall t,\forall(s,s')\in M_3, s<s' \tag{B3}$$

$$z_t^{s,s'} \Leftrightarrow \bigwedge_{r\in D(s,s')} \left(w_{1,t}^{r,s}\wedge w_{2,t}^{r,s}\wedge w_{3,t}^{r,s}\right)\wedge\left(w_{1,t}^{r,s'}\wedge w_{2,t}^{r,s'}\wedge w_{3,t}^{r,s'}\right)$$
$$\forall t,\forall(s,s')\in M_4, s<s' \tag{B4}$$

$$z_t^{s,s'} \Leftrightarrow \bigwedge_{r\in D(s,s')} \left(w_{1,t}^{r,s}\wedge w_{4,t}^{r,s}\right)\wedge\left(w_{1,t}^{r,s'}\wedge w_{4,t}^{r,s'}\right) \qquad \forall t,\forall(s,s')\in M_5, s<s' \tag{B5}$$

$$z_t^{s,s'} \Leftrightarrow \bigwedge_{r\in D(s,s')} \left(w_{2,t}^{r,s}\wedge w_{3,t}^{r,s}\wedge w_{4,t}^{r,s}\right)\wedge\left(w_{2,t}^{r,s'}\wedge w_{3,t}^{r,s'}\wedge w_{4,t}^{r,s'}\right)$$
$$\forall t,\forall(s,s')\in M_6, s<s' \tag{B6}$$

$$z_t^{s,s'} \Leftrightarrow \bigwedge_{r\in D(s,s')} \left(w_{1,t}^{r,s}\wedge w_{2,t}^{r,s}\wedge w_{3,t}^{r,s}\wedge w_{4,t}^{r,s}\right)\wedge\left(w_{1,t}^{r,s'}\wedge w_{2,t}^{r,s'}\wedge w_{3,t}^{r,s'}\wedge w_{4,t}^{r,s'}\right)$$
$$\forall t,\forall(s,s')\in M_7, s<s' \tag{B7}$$

Eqs. (B1) - (B7) can be rewritten as eqs. (41) - (47) using proposition 3.

Proposition 3

Consider the logic eqs. (B1) - (B7) written in a compact form as follows:

$$z_t^{s,s'} \Leftrightarrow \bigwedge_{r\in D(s,s')} \bar{z}_t^{r,s}\wedge\bar{z}_t^{r,s'} \qquad \forall t,\forall(s,s')\in M_k, s<s' \tag{B8}$$

The right hand side of constraint (B8) can be rewritten by eliminating one of the terms $\bar{z}_t^{r,s}$ or $\bar{z}_t^{r,s'}$. Therefore, the right hand side can be written using only scenario s or s'.

Proof

Given the model with constraints eqs. (4) - (40), (B1) - (B7) and (48) - (53), it is enough to show that $\bar{z}_t^{r,s} \Leftrightarrow \bar{z}_t^{r,s'}$. We will prove only forward implication $\bar{z}_t^{r,s} \Rightarrow \bar{z}_t^{r,s'}$ as the backward implication can be proved similarly. Let us assume $\bar{z}_t^{r,s}$ is true, then the requirements in order to make scenario s indistinguishable from other scenarios s' in the same set M_k is satisfied. If they are indistinguishable at time period t, they need to be indistinguishable for all previous time periods τ, $\tau < t$. Also, at $t = 1$, all decisions for all scenarios should be the same because of the first period non-anticipativity constraints (49) - (53). Because the decisions are the same at $t = 1$, the state variables (such as number of facilities or wells) used for finding indistinguishable scenarios at $t = 2$ will be the same, making the decisions at $t = 2$ for different scenarios identical. It can be shown recursively that the values of state variables for scenarios s and s' at time period t are identical, making $\bar{z}_t^{r,s'}$ also true. ■

Appendix C: Mixed integer constraints for logic constraints (36) - (48)

Eq. (36) can be rewritten as,

$$ntw_t^{r,s} \cdot G^{tw} + nsw_t^{r,s} \cdot G^{sw} \geq N_1 - N_1 \cdot w_{1,t}^{r,s} \qquad \forall t, \forall s, \forall r \qquad \text{(C1)}$$

$$ntw_t^{r,s} \cdot G^{tw} + nsw_t^{r,s} \cdot G^{sw} \leq (N_1 - 1) + M \cdot (1 - w_{1,t}^{r,s}) \qquad \forall t, \forall s, \forall r \qquad \text{(C2)}$$

where, M is an upper bound on the number of wells to drill.

Eq. (37) can be rewritten as,

$$ntw_t^{r,s} \cdot G^{tw} + nsw_t^{r,s} \cdot G^{sw} \geq N_2 - N_2 \cdot w_{2,t}^{r,s} \qquad \forall t, \forall s, \forall r \qquad \text{(C3)}$$

$$ntw_t^{r,s} \cdot G^{tw} + nsw_t^{r,s} \cdot G^{sw} \leq (N_2 - 1) + M \cdot (1 - w_{2,t}^{r,s}) \qquad \forall t, \forall s, \forall r \qquad \text{(C4)}$$

where, M is an upper bound on the number of wells to drill.

Eq. (38) can be rewritten as,

$$\sum_{\tau=1}^{t-1} bp_\tau^{r,s} \geq N_3 - N_3 \cdot w_{3,t}^{r,s} \qquad \forall t, \forall s, \forall r \qquad \text{(C5)}$$

$$\sum_{\tau=1}^{t-1} bp_\tau^{r,s} \leq (N_3 - 1) + M \cdot (1 - w_{3,t}^{r,s}) \qquad \forall t, \forall s, \forall r \qquad \text{(C6)}$$

where, M is an upper bound on the number of periods to produce oil.

Eq. (39) can be rewritten as,

$$\sum_{\tau=1}^{t-1} bp_\tau^{r,s} \geq N_4 - N_4 \cdot w_{4,t}^{r,s} \qquad \forall t, \forall s, \forall r \qquad \text{(C7)}$$

$$\sum_{\tau=1}^{t-1} bp_\tau^{r,s} \leq (N_4 - 1) + M \cdot (1 - w_{4,t}^{r,s}) \qquad \forall t, \forall s, \forall r \qquad \text{(C8)}$$

where, M is an upper bound on the number of periods to produce oil.

Eq. (40) can be converted into mixed-integer form as follows,

$$M \cdot bp_t^{r,s} \geq prod_t^{r,s} \qquad \forall t, \forall s, \forall r \qquad \text{(C9)}$$

$$\varepsilon \cdot bp_t^{r,s} \leq prod_t^{r,s} \qquad \forall t, \forall s, \forall r \qquad \text{(C10)}$$

where, M and ε are upper and lower bounds on the amount of oil production per year respectively, if there is production made during that year (i.e. $bp_t^{r,s} = 1$).

Eq. (41) can be converted into mixed-integer form as follows,

$$\sum_{r \in D(s,s')} \left(1 - w_{1,t}^{r,s}\right) + z_t^{s,s'} \geq 1 \qquad \forall t, \forall (s,s') \in M_1, s < s' \qquad \text{(C11)}$$

$$w_{1,t}^{r,s} \geq z_t^{s,s'} \qquad \forall t, \forall (s,s') \in M_1, s < s', \forall r \in D(s,s') \quad \text{(C12)}$$

Eq. (42) can be converted into mixed-integer form as follows,

$$\sum_{r \in D(s,s')} \left(\left(1 - w_{2,t}^{r,s}\right) + \left(1 - w_{3,t}^{r,s}\right)\right) + z_t^{s,s'} \geq 1 \qquad \forall t, \forall (s,s') \in M_2, s < s' \qquad \text{(C13)}$$

$$w_{2,t}^{r,s} \geq z_t^{s,s'} \qquad \forall t, \forall (s,s') \in M_2, s < s', \forall r \in D(s,s') \quad \text{(C14)}$$

$$w_{3,t}^{r,s} \geq z_t^{s,s'} \qquad \forall t, \forall (s,s') \in M_2, s < s', \forall r \in D(s,s') \quad \text{(C15)}$$

Eq. (43) can be converted into mixed-integer form as follows,

$$\sum_{r \in D(s,s')} \left(1 - w_{4,t}^{r,s}\right) + z_t^{s,s'} \geq 1 \qquad \forall t, \forall (s,s') \in M_3, s < s' \qquad \text{(C16)}$$

$$w_{4,t}^{r,s} \geq z_t^{s,s'} \qquad \forall t, \forall (s,s') \in M_3, s < s', \forall r \in D(s,s') \quad \text{(C17)}$$

Eq. (44) can be converted into mixed-integer form as follows,

$$\sum_{r \in D(s,s')} \left(\left(1 - w_{1,t}^{r,s}\right) + \left(1 - w_{2,t}^{r,s}\right) + \left(1 - w_{3,t}^{r,s}\right)\right) + z_t^{s,s'} \geq 1 \qquad \forall t, \forall (s,s') \in M_4, s < s' \qquad \text{(C18)}$$

$$w_{1,t}^{r,s} \geq z_t^{s,s'} \qquad \forall t, \forall (s,s') \in M_4, s < s', \forall r \in D(s,s') \quad \text{(C19)}$$

$$w_{2,t}^{r,s} \geq z_t^{s,s'} \qquad \forall t, \forall (s,s') \in M_4, s < s', \forall r \in D(s,s') \quad \text{(C20)}$$

$$w_{3,t}^{r,s} \geq z_t^{s,s'} \qquad \forall t, \forall (s,s') \in M_4, s < s', \forall r \in D(s,s') \quad \text{(C21)}$$

Eq. (45) can be converted into mixed-integer form as follows,

$$\sum_{r \in D(s,s')} \left(\left(1 - w_{1,t}^{r,s}\right) + \left(1 - w_{4,t}^{r,s}\right)\right) + z_t^{s,s'} \geq 1 \qquad \forall t, \forall (s,s') \in M_5, s < s' \qquad \text{(C22)}$$

$$w_{1,t}^{r,s} \geq z_t^{s,s'} \qquad \forall t, \forall (s,s') \in M_5, s < s', \forall r \in D(s,s') \quad \text{(C23)}$$

$$w_{4,t}^{r,s} \geq z_t^{s,s'} \quad \forall t, \forall (s,s') \in M_5, s < s', \forall r \in D(s,s') \quad \text{(C24)}$$

Eq. (46) can be converted into mixed-integer form as follows,

$$\sum_{r \in D(s,s')} \left(\left(1 - w_{2,t}^{r,s}\right) + \left(1 - w_{3,t}^{r,s}\right) + \left(1 - w_{4,t}^{r,s}\right) \right) + z_t^{s,s'} \geq 1 \quad \forall t, \forall (s,s') \in M_6, s < s' \quad \text{(C25)}$$

$$w_{2,t}^{r,s} \geq z_t^{s,s'} \quad \forall t, \forall (s,s') \in M_6, s < s', \forall r \in D(s,s') \quad \text{(C26)}$$

$$w_{3,t}^{r,s} \geq z_t^{s,s'} \quad \forall t, \forall (s,s') \in M_6, s < s', \forall r \in D(s,s') \quad \text{(C27)}$$

$$w_{4,t}^{r,s} \geq z_t^{s,s'} \quad \forall t, \forall (s,s') \in M_6, s < s', \forall r \in D(s,s') \quad \text{(C28)}$$

Eq. (47) can be converted into mixed-integer form as follows,

$$\sum_{r \in D(s,s')} \left(\left(1 - w_{1,t}^{r,s}\right) + \left(1 - w_{2,t}^{r,s}\right) + \left(1 - w_{3,t}^{r,s}\right) + \left(1 - w_{4,t}^{r,s}\right) \right) + z_t^{s,s'} \geq 1$$

$$\forall t, \forall (s,s') \in M_7, s < s' \quad \text{(C29)}$$

$$w_{1,t}^{r,s} \geq z_t^{s,s'} \quad \forall t, \forall (s,s') \in M_7, s < s', \forall r \in D(s,s') \quad \text{(C30)}$$

$$w_{2,t}^{r,s} \geq z_t^{s,s'} \quad \forall t, \forall (s,s') \in M_7, s < s', \forall r \in D(s,s') \quad \text{(C31)}$$

$$w_{3,t}^{r,s} \geq z_t^{s,s'} \quad \forall t, \forall (s,s') \in M_7, s < s', \forall r \in D(s,s') \quad \text{(C32)}$$

$$w_{4,t}^{r,s} \geq z_t^{s,s'} \quad \forall t, \forall (s,s') \in M_7, s < s', \forall r \in D(s,s') \quad \text{(C33)}$$

Disjunctive eq. (48) can be converted to algebraic constraints using the Big-M method,

$$bt_t^s \leq bt_t^{s'} + M\left(1 - z_t^{s,s'}\right) \quad \forall t, \forall (s,s'), s < s' \quad \text{(C34)}$$

$$bt_t^s \geq bt_t^{s'} - M\left(1 - z_t^{s,s'}\right) \quad \forall t, \forall (s,s'), s < s' \quad \text{(C35)}$$

$$bsf_t^s \leq bsf_t^{s'} + M\left(1 - z_t^{s,s'}\right) \quad \forall t, \forall (s,s'), s < s' \quad \text{(C36)}$$

$$bsf_t^s \geq bsf_t^{s'} - M\left(1 - z_t^{s,s'}\right) \quad \forall t, \forall (s,s'), s < s' \quad \text{(C37)}$$

$$blf_t^s \leq blf_t^{s'} + M\left(1 - z_t^{s,s'}\right) \quad \forall t, \forall (s,s'), s < s' \quad \text{(C38)}$$

$$blf_t^s \geq blf_t^{s'} - M\left(1 - z_t^{s,s'}\right) \quad \forall t, \forall (s,s'), s < s' \quad \text{(C39)}$$

$$dtw_t^{r,s} \leq dtw_t^{r,s'} + M\left(1 - z_t^{s,s'}\right) \quad \forall t, \forall (s,s'), s < s', \forall r \quad \text{(C40)}$$

$$dtw_t^{r,s} \geq dtw_t^{r,s'} - M\left(1 - z_t^{s,s'}\right) \quad \forall t, \forall (s,s'), s < s', \forall r \quad \text{(C41)}$$

$$dsw_t^{r,s} \leq dsw_t^{r,s'} + M\left(1 - z_t^{s,s'}\right) \qquad \forall t, \forall (s, s'), s < s', \forall r \qquad \text{(C42)}$$

$$dsw_t^{r,s} \geq dsw_t^{r,s'} - M\left(1 - z_t^{s,s'}\right) \qquad \forall t, \forall (s, s'), s < s', \forall r \qquad \text{(C43)}$$

References

1. Aseeri. A., Gorman, P., Bagajewicz, M. J., 2004. Financial risk management in offshore oil infrastructure planning and scheduling. *Industrial Engineering and Chemistry Research* 43, 3063-3072.

2. Ahmed, S., 2000. Strategic planning under uncertainty: Stochastic integer programming approaches. Ph.D. thesis, University of Illinois at Urbana-Champaign.

3. Bailey, W. J., Couët, B., Wilkinson, D., 2005. Field Optimization Tool for Maximizing Asset Value. *SPE Reservoir Evaluation & Engineering* 8 (1), 7-21.

4. Begg, S.H., Bratvold, R., Campbell, J.C., Improving Investment Decisions Using A Stochastic Integrated Asset Model, SPE 71414, New Orleans, 2001.

5. Behrenbruch, P., 1993. Offshore oilfield development planning. *Journal of Petroleum Technology* 45 (8), 735-743.

6. Birge, J.R. , Louveaux, F., 1997. Introduction to stochastic programming. New York, NY: Springer.

7. Bratvold, R.B., Begg, S.H., Campbell, J.M., 2002. Would you know a good decision if you saw one? Paper SPE 77509 presented at the 2002 Annual Technical Conference and Exhibition, San Antonio, TX, Sept. 29 – Oct. 2.

8. Carvalho, M., Pinto, J.M., 2006a. A bilevel decomposition technique for the optimal planning of offshore platforms. *Brazilian Journal of Chemical Engineering* 23, 67-82.

9. Carvalho, M., Pinto, J.M., 2006b. Am MILP model and solution technique for the planning of infrastructure in offshore oilfields. *Journal of Petroleum Science and Engineering* 51, 97-110.

10. Cullick A. S, Heath D., Narayanan K., April J., Kelly J. Optimizing Multiple-Field Scheduling and Production Strategy with Reduced Risk, SPE 84239, Denver, 2003.

11. Dimitriadis, A.D.; Shah, N.; Pantelides, C. C., 1997. RTN-based rolling horizon algorithms for medium term scheduling of multipurpose plants. *Computers and Chemical Engineering* 21, Supplement, S1061-S1066.

12. Dixit, A.K., Pindyck, R.S., 1994. Investment under uncertainty. Princeton University Press.

13. Fisher, M. L., 1985. An Applications Oriented Guide to Lagrangian Relaxation. *Interfaces* 15, 10-21.

14. Geary, A., Bertsekas, D. P., 2002. Incremental Subgradient Methods for Nondifferentiable Optimization. Proceedings 1999 IEEE Conference on Decision and Control Conference.

15. Goel, V., Grossmann, I. E., 2004. A stochastic programming approach to planning of offshore gas field developments under uncertainty in reserves. *Computers and Chemical Engineering* 28 (8), 1409–1429.

16. Goel, V., 2005. Stochastic programming approaches for the optimal development of gas fields under uncertainty. Ph.D. thesis, Carnegie Mellon University.

17. Goel V., Grossmann, I. E., El-Bakry, A. S. Mulkay, E. L., 2006. A novel branch and bound algorithm for optimal development of gas fields under uncertainty in reserves. *Computers and Chemical Engineering* 30, 1076-1092.

18. Goel, V., Grossmann, I.E., 2006. A Class of Stochastic Programs with Decision Dependent Uncertainty. *Mathematical Programming*, Ser. B 108, 355–394.

19. Grossmann, I.E., 2002. Review of nonlinear mixed integer and disjunctive programming techniques. *Optimization and Engin*eering 3, 227-252.

20. Haugland, D., Hallefjord, A., Asheim, H., 1988. Models for petroleum field exploitation. European Journal of Operational Research 37 (1), 58-72.

21. Held, H., D.L., Woodruff, 2005. Heuristics for Multi-Stage Interdiction of Stochastic Networks. *Journal of Heuristics* 11, 483-500.

22. Held, H., D.L., Woodruff, 2005. Heuristics for Multi-Stage Interdiction of Stochastic Networks. *Journal of Heuristics* 11, 483-500.

23. Iyer, R. R., Grossmann I. E., 1998. A bilevel decomposition algorithm for long-range planning of process networks. *Industrial and Engineering Chemistry Research* 37, 474-481.

24. Iyer, R. R., Grossmann I. E., Vasantharajan, S., Cullick, A. S., 1998. Optimal planning and scheduling offshore oilfield infrastructure investment and operations. *Industrial and Engineering Chemistry Research* 37, 1380-1397.

25. Jonsbraten, T.W., 1998. Optimization models for petroleum field exploitation. PhD thesis, Norwegian School of Economics and Business Administration.

26. Jonsbraten, T.W., Wets, R.J.B., Woodruff, D.L., 1998. A class of stochastic programs with decision dependent random elements. *Annals of Operations Research* 82, 83-106.

27. Ligero, E.L., Xavier, A.M., Schiozer, D.J., 2005. Value of information during appraisal and development of petroleum fields. 18th International Congress of Mechanical Engineering, Ouro Preto, MG-Brasil, November 6-11.

28. Meister, B., Clark, J. M. C., Shah, N., 1996. Optimisation of oilfield exploitation under uncertainty. *Computers and Chemical Engineering* 20, S1251-S1256.

29. Nedic A., Bertsekas, D. P., 2001. Incremental Subgradient Methods for Nondifferent Optimization. *SIAM Journal on Optimization*, 12, 109-138.

30. Nikolaos V., Sahinidis, 2000. BARON Branch And Reduce Optimization Navigator, User's Manual, Version 4.0. http://archimedes.cheme.cmu.edu/baron/manuse.pdf.

31. Ozdogan, U., Horne, R. N., 2006. *SPE Reservoir Evaluation & Engineering* 9 (2), 135-145.

32. Pflug, G. Ch., 1990. Online optimization of simulated markovian processes. *Mathematics of Operations Research* Res. 15, No.3, 381-395.

33. Raman, R., Grossmann, I.E., 1994. Modeling and computational techniques for logic-based integer programming. *Computers and Chemical Engineering* 18 (7), 563-578.

34. Ruszczynski, A., 1997. Decomposition methods in stochastic programming. *Matematical Programming* (Ser. B) 79, 333-353.

35. Stensland, G., Tjostheim, D. B., 1991. Optimal decisions with reduction of uncertainty over time – an application to oil production. Stochastic Models and Option Values, Elsevier Science Publishers B.V, pp. 267-291.

36. Tarhan, B., Grossmann, I.E., 2008a. A Multistage Stochastic Programming Approach with Strategies for Uncertainty Reduction in the Synthesis of Process Networks with Uncertain Yields. *Computers and Chemical Engineering*, 32, 766–788.

37. Tarhan, B, Grossmann, I.E., Goel, V., 2008b. Computational strategies and sensitivity studies for planning of oil or gas field infrastructures under uncertainty. Working paper.

38. Tsarbopoulou, C., 2000. Optimization of oil facilities and oil production, M.Sc. Dissertation, University College London, London, UK.

39. Ulstein, N. L., Nygreen, B., Sagli, J. R., 2007. Tactical planning of offshore petroleum production. *European Journal of Operational Research* 176, 550-564.

40. Van den Heever, S. A., Grossmann, I. E., 2000a. An iterative aggregation / disaggregation approach for the solution of a mixed integer nonlinear oilfield infrastructure planning model. *Industrial Engineering and Chemistry Research* 39, 1955-1971.

41. Van den Heever, S. A., Grossmann, I. E., Vasantharajan, S., Edwards, K., 2000b. Integrating complex economic objectives with the design and planning of offshore oilfield infrastructures. *Computers and Chemical Engineering* 24, 1049-1055.

42. Van den Heever, S. A., Grossmann, I. E., 2001. A Lagrangean Decomposition Heuristic for the Design and Planning of Offshore Hydrocarbon Field Infrastructures with Complex Economic Objectives. *Industrial Engineering and Chemistry Research* 40, 2857-2875.

43. Vishwanath, K., Peeta, S., Salman, S., 2004. Investing in the links of a stochastic network to minimize expected shortest path length, Institute for Research in the Behavioral, Economic, and Management Sciences, Paper no: 1167.

44. Westerberg, A.W., Shah, J.V., 1978. Assuring a Global Minimum by the Use of an Upper Bound on the Lower (Dual) Bound, *Computers and Chemical Engineering* 2, 83-92.

45. Williams, H.P., Model Building in Mathematical Programming. John Wiley and Sons, New York, New York, 4th edition, 1999.

46. Zabalza-Mezghani, I., Manceau, E., Feraille, M., Jourdan, A., 2004. Uncertainty management: From geological scenarios to production scheme optimization. *Journal of Petroleum Science and Engineering*, 44:11–25.